Binomial Theorem

2) $(x+3y)^4 = 1x^4 \quad +4x^3 \cdot 3y \quad +6x^2(3y)^2 \quad +4x(3y)^3 \quad +1(3y)^4$

$$= x^4 + 12x^3y + 6x^2 \cdot 9y^2 + 4x \cdot 27y^3 + 81y^4$$

$$= x^4 + 12x^3y + 54x^2y^2 + 108xy^3 + 81y^4$$

$$\binom{n}{r} = \frac{n!}{(n-r)!\,r!}$$

FACTORIAL

$0! = 1$

If $n \neq 0$, $n! = n(n-1)(n-2)\cdots 3\cdot 2\cdot 1$

n is a whole number

$$5! = 5\cdot 4\cdot 3\cdot 2\cdot 1 = 120$$

$$\binom{4}{0} = \frac{4!}{(4-0)!\,0!} = \frac{4!}{4!\,0!} = 1$$

$$\binom{4}{1} = \frac{4!}{(4-1)!\,1!} = \frac{4\cdot\cancel{3\cdot 2\cdot 1}}{\cancel{3\cdot 2\cdot 1}\cdot 1} = 4$$

$$\binom{4}{2} = \frac{4!}{(4-2)!\,2!} = \frac{\cancel{4}\cdot 3\cdot 2\cdot 1}{\cancel{2\cdot 1\cdot 2\cdot 1}} = 6$$

$$\binom{4}{3} = \frac{4!}{(4-3)!\cdot 3!} = \frac{4\cdot\cancel{3\cdot 2\cdot 1}}{1\cdot\cancel{3\cdot 2\cdot 1}} = 4$$

$$\binom{4}{4} = 1$$

Find the 3rd term of $(6x+7y)^4 = \underset{\binom{4}{0}}{1} \underset{\binom{4}{1}}{+4} \underset{\binom{4}{2}}{+6(6x)^2(7y)^2} \underset{\binom{4}{3}}{+4} \underset{\binom{4}{4}}{+1}$

3-1

$$\binom{4}{2}(6x)^{4-2}(7y)^2 = 6(6x)^2(7y)^2$$

$$= 6\cdot 36x^2\cdot 49y^2$$

$$= 10584x^2y^2$$

one less than the number of the term

3-1

3rd term

On calculator $\binom{n}{r} = {}_nC_r$

ex: $\binom{7}{3}$ 7 MATH PRB nCr 3 enter

35

j^{th} term of $(a+b)^n$

$$\binom{n}{j-1}a^{n-(j-1)}b^{j-1}$$

6^{th} term of $(x+3y)^8$ $n=8$

$j=6$

$$\binom{8}{5}x^{8-5}(3y)^5 = 56x^3\cdot 243y^5 = 13{,}608x^3y^5$$

3^{rd} term of $(2x-5y)^{11}$

$2x+(-5y)$

$$\binom{11}{2}(2x)^{9}(-5y)^{2} = 55\cdot 512x^9\cdot 25y^2 = 704{,}000x^9y^2$$

Assignment

1) 3rd term in $(3x-2)^9$
2) 10th term in $(x+y)^{15}$
3) 8th term in $(5x-6y)^7$
4) 2nd term in $(6x^2+5y^2)^{10}$
5) 4th term in $(x^3-3y^2)^6$