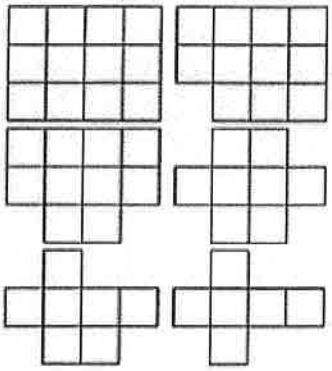
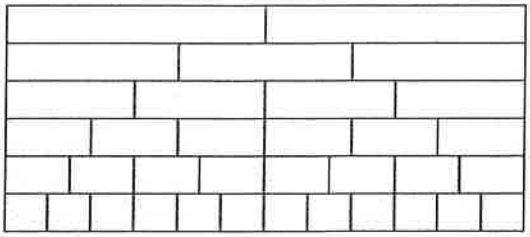
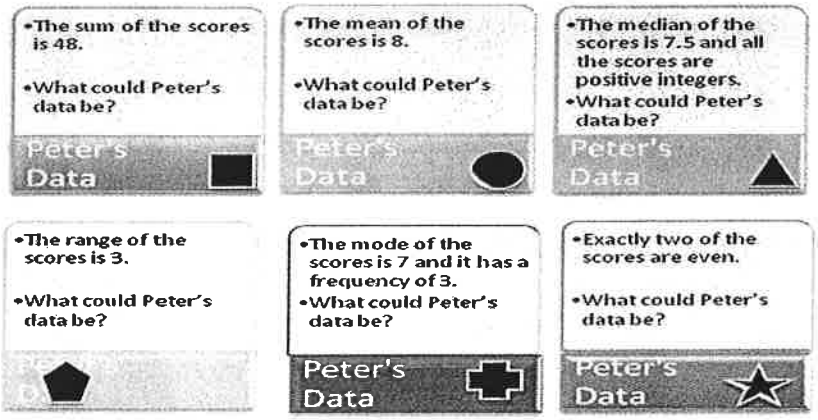
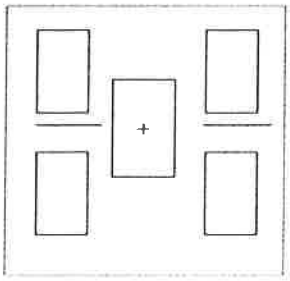
Types of tasks

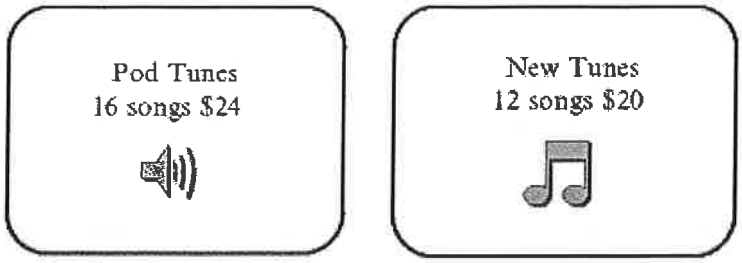
Representational Tasks	Contextualized Tasks	Content Specific Open-Ended Tasks
• Use models and representations to develop mathematical understanding • Often game-based • Includes sorting and classifying activities	• A mathematical problem embedded in a real situation • Requires application of the mathematics within a contextualized, practical problem	• Multiple possible answers • Choice in strategy and solution type • Prompt insights into specific mathematics concepts through seeing and discussing a range of possible answers
Find the perimeter and area of the following shape.  Explain the relationship between the perimeter and the area of these shapes. *Source: Sullivan, Clarke & Clarke, 2013, ch.9*	Ms. Brown's class will raise rabbits for their spring science fair. They have 24 feet of fencing with which to build a rectangular rabbit pen in which to keep the rabbits. a.) If Ms. Brown's students want their rabbits to have as much room as possible, how long would each of the sides of the pen be? b.) How would go about determining the pen with the most room for any amount of fencing? Organize your work so that someone else who reads it will understand it. *Source: Stein, Smith, Henningsen, & Silver, 2000, pg. 2*	The perimeter of rectangle is 20 cm. What might be the area of the rectangle? Show and explain how you know. *Source: Sullivan, Clarke & Clarke, 2013, ch.6*

Representational Tasks

Examples	• Use models and representations to develop mathematical understanding • Often game based • Sorting and classifying activities
Color in Fractions	**Materials Needed:** • Numerator die labeled 1, 2, 2, 3, 3, and 4 • Denominator die labeled 2, 3, 4, 6, 8, and 12 • Fraction Wall  Roll the two dice and create a fraction. Color in the section of the wall that matches the fraction you rolled, or an equivalent faction to the fraction rolled on the dice. For example, if you roll 2 and 4, you can color any combination of $\frac{2}{4}$ (e.g., $\frac{2}{4}$ of one line, or $\frac{4}{8}$ of one line or $\frac{1}{4}$ of one line and $\frac{2}{8}$ of another). If a player is unable to use his/her turn, he/she passes. The first player who colors in the whole wall is the winner.
Clues on a Card	Use the clue cards to reconstruct Peter's data set. Write a story to match Peter's data. 
Fractions Close to 1	Place number cards (choosing from 1, 3, 4, 5, 6, and 7) in the boxes to make fractions so that when you add them, the answer is as close to one as possible, but not equal to one. Each card can be used only once. Explain your strategy and how you know the sum is as close to one as possible. 

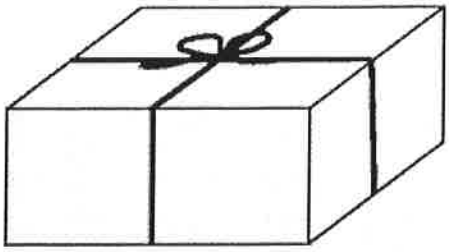
Referenced in Sullivan, P, Clarke, D., Clarke, B., (2013.) *Teaching with Tasks for Effective Mathematics Learning.* New York: Springer

Contextualized Tasks

Examples	• A mathematical problem embedded in a real situation • Requires application of the mathematics within a contextualized, practical problem
Music Cards	Which card is a better value? Explain your choice. 
Making Cookies	A recipe for chocolate chip cookies makes 4 dozen cookies and calls for the following ingredients: • $1\frac{1}{2}$ c margarine • $1\frac{3}{4}$ c sugar • 2 t vanilla • $3\frac{1}{4}$ c flour • 1 t baking powers • $\frac{1}{4}$ t salt • 8 oz. chocolate chips a.) How much of each ingredient is needed to make 3 recipes? Show how you know. b.) How much of each ingredient is needed to make ¾ of a recipe? Show how you know.
Block of Land	Suppose you receive the e-mail below from a friend. What would your answer be? Explain. Hi Doug, Can I call on your math expertise? If, on paper a block of land is 2 cm x 5.8 cm, and the overall dimensions are 4,768 square meters, how do I work out the actual length and width of the block of land? Hope you can help. Thanks, Peter

Referenced in Sullivan, P, Clarke, D., Clarke, B., (2013.) *Teaching with Tasks for Effective Mathematics Learning.* New York: Springer

Content Specific Open-Ended Tasks

Examples	• Multiple possible answers • Choice in strategy and solution type • Prompt insights into specific mathematics concepts through seeing and discussing range of possible answers
Wrap the Present	You need to wrap present in a box. You have 1 meter of ribbon. The bow at the top will use 30 cm of this. What might be the dimensions of the box? Explain how you know. 
Looking for 3 More	Four people in this room have an average height of 148 cm. You are one of those. Find the other three. Explain your approach.
Volume of a Box	A rectangular sheet is 20 cm long and 16 cm wide. You cut squares out of each of the corners. You then fold up the sides to make an open-top box. Calculate the volume of some boxes that can be made from that card.
Missing Numbers	What might be the missing numbers, __ __ x 8__ = ___ ___ 0 Justify your solution.

Referenced in Sullivan, P, Clarke, D., Clarke, B., (2013.) *Teaching with Tasks for Effective Mathematics Learning.* New York: Springer

Analyzing Levels of Task Cognitive Demand

Low-Level Cognitive Demand
Memorization Tasks • Involve either producing previously learned facts, rules, formulae, or definitions, or memorizing • Are routine, involving exact reproduction of previously learned procedure • Have no connection to related concepts
Procedures without Connections Tasks • Specifically call for the use of procedure • Are straightforward with little ambiguity about what needs to be done and how to do it • Have no connection to related concepts • Focus on producing correct answers, rather than on developing mathematical understanding • Require no explanations or explanations focus solely on describing the procedure
High-Level Cognitive Demand
Procedures with Connections Tasks • Focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas • Suggest general procedures that have close connections to underlying conceptual ideas • Are usually represented in multiple ways (e.g., visuals, manipulatives, symbols, problem situations) • Require students engage in conceptual ideas that underlie the procedures in order to successfully complete the task
Doing Mathematics Tasks • Require complex and non-algorithmic thinking (i.e., non-routine-there is not a predictable, known approach) • Require students to explore and to understand the nature of mathematical concepts, processes, or relationships • Demand self-monitoring or self-regulation of cognitive processes • Require students to access relevant knowledge in working through the task • Require students to analyze the task and actively example task constraints that may limit possible strategies and solutions • Require considerable cognitive effort

Referenced from Bay-Williams, J.M. McGatha, M., Kobbet, B., & Wray, J. (2014). *Mathematics Coaching: Resources and Tools for Coaches and Leaders, K-12.* Boston: Pearson

Strategies for Transforming Tasks from low-level cognitive demand to high-level

Low-Level Cognitive Demand	High-Level Cognitive Demand
If I have two pennies, a nickel, two dimes, and a quarter, how money do I have? *Source: Hull, Miles, & Balka, 2014, pg. 23*	I have 5 coins in my pocket. The coins may only be pennies, nickels, dimes, or quarters. If I reach into my pocket and pull out three coins, how much money might I have in my hand? *Source: Hull, Miles, & Balka, 2014, pg. 23*
Martha was re-carpeting her bedroom which was 15 feet long and 10 feet wide. How many square feet of carpeting will she need to purchase? *Source: Stein, Smith, Henningsen, & Silver, 2000, pg. 1*	Ms. Brown's class will raise rabbits for their spring science fair. They have 24 feet of fencing with which to build a rectangular rabbit pen in which to keep the rabbits. a.) If Ms. Brown's students want their rabbits to have as much room as possible, how long would each of the sides of the pen be? b.) How long would each of the sides of the pen be if they had only 16 feet of fencing? c.) How would go about determining the pen with the most room for any amount of fencing? Organize your work so that someone else who reads it will understand it. *Source: Stein, Smith, Henningsen, & Silver, 2000, pg. 2*

Working Backward Strategy

Process	Example
Identify a topic	The topic for tomorrow is averages.
Think of a low-level question and write down the answer	A low level question might be: The children in the Smith family are aged 3, 8, 9, 10, and 15. What is their average age?
Make up a question that includes or addresses the answer to the original question.	A high-level question could be: There are five children in a family. Their average age is 9. How old might the children be?

Adapting a Standard Questions

Process	Example
Identify a topic	The topic for my lesson is subtraction.
Think of a low-level question and write down the answer	A low level question might be: Find the difference 731 – 256.
Adapt it to make it a high-level question.	A high-level question could be: Arrange the digits in the problem below so the difference is between 100 and 200. 731 – 256 =

Adapted from Sullivan, P, Clarke, D., Clarke, B., (2013.) *Teaching with Tasks for Effective Mathematics Learning.* New York: Springer