Common Core Math in Kindergarten

The main focus in Kindergarten is very basic number sense. Of course they'll work on counting as part of this. One aspect that will be new for some classrooms is counting starting from numbers other than one. This helps with addition and subtraction later.

Kindergarteners will compare groups of things to decide which is bigger. They will combine groups together or take some away from a group. Eventually they'll use written numbers to describe what's going on.

Kindergarteners will usually have "rug time" discussion of math as well as play games.
A change (for some) is that all of this investigation is carefully directed to develop skills important for later grades.

One of the most important skills in math that students begin in Kindergarten is putting things together and taking them apart in various ways. They'll think about different ways that a number can be made from two other numbers as they begin to think about addition and subtraction and the geometry Kindergarteners learn reinforces this idea of putting together and taking apart, too. For example, students may be asked to make two triangles from a square or to put together shapes to form a new one.

Examples:

The ideas in "My Book of Five" (see reverse) help kids understand what it means to add and subtract. An important application of this idea comes in representing the "teen" numbers as ten and some more ones (So that 13 means 10 and 3 more ones) because it is the foundation for regrouping (what most of learned as ``borrowing and carrying"). Recognizing the various combinations of numbers that "make up" the numbers from 1 to 10 is a critical building block in learning multi-digit arithmetic.

Tips for Parents:

Even though you may not have been taught math in this way, you can still help your child.

- If you count with them, work on starting from any given number.
- Play games that encourage breaking apart numbers in different ways.
- For teen numbers, you may even count in the unit-form way that emphasizes the ten. (e.g. 8, 9, ten, ten-one, ten-two, ten three, ...) as well as with standard names.

Provided by Dev Sinha & Tricia Bevans of the
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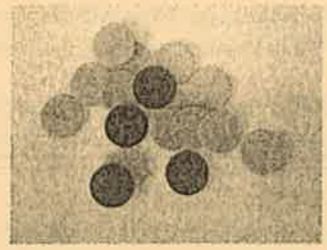
My book of five
https://www.illustrativemathematics.org/illustrations/1408



MATERIALS

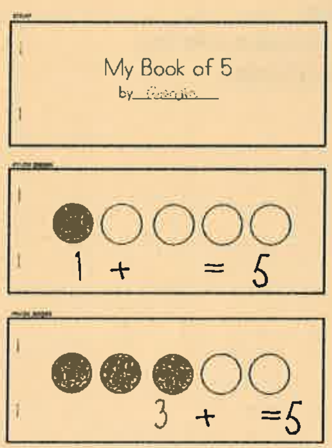
- Double sided counters
- Markers that are the same colors as the counters
- Teacher-made "My Book of 5" (see below for detailed directions)

ACTION

Students will be given double sided counters/dots (see picture of counters, above). It is important for the markers to match the colors on the counters.

Students take five counters in their cupped hands (or a cup), shake them around, pour them onto the desk. Next, they count how many counters are yellow and how many are red. Students then record the numbers in their book and write a corresponding equation. For example, if the counters landed so that 1 was yellow and 4 were red, then the student would draw one yellow dot and four red dots and then write "1+4=5" under the drawing. The student would then collect the counters and roll them again. For each combination of colors, the students record with a picture and an equation. Students continue until they fill their book of 5. The teacher can choose how many pages to put in, somewhere between five and eight is a good number so that students get a chance to see multiple combinations.

After the students have completed their books, the teacher should have a whole-group discussion to make the number relationships explicit. One way to do this is to write each of the two addends into a table and to discuss possible patterns and reasons for the pattern. The teacher can ask specific questions such as, "What do you notice about the numbers in the table?" Or "Why is it that as one number gets bigger, the other number gets smaller?"



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Common Core Math in 1st Grade

The main ideas in first grade are addition and subtraction up to twenty, and starting to make larger numbers out of tens and ones. In the Common Core, kids will not only learn their number facts, but see them as related. This will help them not only learn these facts, but to build number sense.

For example, a child might learn their "doubles"–like $8 + 8 = 16$ –and from there know close facts such as $8 + 7 = 15$ because it must be one less than $8 + 8$. Another child might prefer to see $8 + 7$ as $8 + 2 + 5$, and then see that as $10 + 5$ to get 15. This last approach of "making a ten" is key. Finding it this way will help kids remember it and will also be important for knowing the rules of arithmetic and eventually algebra.

Kids will be working in concrete ways with tens and ones –often with blocks, definitely with pictures—so that they know what it means make a ten or break one up. This process is called "regrouping" (we have called it than carrying or borrowing in the past, but are we really `borrowing' if we never get it back?) to emphasize that the value of the number hasn't changed.
Eventually kids will be proficient with pencil-and-paper and even mental math, but using pictures or objects gives them a firm foundation for what they're doing.

Another small but important change is that kids won't just see problems like $3 + 2 = 5$ but also $5 = 3 + 2$ and even $3 + 2 = 1 + 4$. Well-established research suggests the importance of activities like this to lay a proper understanding of the equal sign.

Examples:

The game "Kiri's Mathematics Matching Game" (see reverse) is like Memory, though one can even start with all cards face up as one is learning the game. The idea is to look for two numbers which add **or** subtract to give a target. So if the target is 6 and you turn over a 4 first you can look for 2 next, because $2 + 4 = 6$, or 10 because $10 - 4 = 6$. To figure out what you need to turn over, you can use the relationship between addition and subtraction, which is what the game is really about.

And far from the Common Core being "one size fits all", this shows that even kids (in this case a 4J kid) can help create Common Core materials!

Tips for Parents:

Here are some ideas for reinforcing the math at home.

- Talking about arithmetic out loud as it comes up in daily life is wonderful.
  "There are six of us at dinner and two cups already out; how many more cups do we need."
  If you use cash, talking through money is terrific.
- There are many good games that promote good number sense, without your kids even noticing. For example, if you play the card game "War" but use two cards instead—so my $5 + 3 = 8$ beats your $2 + 5 = 7$ –even better than just doing the addition in this case is reasoning that $5+3$ wins because both have 5's but the three is greater than the 2.
- If you want to give kids skill practice, it is better to have activities which encourage reflection. A website or worksheet which has kids do a "plus two" right next to a "plus three" will encourage them to make connections that reinforce recall.

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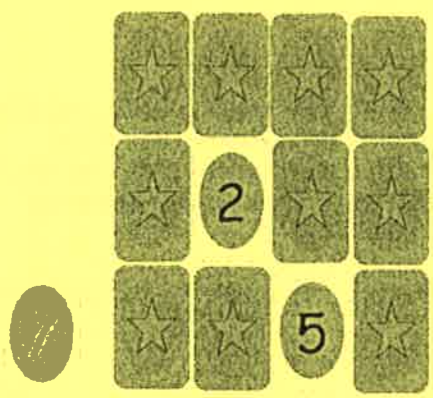
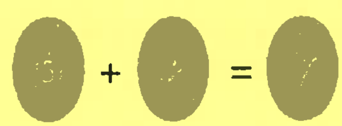
Kiri's Mathematics Matching Game

https://www.illustrativemathematics.org/illustrations/991

- Students can play in groups of 2-4.
- An array of cards (twelve to twenty in total) is placed face down and one card, called the target card, is put face up.
- The students take turns flipping over two cards, one at a time.
- If the sum or difference of the values on the two cards equals the value on the target card, the student who exposed those cards should say a number sentence to express the relationship. If they are correct, the three cards are removed and replaced so there is again a full array.
- If a student does not combine the values of flipped cards to make the value on the target card, then it is the next student's turn.
- In the no-memory-needed version of the game, all chosen cards are left face up (after an unsuccessful turn) and may be used to make matches. In the light-memory version, cards are left face up until there is a match, after which all are put face down. In the memory version, cards are put face down after an unsuccessful turn before the next player's turn.





In all versions, students must engage basic addition and subtraction facts. In the memory version, after a student has turned over one card, in order to know whether there is a match using cards they've seen, they need to to solve equations of the form
$\square+b=c$, $b+\square=c$, $\square-b=c$ and $b-\square=c$.

Students could also be asked to record the number sentences they make.
Teachers could make cards, or have students make them, or use numbered cards from a standard deck or by taking cards from other games. Zeros would be appropriate, and "wilds" could also naturally be incorporated. The target card values should be up to *20* to fully meet the standard (with target cards kept separately).

To extend, and incorporate 1.OA.7 into this activity, there could be two target cards to match in total or difference and/or students could flip over three cards and possibly use all of them.
Note: This game was invented by Kiri, a first grader (now a fourth grader) at Edison Elementary.

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Common Core Math in 2nd Grade

Second graders will continue their work understanding the way our number system works using place values of ones, tens, hundreds, etc. They'll recognize that the 3 in the number 357 represents 3 hundreds rather than "just being a three" and that 12 tens is the same as 1 hundred and 2 tens. Later this will make it clear that adding two hundred to 357 is just a matter of adding 2 to the 3 in the hundreds place.

Kids will work on skip counting by various numbers including tens and hundreds both to increase skill for addition and subtraction using these place values but also as a foundation for multiplication.

While second graders will continue to use many different strategies for adding and subtracting, they use their understanding of the way numbers are built to move toward methods that will always work quickly and accurately.

Geometric concepts they're studying at the same time reinforce the number sense they're working on, provide real world contexts, and give a good foundation for understanding more advanced concepts. For instance, you'll notice that students work with measuring lengths. They might add two different lengths together or compare the lengths of two objects (which would require subtraction). Using bar graphs, clocks, or money they might practice these same skills. In second grade they also do things like partition rectangles into squares and other equal shapes in preparation for understanding both multiplication and fractions.

Examples:

Bundling and Unbundling https://www.illustrativemathematics.org/illustrations/144 (see reverse)

The first part of this task is straightforward, but in part b) of this task, the kids have to think a bit more. They're breaking apart the number 14 tens into 10 tens and 4 tens. Then, they recognize that the group of 10 tens can be "bundled" into a group of 1 hundred. This is just what they will need to understand in order to add something like 152 + 91 using the standard algorithm where we line up the ones and the tens and the hundreds and add in columns. Adding 2 and 1 in the ones place is straightforward, but when they add the 5 and 9 in the tens place, the 14 they get will have to be regrouped (or "carried").

Tips for Parents:

- Practice in everyday situations. For example, ask your child to compare the price of two different items and decide how much you would save. Count by 2's, 3's, 4's, etc. to figure out how many there are of something rather than counting one at a time.

- You may find that there are methods of writing basic arithmetic that are unfamiliar to you. Often, these are just ways of recording more of the thinking that goes into the math. Try to understand the process yourself, checking in with the teacher if need be. If you do want to share the way you learned make sure you can also explain the thinking around it as well as how it relates to the ways things are being done in class.

- Have your child explain how she found an answer using words or pictures, sometimes even if the process is easy for her.

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Bundling and Unbundling.
https://www.illustrativemathematics.org/illustrations/144

Make true equations. Write one number in every space. Draw a picture if it helps.

a. 1 hundred + 4 tens = ______ ;   4 tens + 1 hundred = ______
b. 14 tens = 10 tens + _____ tens; 14 tens = _____ hundred + 4 tens; 14 tens = _____ ones
c. 7 ones + 5 hundreds = ______
d. 8 hundreds = ______
e. 106 = 1 hundred + _____tens + _____ones;   106 = _____tens + _____ones ;
   106 = _____ones
f. 90 + 300 + 4 = ______

Commentary

Students determine the number of hundreds, tens and ones that are necessary to write equations when some digits are provided. Student must, in some cases, decompose hundreds to tens and tens to ones. The order of the summands does not always correspond to the place value, making these problems less routine than they might

Solutions

a. 140, 140.The first problem asks for the same number (140) in different ways. This emphasizes that order doesn't matter in addition – yet order is everything when using place-value notation.

b. 14 tens = 10 tens + 4 tens14 tens = 1 hundred+4 tens14 tens = 140.In this problem, the base-ten units in 140 are bundled in different ways. In the first line, "tens" are thought of as units: 14 things = 10 things + 4 things.

c. 507.By scrambling the usual order, the third problem requires students to link the values of the parts with the order of the digits in the positional system. Also, to encode the quantity, the student will have to think: "no tens," emphasizing the role of 0.7 ones + 5 hundreds = 507

d. 800.In the fourth problem, the zeros come with a silent "no tens and no ones": 8 hundreds = 800

e. 106 = 1 hundred + 0 tens + 6 ones106 = 10 tens + 6 ones106 = 106 onesIn this problem, the base-ten units in 106 are bundled in different ways. This is helpful when learning how to subtract in a problem like 106 – 34 by thinking about 106 as 100 tens and 6 ones.

f. 394. The sixth problem is meant to illustrate the notion that If the order is always given "correctly," then all we do is teach students rote strategies without thinking about the size of the units or how to encode them in positional notation.90 + 300 + 4 = 394

Common Core Math in 3rd Grade

Back in the good old days, third grade math was all about multiplication. In the Common Core, that's what it is still about!

A key change is that now we want students to apply their multiplication skills to more story problems, as well as connect the multiplication facts to one another. For example, if a child knows their "times fours", that can be used to help recall or figure out their "times eights": Since $3 \times 4 = 12$, then 3×8 must be twice that or 24. Some, but not all kids, have used these kinds of strategies in the past. Now they will be used widely, and they will all be discussed so the kids who notice these kinds of things learn to not just see it but describe what and why.

Kids will see pictures explaining these connections (see example below). Students will also do multiplication and division together more, rather than seeing them separately. So, for example, soon after students learn that $4 \times 6 = 24$ they'll learn it also means that $24 \div 4 = 6$ and $24 \div 6 = 4$. Kids will also be mastering addition and subtraction in the hundreds. This will mean not only learning the standard way, but figuring out short cuts and alternate approaches and talking about why they work. For many reasons we'd like to see kids see an addition such as $398 + 15$ and not have to "line it up" to add but instead say, "well, if we give two of the 15 to the 398 that makes 400 so the answer is 413" or "if we look on the number line, only two steps are needed to get to 400, and then 13 steps more would be 413.

Examples:

Eureka Math: Demonstrating the Commutativity of Multiplication (see reverse)
https://www.engageny.org/resource/grade-3-mathematics-module-1

Here we see third graders using pictures of neatly organized objects called rectangular arrays (or just arrays). In the Common Core, second graders will begin to use arrays so they will already be familiar. In this worksheet, students use these arrays to see why we get the same amount when we calculate $2 x 6$, (that is, two sixes) and 6×2 (that is, six twos). Later they fill in $2 \times 9 = 9 \times$ _____. Here, instead of having two problems to evaluate and get the answer of eighteen, students see these as directly related. This is emphasizing how arithmetic follows rules which eventually become the rules of algebra.

Tips for Parents:

If you practice multiplication facts, try to highlight related facts especially when your child cannot recall one. For example, if they don't remember 6×6 right away, you can ask "do you remember 5×6?" If they do, then remind them (if needed) that 6×6 is just six more.

Be patient with the rectangular arrays and other unfamiliar approaches. No method is perfect, but for many students and teachers their use has already proven to be more effective than what we are doing in the past.

It should be fine to show your child the standard "line them up" ways to add and subtract (and they will see them in class too!) but realize that they may need to provide an alternate approach, especially when the standard way isn't as efficient as some meaningful shortcut.

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Eureka Math Module 1 Lesson 7 (excerpt)

1. a. Count by 2 six times.

   _____, _____, _____, _____, _____, _____

   b. Draw an array that matches your count-by.

   c. Write a multiplication sentence that represents the total number of objects in your array.

   _____ × _____ = _____

2. a. Count by 6 two times.

   _____, _____

   b. Draw an array that matches your count-by.

   c. Write a multiplication sentence that represents the total number of objects in your array.

   _____ × _____ = _____

3. a. Compare your work in Problems 1 and 2. Turn your paper as you study the arrays to look at them in different ways.

   b. Why are the factors in your multiplication sentences in a different order?

Write and solve a different multiplication sentence to describe each array.

____

____

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Common Core Math in 4th Grade

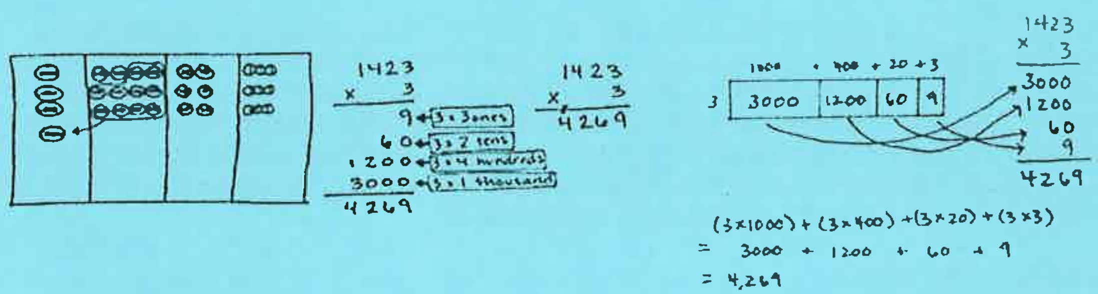
The two most important areas of focus for this grade are skill with multiplication and division and building understanding of fractions.

Fourth Graders will develop understanding of and fluency with multi-digit multiplication and division. Eventually, they should be comfortable with methods for multiplication and division that work quickly and accurately. This includes the usual procedures, as well as some which could be faster in some cases or more understandable to the students. For example, 35 x 12 = 35 x 2 x 6 = 70 x 6 = 420. This not only helps when calculator or pencil-and-paper are not available, but helps to prepare for algebra. To be sure these processes work, and to better prepare for algebra, the students will use pictures and other methods to explain why they work.

Fractions is another key element of 4th grade math. To understand why fractions have many names for the same number—for example $\frac{1}{2}$ is the same as $\frac{2}{4}$ is the same as $\frac{3}{6}$ and so on—students will use pictures, instead of "canceling", which doesn't really have any meaning for kids at that point, or using fraction multiplication, which would be using an advanced topic for more basic understanding.

Examples:

From Eureka Math: Grade 4 Module 3 Topic C Overview
https://www.engageny.org/resource/grade-4-mathematics-module-3-topic-c-overview



Each of these descriptions of how to calculate 1423 times 3 is useful in different ways.
The first uses place value (the meaning of ones, tens, hundreds and thousands) and connects multiplication to addition. The middle two descriptions are expanded and condensed versions of the standard algorithm. The last uses area to represent the multiplication and connects the other descriptions with ideas needed in algebra. Students will learn to see the connections between these methods both to check their work and to reinforce why each process works.

Tips for Parents:

- Communicate with your child's teacher if you are regularly unable to help your child with unfamiliar multiplication or division methods.

- Do math in everyday settings. Encourage your child to recognize fraction equivalence in activities like cooking, for example "I can put in one cup and a half cup of milk or three half-cups of milk". There are lots of multiplication and division examples, for example estimating how many candies they'll get from trick-or-treating.

- Especially if your child catches on to procedures quickly, make sure he can explain why something makes sense.

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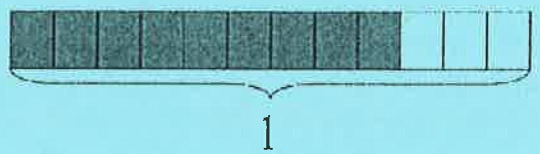
Explaining Fraction Equivalence with Pictures

https://www.illustrativemathematics.org/illustrations/743

a. The rectangle below has length 1. What fraction does the shaded part represent?

b. The rectangle below has the same length as the rectangle above. What fraction does the shaded part represent?

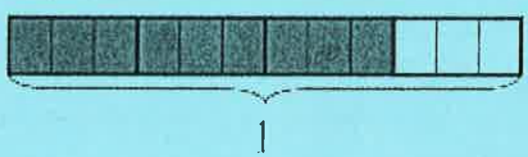


c. Use the pictures to explain why the two fractions represented above are equivalent.

Commentary: The purpose of this task is to provide students with an opportunity to explain fraction equivalence through visual models in a particular example. Part c) should be approached as a discussion before students are asked to write an explanation. Students can talk generally about the relationship between the pictures ("Each of the larger pieces is broken up into 3 little pieces"), which can then be refined and connected to the appropriate operations ("There are three times as many smaller pieces as bigger pieces"). Students will need more opportunities to think about fraction equivalence with different examples and models, but this task represents a good first step.

Solutions: a) ¾   b) 9/12

c. Three pieces in the bottom rectangle have the same size as 1 piece in the top rectangle. We can even show this by darkening the lines around groups of three small pieces in the rectangle that represents $\frac{9}{12}$:



When we make groups of three in the bottom rectangle, there are 3 groups of 3 shaded pieces and 4 groups of 3 in the whole rectangle. Using these groups, we see that

$$\frac{9}{12} = \frac{(3 \times 3)}{(4 \times 3)} = \frac{3}{4}$$

of the bottom rectangle is shaded. Since the shaded portion is the same in each case but we just look at it in a different way and describe it with a different fraction, the fractions are equal. So

$$\frac{9}{12} = \frac{3}{4}$$

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Common Core Math in 5th Grade

Fifth grade in the Common Core is when students finish having arithmetic as a focus, though in later grades there will be plenty of opportunity to continue practicing these skill -for example, dividing numbers when computing proportions.

This year students will learn to add fractions. This is a complicated process, and some curricula even suggest using elaborate gimmicks to remember it. In the Common Core, students will have a firm grounding in the number line, in renaming fractions (e.g. $\frac{2}{3}$ is also $\frac{4}{6}$) and in adding fractions with the same denominator ($\frac{3}{8}+\frac{2}{8}=\frac{5}{8}$). All of this will make addition of fractions a process that makes sense rather than something to remember using tricks which use pictures of X's or butterfly wings. This type of reasoning also helps to *apply* fraction arithmetic correctly. Many of us remember that to multiply $\frac{2}{3}\times\frac{4}{5}$ you "multiply across," but struggle to know if one *should* multiply in a real world context. A key is that $\frac{2}{3}\times\frac{4}{5}$ is what you get when you split $\frac{4}{5}$ of something into three equal pieces and take two of those. Students will use pictures to reason about problems, as many good problem-solvers often do. From these they will be able to know whether to multiply or divide, and have a sense for what a reasonable answer should be.

Students will use similar reasoning about whole numbers and decimals—using sketches, examples, and properties which have been carefully developed, so these arithmetic skills will provide a strong base for algebra.

Examples:

Video Game Scores (see reverse) https://www.illustrativemathematics.org/illustrations/590

In this task, students connect a "real-life" situation to arithmetic with many steps. The students don't have to evaluate the scores, though a teacher could ask them to if necessary. The more important part of the activity is to have students work on their mathematical language skills to interpret expressions in the context of the problem. This gives some great practice leading up to using variables as in algebra. One can just change the task a bit—an unknown amount of bonus points, for example—and it is a good algebra activity.

Tips for Parents:

- It is likely that your child is learning in a way you didn't so you can't just figure out in a minute what's going on. That presents a great opportunity: ask your child to explain some math to you! Communicating reasoning is a skill we want children to have, and it rarely happens enough.

- Kids at this point will likely have a strong sense of how "good" they are at math, usually based on how quickly they can calculate. Challenge this! Many of the best mathematicians are slow at calculation, but take time to truly understand a problem. Understanding will eventually be a struggle for everyone in some math class. Just as a musician doesn't expect to play every new piece well, a math learner won't understand every concept right away but can progress until they get there.

- High achievers may be ready to use variables to more deeply reflect on the arithmetic they learn. If they see exactly why three fourths and two fourths makes five fourths (on the number line, especially), and similarly nine fourths and two fourths makes eleven fourths and so on, then they could also say that n fourths and two fourths makes $n+2$ fourths. In symbols, that's $\frac{n}{4}+\frac{2}{4}=\frac{n+2}{4}$. This deeper reflection on fraction arithmetic is much more beneficial than rushing though the rules of arithmetic in an accelerated track.

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Video Game Scores https://www.illustrativemathematics.org/illustrations/590

Eric is playing a video game. At a certain point in the game, he has 31500 points. Then the following events happen, in order: He earns 2450 additional points. He loses 3310 points. The game ends, and his score doubles.

> Write an expression for the number of points Eric has at the end of the game. Do not evaluate the expression. The expression should keep track of what happens in each step listed above.
>
> Eric's sister Leila plays the same game. When she is finished playing, her score is given by the expression $3(24500+3610)-6780$. Describe a sequence of events that might have led to Leila earning this score.

Commentary: Standard 5.OA.2 asks students to "Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them." This task asks students to exercise both of these complementary skills, writing an expression in part (a) and interpreting a given expression in (b). The numbers given in the problem are deliberately large and "ugly" to discourage students from calculating Eric's and Leila's scores. The focus of this problem is not on numerical answers, but instead on building and interpreting expressions that could be entered in a calculator or communicated to another student.

Solution:

a. When Eric earns 2450 additional points, his score becomes 31500 + 2450.When he loses 3310 points, his score becomes (31500 + 2450) - 3310. (Note that this can also be written without the parentheses.)When Eric's score doubles, the score becomes 2 x ((31500 + 2450) - 3310), which can also be written 2(31500 + 2450 - 3310).

b. Here is a possible sequence of events that might lead to the score given: At a certain point in the game, Leila has 24500 points. She earns 3610 additional points. Her score triples. She loses 6780 points.

c. Note that the order of the steps is important; rearranging the steps will likely lead to a different expression and a different final score.

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