Set B #9
(2005B #4c)

Question 4

Solution

Part (a):

Step 1: Identify appropriate confidence interval by name or by formula.

One sample confidence interval for a mean (of the differences)

$$OR \quad \bar{x}_d \pm t^*_{n-1}\frac{s_d}{\sqrt{n}}$$

Step 2: Check appropriate conditions.

Assume the population of differences in growth is normally distributed. The information provided in the stem of the problem suggests that this condition is met. Because the 24 seeds were randomly chosen and randomly assigned to the containers, the differences are independent.

Step 3: Correct mechanics.

The 95% confidence interval for the mean difference in growth is

$$-2.015 \pm 2.201\frac{1.163}{\sqrt{12}} = -2.015 \pm (2.201)(0.336) = -2.015 \pm 0.7389$$

or (-2.7539, -1.2761).

Step 4: Interpret the confidence interval in context.

We are 95% confident that the mean difference in the growth of the untreated and treated seeds is between –2.7539 and –1.2761.

Part (b):

Step 1: Identify a correct pair of hypotheses.

$H_0 : \mu_d = 0$ versus $H_a : \mu_d \neq 0$, where μ_d is the mean difference in the untreated and treated seeds.

Step 2: State the correct conclusion in context.

Since the 95% confidence interval does not include zero, the null hypothesis can be rejected at the $\alpha = 0.05$ significance level. In other words, we have statistically significant evidence at the $\alpha = 0.05$ level that there is a mean difference in the growth of untreated and treated seeds.

Question 4 (continued)

Scoring

Each part is scored as essentially correct (E), partially correct (P), or incorrect (I).

Part (a) is essentially correct (E) if all three of steps 1, 3, and 4 of the confidence interval solution are correct.

Part (a) is partially correct (P) if two of the three steps are correct.

Part (a) is incorrect (I) if only one of the steps is correct.

Notes:

1. Step 2 may be omitted since this information is provided in the stem.
2. In step 3, other confidence levels may be used, e.g.,
   - 90% C.I. is $-2.015 \pm 1.796\frac{1.163}{\sqrt{12}} = -2.015 \pm 0.6030$ or (-2.618, -1.412)
   - 99 % C.I. is $-2.015 \pm 3.106\frac{1.163}{\sqrt{12}} = -2.015 \pm 1.0428$ or (-3.0578, -.9722)
3. In step 4, a correct interpretation of the confidence level cannot substitute for a correct interpretation of the confidence interval in context.
4. If a two-sample procedure is used, the highest possible score is (P). The 95% confidence interval for the difference in the two means is $(\bar{x}_1 - \bar{x}_2) \pm t^*_{\min\{n_1-1, n_2-1\}}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} =$

   $(15.989 - 18.004) \pm 2.201\sqrt{\frac{(1.098)^2}{12} + \frac{(1.175)^2}{12}} = -2.015 \pm 1.0218$ or $(-3.037, -0.993)$.
5. The incorrect two-sample confidence intervals from the calculator are:
   - 95% C.I. is $(-2.978, -1.052)$
   - 99% C.I. is $(-3.324, -0.706)$

Question 4 (continued)

Part (b) is essentially correct (E) if the correct conclusion is stated in context and justified using the confidence interval constructed in part (a). Statements of the null and alternative hypotheses are not required.

Part (b) is partially correct (P) if the student:
- Does not provide a conclusion in the context of the problem; OR
- Provides a conclusion in the context of the problem but the justification is weak; OR
- Uses a completely correct application of an appropriate hypothesis test to justify a correct conclusion but does not refer to the confidence interval in part (a).

Part (c) is incorrect (I) if the student:
- Provides a conclusion that is inconsistent with the interval provided in part (a); OR
- Provides a correct conclusion with no justification; OR
- Ignores the confidence interval in part (a) and fails to correctly conduct an appropriate hypothesis test to justify a conclusion.

Note: If the student uses a two-sample confidence interval or some other incorrect confidence interval in part (a), the solution will be scored relative to the reported interval. The student will not be penalized in part (b) for an incorrect solution to part (a).

4 Complete Response (EE)

Both parts are essentially correct

3 Substantial Response (EP or PE)

One part essentially correct and the other part partially correct

2 Developing Response (EI, IE, or PP)

One part essentially correct and the other part incorrect
OR
Both parts partially correct

1 Minimal Response (PI or IP)

One part is partially correct