# AP® STATISTICS
## 2009 SCORING GUIDELINES (Form B)

### Question 3

### Intent of Question

The primary goals of this question were to assess students' ability to (1) state and check appropriate conditions for inference in a study, (2) state the appropriate hypotheses for a statistical test, (3) identify and compute the appropriate test statistic, and (4) make a conclusion in the context of the problem.

### Solution

Let A represent the cardiopump treatment, and let B represent the CPR treatment.

Let $p_A$ = proportion of patients who will survive at least one year if treated with the cardiopump.
Let $p_B$ = proportion of patients who will survive at least one year if treated with CPR.

### Part (a):

Step 1: State the conditions for inference.

The conditions required for a two-sample $z$ test of equal proportions for an experiment are:
1. Random assignment of treatments to subjects
2. Sufficiently large sample sizes

Step 2: Check the conditions.

1. If we assume that the relevant characteristics of people who have heart attacks on even-numbered and odd-numbered days are comparable, randomly assigning one treatment to be given on even-numbered days and the other to be given on odd-numbered days is a reasonable approximation to randomly assigning the two treatments to the available subjects.
2. The large sample condition is met because all of the following are at least 5 (or 10):

$$n_A \hat{p}_A = 37 \geq 5 \text{ or } 10, \ n_A(1 - \hat{p}_A) = 717 \geq 5 \text{ or } 10$$
$$n_B \hat{p}_B = 15 \geq 5 \text{ or } 10, \ n_B(1 - \hat{p}_B) = 731 \geq 5 \text{ or } 10$$

### Part (b):

Step 1: State a correct pair of hypotheses.

$$H_0 : p_A - p_B = 0 \ (\text{or } p_A = p_B)$$
$$H_a : p_A - p_B > 0 \ (\text{or } p_A > p_B)$$

Step 2: Identify a correct test by name or by formula.

Two-sample $z$ test for proportions

*OR*

$$z = \frac{\hat{p}_A - \hat{p}_B}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_A} + \frac{1}{n_B}\right)}} = \frac{\hat{p}_A - \hat{p}_B}{\sqrt{\frac{\hat{p}(1-\hat{p})}{n_A} + \frac{\hat{p}(1-\hat{p})}{n_B}}} \quad \text{where } \hat{p} = \frac{n_A\hat{p}_A + n_B\hat{p}_B}{n_A + n_B}.$$

Step 3: Correct mechanics, including the value of the test statistic and $p$-value (or rejection region).

$$\hat{p}_A = \frac{37}{754} \approx 0.049 \qquad \hat{p}_B = \frac{15}{746} \approx 0.020 \qquad \hat{p} = \frac{37 + 15}{754 + 746} = \frac{52}{1500} \approx 0.035$$

$$z = \frac{\dfrac{37}{754} - \dfrac{15}{746}}{\sqrt{\dfrac{52}{1500}\left(1 - \dfrac{52}{1500}\right)\left(\dfrac{1}{754} + \dfrac{1}{746}\right)}} \approx 3.066$$

The $p$-value is 0.0011.

Step 4: State a correct conclusion in the context of the problem, using the result of the statistical test.

Because the $p$-value of 0.0011 is very small, that is, less than any reasonable significance level such as $\alpha = 0.01$, or $\alpha = 0.05$, we reject the null hypothesis. We have strong evidence to support the conclusion that the proportion of patients who survive when treated with the cardiopump is higher than the proportion of patients who survive when treated with CPR; that is, the survival rate is higher for patients treated with the cardiopump. (*OR*, If all of these patients had been assigned the cardiopump, we have strong evidence that the survival rate would be higher than if all of these patients had been assigned CPR.)

## Scoring

This problem is scored in four sections. Section 1 consists of part (a). Section 2 consists of part (b), step 1. Section 3 consists of part (b), steps 2 and 3. Section 4 consists of part (b), step 4. Section 1 is scored as essentially correct (E), partially correct (P), or incorrect (I). Sections 2, 3, and 4 are each scored as essentially correct (E) or incorrect (I).

**Section 1** [part (a)] is scored as follows:

Essentially correct (E) if the student correctly states and addresses randomization with reasonable justification *AND* correctly checks that the numbers of successes and failures are at least 5 or 10.

Partially correct (P) if the student correctly states and checks only one condition, *OR* if the student states both conditions correctly but checks neither of them.

Incorrect (I) otherwise.

**Section 2** [part (b), step 1] is scored as follows:

Essentially correct (E) if the student states a correct pair of hypotheses.

*Note:* The hypotheses may be stated using words or using transparent variable notation such as $p_{CPR}$ and $p_{CP}$, even if the parameters are not defined.

Incorrect (I) otherwise.

*Note:* Hypotheses that clearly address sample proportions are incorrect. It must be clear that the hypotheses are not about the sample proportions.

**Section 3** [part (b), steps 2 and 3] is scored as follows:

Essentially correct (E) if the student identifies a correct test and includes correct mechanics.

*Note:* The mechanics are considered correct if the student uses a pooled test procedure or an unpooled test procedure, as long as the correct value of $z$ is calculated.

Incorrect (I) otherwise.

*Note:* If the student writes the formula for the unpooled version but gets the correct values for the test statistic and $p$-value for the pooled version, section 3 is scored as incorrect.

**Section 4** [part (b), step 4] is scored as follows:

Essentially correct (E) if the student states a correct conclusion in the context of the problem.

*Note:* A correct conclusion in context must explicitly state that the cardiopump survival rate is significantly higher than the CPR survival rate.

Incorrect (I) otherwise.

*Notes:*
- If the $p$-value in section 3 is incorrect but the conclusion is consistent with the computed $p$-value, section 4 can be considered correct.
- In section 4 if both an $\alpha$ and a $p$-value are given together, the linkage between the $p$-value and the conclusion is implied. If no $\alpha$ is given, the solution must be explicit about the linkage by giving a correct interpretation of the $p$-value or explaining how the conclusion follows from the $p$-value.
- If, instead of a one-sided test, a student correctly performs a two-sided test (chi-square test for homogeneity of proportions or a two-sided $z$ test for comparing two proportions), the final score drops automatically by 1 point.

### Question 3 (continued)

Each essentially correct (E) response counts as 1 point, and a partially correct (P) response in part (a) counts as ½ point.

**4**   **Complete Response**

**3**   **Substantial Response**

**2**   **Developing Response**

**1**   **Minimal Response**

If a response is between two scores (for example, 1½ points), use a holistic approach to determine whether to score up or down, depending on the strength of the response and communication.