but first, let's fix some poor wording on yesterday's notes

4. **Construct the interval:**

General Formula: Pt. Estim ± MOE

Specific Formula: $b \pm t^* \cdot SE_b$

df = n−2 = 30−2 = 28

Work: $-1.517 \pm 2.048 \times 1.33$ given

$(-4.24, 1.21)$

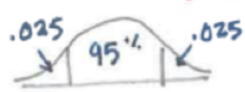


TABLE B or invT } $t^* = 2.048$

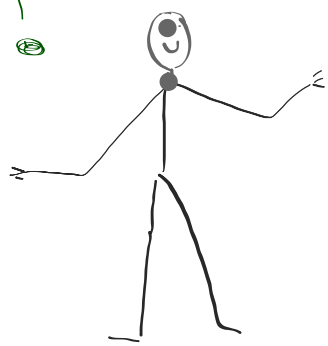
5. **Conclude:**

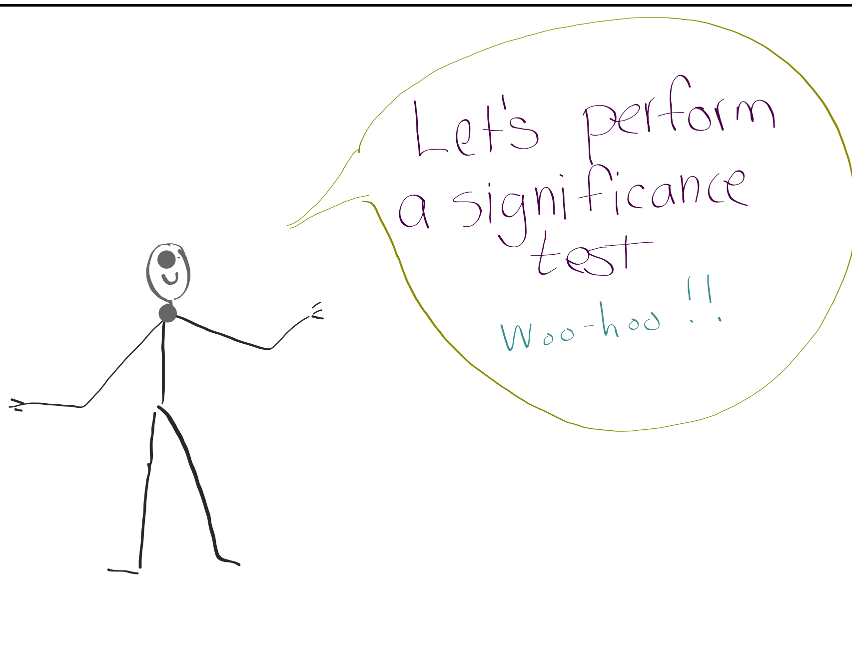
We are 95% confident that the interval from (−4.24, 1.21) captures the ~~true~~ slope of the population regression line relating y = score and x = row

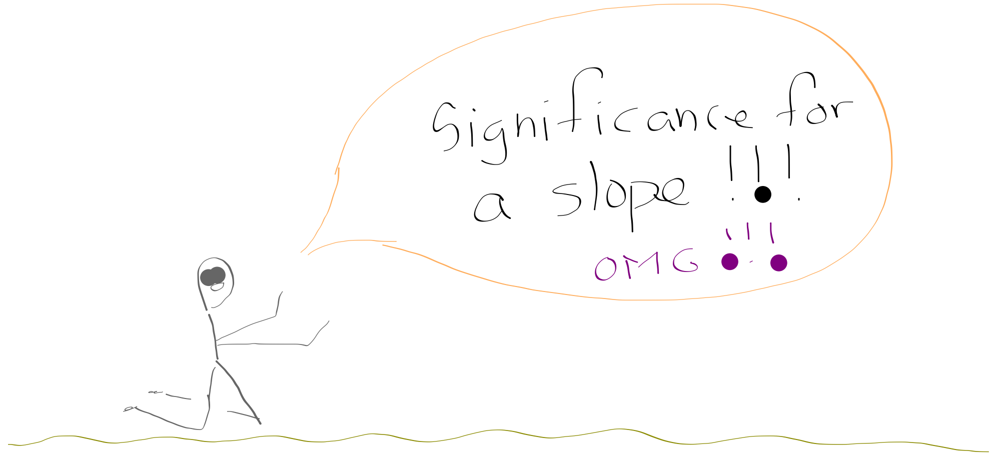
New

Today:

The last inference procedure of AP Stats!







Why do we perform a significance test for a slope?

When data from a random sample or a randomized experiment suggests that a linear association exists between two variables, there are two possible explanations for why the slope differs from 0.

there is really no association and we got a non-zero slope due to sampling variability (or chance variation due to random assignment)

OR

there really is an association

[we do a significance to see which explanation is more plausible]

Lesson 12.1 Day 3

How does GPA relate to The ACT score?

A counselor is wondering if there is a relationship between GPA and ACT score among 101 students that were applying to schools outside the state. She took a random sample of 9 out of the 101 students and recorded their GPA and ACT score. The data are below.

Student #	83	69	96	89	57	13	24	37	91
GPA	3.7	2.3	4.0	3.8	3.0	1.8	2.0	2.3	3.9
ACT	23	20	35	33	22	13	17	20	29

1. **Before even looking at any data, shat relationship would you expect GPA and ACT score to have? Explain.**

One would think....

Positive, Linear, relationship.

As GPA goes up we would expect ACT scores to rise.

Here is the minitab ouput as well as graphs of the data.

Predictor	Coef	SE Coef	T	P
Constant	1.201	0.0874	13.72	0
GPA	7.507	1.29	5.82	0.0006511
S = 3.252686	R-Sq = 82.8%		R-Sq(adj) = 76.5%	

2. Find the LSRL for the data.

Here is the minitab ouput as well as graphs of the data.

Predictor	Coef	SE Coef	T	P
Constant	1.201	0.0874	13.72	0
GPA	7.507	1.29	5.82	0.0006511
S = 3.252686	R-Sq = 82.8%		R-Sq(adj) = 76.5%	

y-int

slope

SE b_1

2. Find the LSRL for the data.

$$\widehat{ACT} = 1.201 + 7.507(GPA)$$

predicted ACT score

3. Do the data provide significant evidence that there is a positive linear relationship between GPA and ACT?

STATE:

Parameter: Statistic:

H_0:

H_a: Sign. Level:

STATE:

Parameter: where β_1 is the slope of the population regression line relating y=ACT score to x=GPA

Statistic: $b_1 = 7.507$

H_0: $\beta_1 = 0$

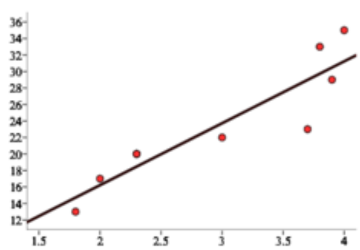
one sided test H_a: $\beta_1 > 0$

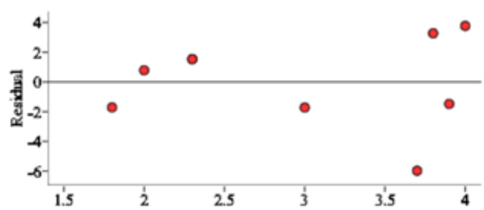
↑ we expected a positive relationship

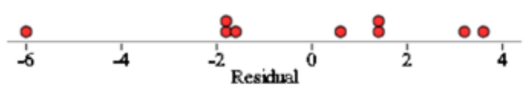
Sign. Level: $\alpha = 0.05$

$$\mu_y = \alpha + \beta x$$

$$\hat{y} = a + bx$$







PLAN: Name of procedure:

Check conditions:

(1) Linear:

(2) Independent:

(3) Normal:

(4) Equal SD:

(5) Random:

PLAN: Name of procedure: One Sample t test for β_1

Check conditions:

✓ (1) Linear: Scatter plot shows a linear relationship and there are no leftover patterns in the residual plot.

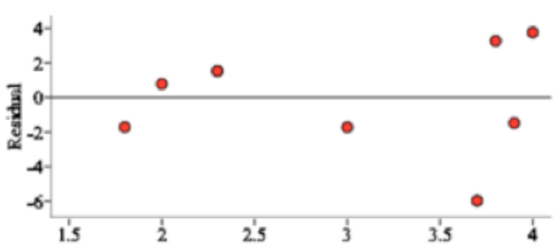
✓ (3) Normal: Dot plot shows no strong skewness or clear outliers

✓ (5) Random: "Rand. sample of 9"

(2) Independent: $9 < \frac{1}{10}(101) = 10.1$

(4) Equal SD: Residual plot shows a fairly equal amount of scatter around horiz. line $x=0$ for all x-values.

If residuals are small for certain values of the explanatory variable and large for others, then the SD of the response variable is <u>not</u> the same for all values of the explanatory variable, thereby violating the "equal SD condition.



DO: General Formula:

Specific Formula:

Picture:

Work:

Test statistic:

P-value:

DO: General Formula: Test Stat. $= \dfrac{\text{stat} - \text{Null}}{SD}$

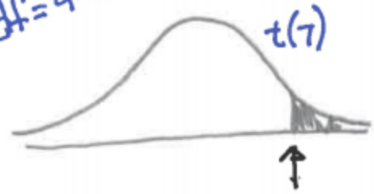
Specific Formula: $t = \dfrac{b_1 - \beta_1}{\cancel{SE_{b_1}}}$ S_b

hypothesized slope

Work:

$t = \dfrac{7.507 - 0}{1.29} = 5.819$

$df = 9 - 2 = 7$



Test statistic: $t = 5.819$

P-value: P-Value = 0.000326

tcdf[ ]

Here is the minitab ouput as well as graphs of the data.

Predictor	Coef	SE Coef	T	P
Constant	1.201	0.0874	13.72	0
GPA	7.507	1.29	5.82	0.0006511
S = 3.252686	R-Sq = 82.8%		R-Sq(adj) = 76.5%	

y-int

slope

SE b_1

Because the computer output p-value is for a two-sided test, and there is some evidence for H_a, we cut it in half

$0.0006511/2 = 0.000326$

CONCLUDE:

CONCLUDE:
Because the P-Value of 0.000326 $< \alpha = .05$, we reject H_0.
There is convincing evidence of a positive linear relationship between ACT score and GPA.....

Association still does not imply causation, even if the association is significant.

Lesson 12.1: Day 3: Significance Test for Slope

Important ideas:

Hypotheses

FORMULAS

Conditions

Lesson 12.1: Day 3: Significance Test for Slope

slope

Important ideas:

Hypotheses

$H_0: \beta_1 = 0$

$H_a: \beta_1 \; > \; \neq \; < \; 0$

Conditions

L.I.N.E.R.

See prev. notes

FORMULAS

One sample t test for β_1

DO: test stat $= \dfrac{b_1 - \beta_1}{SE_{b_1}}$ S_b

$df = n - 2$

P-Value $= t_{cdf}(\text{Lower}, \text{Upper}, df)$

t and two sided P-Value typically given on computer output

TI APP LinRegTTest

Check your understanding

IQ and Crying

H. Armstrong Roberts/ ClassicStock/Archive Photos/ Getty Images

IQ and Crying: Infants who cry easily may be more easily stimulated than others. This may be a sign of higher IQ. Child development researchers explored the relationship between the crying of infants 4 to 10 days old and their later IQ test scores. A snap of a rubber band on the sole of the foot caused the infants to cry. The researchers recorded the crying of 38 infants. They measured the crying intensity by the number of peaks in the most active 20 seconds. They later measured the children's IQ at age three years using the Stanford–Binet IQ test.

Here is computer output from a least-squares regression analysis of these data. Do these data provide convincing evidence at the α = 0.05 level of a positive linear relationship between count of crying peaks and IQ in the population of infants? Assume conditions have been met.

Regression Analysis: IQ versus Crycount

Predictor	Coef	SE Coef	T	P
Constant	91.268	8.934	10.22	0.000
Crycount	1.4929	0.4870	3.07	0.004

S = 17.50   R-Sq = 20.7%   R-Sq(adj) = 18.5%

Regression Analysis: IQ versus Crycount

Predictor	Coef	SE Coef	T	P
Constant	91.268	8.934	10.22	0.000
Crycount	1.4929	0.4870	3.07	0.004

S = 17.50   R-Sq = 20.7%   R-Sq(adj) = 18.5%

STATE

$H_0: \beta = 0$

$H_a: \beta > 0$

$\alpha = 0.05$

Regression Analysis: IQ versus Crycount

Predictor	Coef	SE Coef	T	P
Constant	91.268	8.934	10.22	0.000
Crycount	1.4929	0.4870	3.07	0.004

S = 17.50 R-Sq = 20.7% R-Sq(adj) = 18.5%

STATE

$H_0: \beta = 0$

$H_a: \beta > 0$

$\alpha = 0.05$

β

$\cancel{\beta_1}$ = slope of pop. regression line relating y = IQ score to x = count of crying peaks in pop. of infants.

$b_1 = 1.4929$

Regression Analysis: IQ versus Crycount

Predictor	Coef	SE Coef	T	P
Constant	91.268	8.934	10.22	0.000
Crycount	1.4929	0.4870	3.07	0.004

S = 17.50 R-Sq = 20.7% R-Sq(adj) = 18.5%

PLAN one sample t test for β_1

DO

$$t = \frac{b - \beta}{S_b} \quad \left(\cancel{SE_{b_1}}\right)$$

$$t = \frac{1.4929 - 0}{.4870}$$

$$= 3.07$$

Regression Analysis: IQ versus Crycount

Predictor	Coef	SE Coef	T	P
Constant	91.268	8.934	10.22	0.000
Crycount	1.4929	0.4870	3.07	0.004

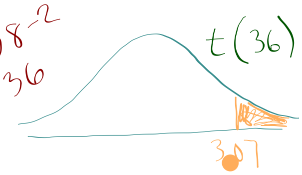
S = 17.50 R-Sq = 20.7% R-Sq(adj) = 18.5%

PLAN one sample t test for β_1

DO

$$t = \frac{b_1 - \beta_0}{SE_{b_1}}$$

$df = 38-2$
$= 36$



$$t = \frac{1.4929-0}{.4870}$$

$= 3.07$

$P = 0.002$

for Signif. testing
you can use

Regression Analysis: IQ versus Crycount

Predictor	Coef	SE Coef	T	P
Constant	91.268	8.934	10.22	0.000
Crycount	1.4929	0.4870	3.07	0.004

S = 17.50 R-Sq = 20.7% R-Sq(adj) = 18.5%

P-Value from two sided test

CONCLUDE Because the P-Value of 0.002 < $\alpha = .05$, we reject H_0. There is convincing evidence of a positive linear relationship between IQ and crying peaks

LCQ

Reviews

12.1... 15, 23-28, 29

study pp. 782-786