

Turn in Take Home  
LCQ

①

②

Warm Up  
AP MC Practice  
- See if you can get  
2 or more correct.

**EXPERIMENTAL DESIGN 12**

Sampling error occurs

- (A) when interviewers make mistakes resulting in bias.
- (B) when interviewers use judgment instead of random choice in picking the sample.
- (C) when samples are too small.
- (D) because a sample statistic is used to estimate a population parameter.
- (E) in all of the above cases.

**Different samples give different statistics, all of which are estimates for the same population parameter, and so error, called sampling error, is naturally present.**

**Answer: D**

## II

### EXPERIMENTAL DESIGN 9

A sales representative wishes to survey her client base of 47 companies. She has 47 business cards, all of the identical size, from her contacts in the companies, and decides to drop them all in a small box, shake them up, and reach in to pick 5 cards for her sample. This procedure is an example of which type of sampling?

- (A) Cluster      (B) Convenience      (C) Simple random  
(D) Stratified      (E) Systematic

**Answer: (C)** A simple random sample (SRS) is one in which every possible sample of the desired size has an equal chance of being selected. In this case, every possible sample of five companies has an equal chance of being selected. Note that even though it is also true that each company has an equal chance of being selected, this by itself would not ensure that we have an SRS.

## III

Each of the 30 major league baseball teams carries a 40-person roster. A sample of 60 players (5 percent of all 1,200 players) is to be randomly selected to undergo drug tests. To do this, each team is instructed to put their 40 names in a hat and randomly draw two names. Will this method result in a simple random sample of the 1,200 baseball players?

- (A) Yes, because each player has the same chance of being selected.  
(B) Yes, because each team is equally represented.  
(C) Yes, because this is an example of stratified sampling, which is a special case of simple random sampling.  
(D) No, because the teams are not chosen randomly.  
(E) No, because not each group of 60 players has the same chance of being selected.

**Answer: (E)** In a simple random sample, every possible group of the given size has to be equally likely to be selected, and this is not true here. For example, with this procedure, it will be impossible for all the Cubs to be together in the final sample. This procedure is an example of stratified sampling, but stratified sampling does not result in simple random samples.

# 10.2 Day 2

Today, you will continue to deal with *differences of means* ( $\mu_1 - \mu_2$ ), but instead of creating confidence intervals you will switch back to conducting a hypotheses test instead.

## 10.2 Day 2

### Incorporating Components of a Hypothesis Test for a Difference of Two means

1. **Which version of the exam is harder?** A college professor has created two different versions of a final exam. She wonders if the difficulty is the same for each version. To find out, she randomly assigns 80 student volunteers to two groups: 40 students take Version A, and 40 students take Version B. Here are the results:

Work up through part d

Version	Mean	SD
A	84.2	8.9
B	79.9	12.3

Do these data give convincing evidence at the  $\alpha = 0.05$  significance level of a difference in the true mean score on the final exam for Version A and Version B for students like the ones in the study?

Need your AP Formula sheet

(a) State appropriate hypotheses for performing a significance test. Be sure to define the parameters of interest. Try to incorporate subscripts that go with the context of the situation.

$$H_0 : \mu_A - \mu_B = 0$$

$$H_a : \mu_A - \mu_B \neq 0$$

where  $\mu_A$  = the true mean score on Version A of the final exam for students like the ones in the study and  $\mu_B$  = the true mean score on Version B of the final exam for students like the ones in the study.

(b) Good news. The Random, 10%, and Normal/Large Sample conditions are exactly the same for significance tests for a difference in means as they were for a confidence interval for a difference in means. WooHoo! Go ahead and check the conditions.

**Random:** The 80 subjects were randomly assigned to Version A or Version B. ✓

**Normal/Large Sample:**  $n_A = 40 \geq 30$  and  $n_B = 40 \geq 30$ . ✓

(c) The table of information has been updated below to include the standard deviation and the sample size of each of the randomly assigned groups.

$$\bar{X}_A - \bar{X}_B =$$

Version	Number of students	Mean	SD
A	40	84.2	8.9
B	40	79.9	12.3

Explain why the sample results give some evidence for the alternative hypothesis.

(c) The table of information has been updated below to include the standard deviation and the sample size of each of the randomly assigned groups.

Version	Number of students	Mean	SD
A	40	84.2	8.9
B	40	79.9	12.3

Explain why the sample results give some evidence for the alternative hypothesis.

The observed difference in the sample means is  $\bar{X}_A - \bar{X}_B = 84.2 - 79.9 = 4.3$ , which gives some evidence in favor of  $H_a$  because  $4.3 \neq 0$

- (d) Looking only at your AP Formula Sheet, attempt to find/create the specific formula for the standardized test statistic that you will need in this situation. Try to avoid looking at other sources for now. Mr. Cedarlund will verify the formula.



### III. Sampling Distributions and Inferential Statistics

$$(\bar{x}_A - \bar{x}_B)$$

$$\mu_A - \mu_B = 0$$

Standardized test statistic:  $\frac{\text{statistic} - \text{parameter}}{\text{standard error of the statistic}}$

Confidence interval:  $\text{statistic} \pm (\text{critical value})(\text{standard error of statistic})$

#### Standard Error\* of Sample Statistic

$$s_{\bar{x}} = \frac{s}{\sqrt{n}}$$

$$s_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$t = \frac{(\bar{x}_A - \bar{x}_B) - (\mu_A - \mu_B)}{\sqrt{\frac{s_A^2}{n_A} + \frac{s_B^2}{n_B}}}$$

- (d) Looking only at your AP Formula Sheet, attempt to find/create the specific formula for the standardized test statistic that you will need in this situation. Try to avoid looking at other sources for now. Mr. Cedarlund will verify the formula.

$$t = \frac{(\bar{x}_A - \bar{x}_B) - 0}{\sqrt{\frac{s_A^2}{n_A} + \frac{s_B^2}{n_B}}} \quad \text{or} \quad t = \frac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$\mu_A - \mu_B$   
 $\downarrow$   
 $0$

- (e) First calculate the standardized test statistic and the P-value using Option 2  
 Option 2: Use the smaller degrees of freedom. Then use Table B or  $t_{cdf}$  to calculate the P-value

- (f) Now Re-calculate the standardized test statistic and the P-value using Option 1  
 (Option 1: Use 2-SampTTest on your calculator. Report the  $t$  statistic, P-value, and the df.)

(e) First calculate the standardized test statistic and the P-value using Option 2

(Option 2: Use the smaller degrees of freedom. Then use Table B or  $t_{cdf}$  to calculate the P-value)

$$t = \frac{(84.2 - 79.9) - 0}{\sqrt{\frac{8.9^2}{40} + \frac{12.3^2}{40}}} = 1.79 \rightarrow df = 39 \rightarrow \text{TABLE B} \rightarrow \text{P-Value between } .025 \text{ and } .05$$

Two tailed  
 $2(.025)$  and  $2(.05)$   
 $.05 < \text{P-Value} < 0.10$

OR  $t_{cdf}[\text{lower } 1.79, \text{upper } 1000, 39] \times 2 = .081$  😊

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(f) Now Re-calculate the standardized test statistic and the P-value using Option 1

(Option 1: Use 2-SampTTest on your calculator. Report the  $t$  statistic, P-value, and the df.)

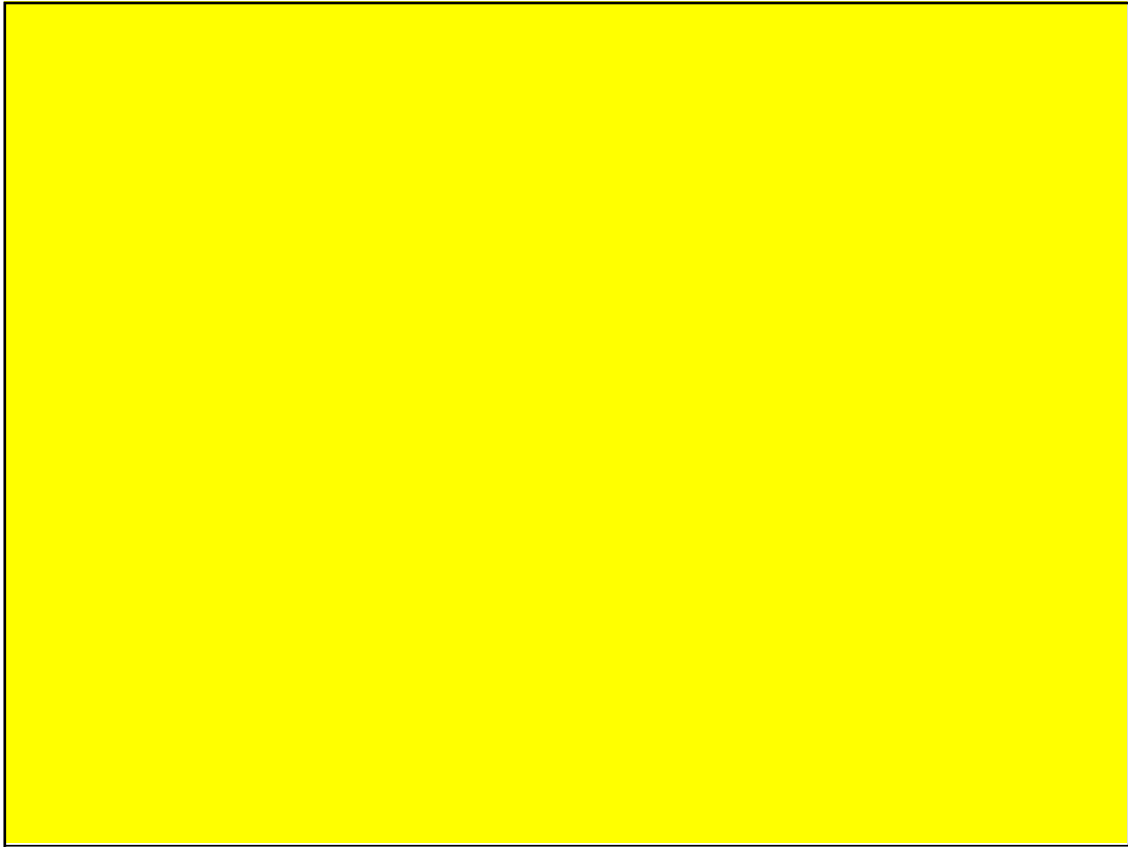
2-SampTTest gives  $t = 1.79$  and P-value = .078 using  $df = 71.05$



(g) What conclusion would you make?

(g) What conclusion would you make?

Because the P-value of  $0.078 > \alpha = .05$ , we fail to reject  $H_0$ .  
There is not convincing evidence of a difference in the true mean score on Version A and Version B of the final exam for students like the ones in the study.



## Requirements for **Hypotheses Tests** for a difference of Means

Conditions

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$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

↑  
Test  
statistic

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↗  
Test  
statistic

$$P\text{-Value} = t_{cdf}\left(\begin{matrix} t & t \\ \text{lower, upper,} & \end{matrix} df\right)$$

or table B

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calculator shortcut 2 Samp T Test

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↑  
Test  
Statistic

$$P\text{-Value} = t_{cdf}\left(\begin{matrix} t \\ \text{lower, upper, } \end{matrix} df\right)$$

or table B

Calculator Shortcut

2 Samp T Test gives  $t = \underline{\hspace{2cm}}$   
and P-Value =  $\underline{\hspace{2cm}}$  using  $df = \underline{\hspace{2cm}}$

### State

- State Hypotheses (using subscripts and be informative and helpful)
- Give significance level
- Define both parameters

### Plan

- Identify the procedure
- State and check conditions (In experiments, we are not sampling w/o replacement so don't check 10% cond.)

**Do**

- Give statistics
- Give standardized test statistics
- Give P-value

Option 1: example: `2SampTTest` gives  $t = 3.27$  and  $P - \text{Value} = 0.076$  using  $df = 13.2$

Option 2: example:  $df = 12$

Or with technology: `tcd` (lower: 3.27, upper: 1000,  $df: 12$ ) = 0.065

Table B:  $P - \text{Value}$  is between 0.05 and 0.10

**Conclude**

Two-sentence structure (as usual)

- Compare P-value to significance level, then reject or fail to reject  $H_0$ .
- Evidence for  $H_a$  in context. (be clear about order)

It's time for :

"HOT POTATO  
INFERENCE"

- ① PAIRS
- ② ONE paper
- ③ ONE person starts with it  
The other observes/coaches
- ④ Until...  
Mr. C says.. Rotate!!!

### Put it All Together - **Does labeling menus reduce calories?**

According to a Stanford Business article, Americans may eat fewer calories at restaurants if the calories of the food items are labeled on the menu. To investigate this, researchers compared Starbucks receipts from locations where the menus were labeled to receipts from stores where the menus were not labeled. A random sample of 30 receipts from stores with the menus labeled had an average number of calories of 225 calories with a standard deviation of 100 calories. A random sample of 40 receipts from stores without menus labeled showed an average of 265 calories per receipt with a standard deviation of 75 calories. Does this provide convincing evidence that the average calories per receipt at Starbucks with a labeled menu is less than at a Starbucks without labeled menus?

**Each group to write a  
portion of the solution**

## Put it All Together - Does labeling menus reduce calories?

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L1	L2	L3	Z
118	124	-----	
126	98		
126	110		
120	140		
129	110		
-----			

L2(6) =

Entering Data  
for 2-SampTTest

```
2-SampTTest
Inpt: Data Stats
x1: 3.2
sx1: 2.7
n1: 10
x2: 2
sx2: 2.62
n2: 10
```

```
2-SampTTest
Inpt: Data Stats
List1: L1
List2: L2
Freq1: 1
Freq2: 1
μ1: ≠ μ2 < μ2
Pooled: Yes
```

```
2-SampTTest
μ1 > μ2
t = .9888665951
p = .1856713115
df = 4.650964126
x1 = 123.8
x2 = 116.4
```



**10.2....**51-57(odd), 67, 69-72  
and study pp. 673-682 !

## Put it All Together - Significance Test for a Difference in Means

How quickly do synthetic fabrics such as polyester decay in landfills? A researcher buried polyester strips in the soil for different lengths of time, then dug up the strips and measured the force required to break them. Breaking strength is easy to measure and is a good indicator of decay. Lower strength means the fabric has decayed more. For one part of the study, the researcher buried 10 strips of polyester fabric in well-drained soil in the summer. The strips were randomly assigned to two groups: 5 of them were buried for 2 weeks and the other 5 were buried for 16 weeks. Here are the breaking strengths in pounds:

Group 1 (2 weeks)	118	126	126	120	129
Group 2 (16 weeks)	124	98	110	140	110

Do the data give convincing evidence that polyester decays more in 16 weeks than in 2 weeks, on average

Group 1 (2 weeks)	118	126	126	120	129	$\bar{x}_1 = 123.8$	$s_1 = 4.60$
Group 2 (16 weeks)	124	98	110	140	110	$\bar{x}_2 = 116.4$	$s_2 = 16.09$

Do the data give convincing evidence that polyester decays more in 16 weeks than in 2 weeks, on average

STATE

$$H_0: \mu_1 - \mu_2 = 0$$

$$H_a: \mu_1 - \mu_2 > 0$$

where  $\mu_1$  = true mean breaking strength for polyester fabric buried for 2 weeks

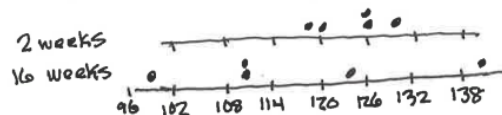
$\mu_2$  = true mean breaking strength after 16 weeks

PLAN

Two-sample t test for  $\mu_1 - \mu_2$

Random strips randomly assigned to the treatments ✓

Normal/Large Counts Dot plots show no strong skewness and no outliers ✓



DO

$$\bar{x}_1 = 123.8$$

$$\bar{x}_2 = 116.4$$

$$s_1 = 4.60$$

$$s_2 = 16.09$$

$$n_1 = 5$$

$$n_2 = 5$$

2 Samp T Test gives  $t = 0.99$   
and P-Value = 0.1857 using  $df = 13.2$

CONCLUDE

Because the P-value of 0.1857  $>$   $\alpha = 0.05$ , we fail to reject  $H_0$ .  
We do not have convincing evidence that the true mean breaking strength of polyester fabric that is buried for 2 weeks is greater than the same fabric that is buried for 16 weeks.