TURN-IN THE FRQ #2
(from PPC UNit 4)

even if not completely finished

The Power Of a Test

The Power of a Test

please! I hope it works.

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Researchers would make a Type II error if they failed to find convincing evidence for *Ha* based on the sample data.

The Power of a Test

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which could be frustrating if the researchers are pretty sure their idea has merit

The Power of a Test

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How likely is a significance test to *avoid* a Type II error? We refer to this probability as the **power** of the test.

Usually desirable.

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How likely is a significance test to *avoid* a Type II error? We refer to this probability as the **power** of the test.

The **power** of a test is the probability that the test will find convincing evidence for H_a when a specific alternative value of the parameter is true.

their research hypothesis

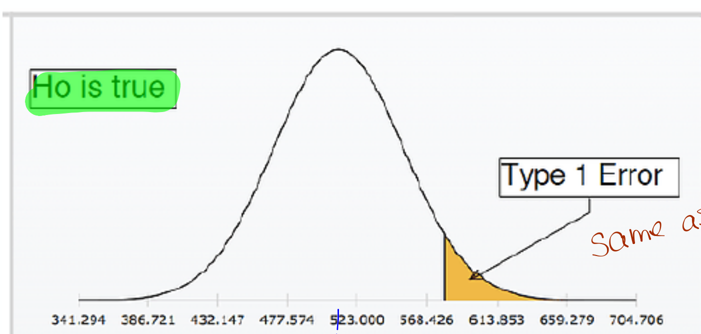
Power is the probability of correctly rejecting the null hypothesis

Power Calculations are usually done prior to collecting data

[Analagous to calculating the sample size to achieve a desired margin of error]

LAPTOPS

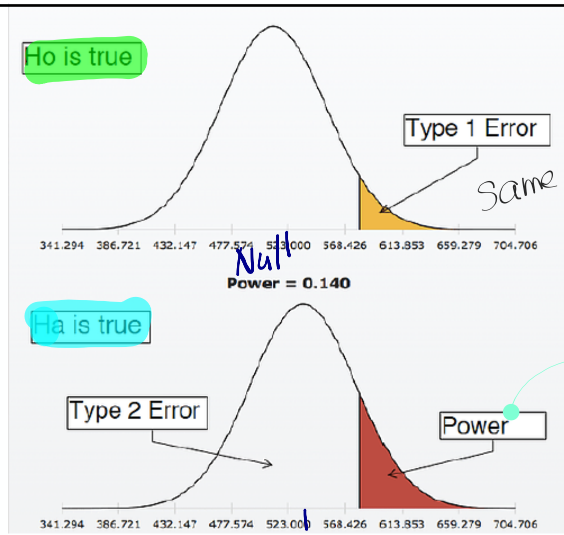
The Power of A Significance Test?



same as α

Ho

DISTRIBUTION IF Ha is true



Same as α

Null

Prob. of finding convincing evidence that Ha is true when Ha really is true.

Alternative Null

The national mean score on the math portion of the SAT is 511 with a standard deviation of 120. Suppose we believe the students at SHS have a higher mean than the national average. To find out, we take a random sample of 8 students and find their average. We will then use the data to conduct a significance test with α = 0.05.

1. Write the appropriate hypotheses for the significance test. Be sure to define the parameter of interest.

$H_0: \mu = 511$

$H_a: \mu > 511$

μ = true mean math SAT score at Sheldon.

511
120
8
.05
535

Suppose the mean math SAT score at SHS is 535 (*alt. μ*). Go to our textbook website and open the "Statistical Power" applet. Enter all of this information into the fields on the left of the applet. You'll notice a value called "Power". This is the probability that the significance test will find convincing evidence against the null with the information you've entered.

2. What is the **Power** (*or probability*) that the test will find convincing evidence against the null hypothesis?

Interpret this value in context.

Suppose the mean math SAT score at SHS is 535 (*alt. μ*). Go to our textbook website and open the "Statistical Power" applet. Enter all of this information into the fields on the left of the applet. You'll notice a value called "Power". This is the probability that the significance test will find convincing evidence against the null with the information you've entered.

2. What is the **Power** (*or probability*) that the test will find convincing evidence against the null hypothesis?

Power = .140

Interpret this value in context.

If the mean score at SHS is 535, there is a 14% probability of the test giving convincing evidence for Ha ($\mu > 511$)

3. We want to **increase** the power of our test. How could we adjust each of the following factors to increase our power? Use the applet to explore each.

a. Sample size:

b. Alpha level:

c. Alternative μ:

3. We want to **increase** the power of our test. How could we adjust each of the following factors to increase our power? Use the applet to explore each.

a. Sample size: Increase the sample size.

b. Alpha level: Increase α

c. Alternative μ: Increase the distance between alternative μ and null value

Power of a Test

The **power** of a test is the probability that the test will find convincing evidence for H_a when a specific alternative value of the parameter is true *(and the null value, H_0, is not true)*

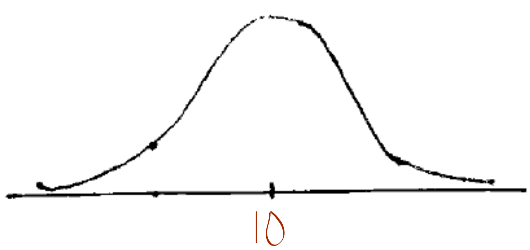
Suppose a researcher wants to show that a particular treatment shortens the duration of an ailment (cold for example)

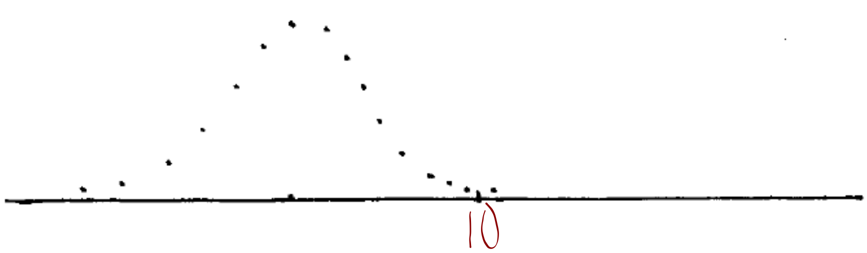
shorten from 10 day average

H_0: $\mu = 10$ days H_a: $\mu < 10$ days

Sampling distribution if H_0 is true

H_0 true

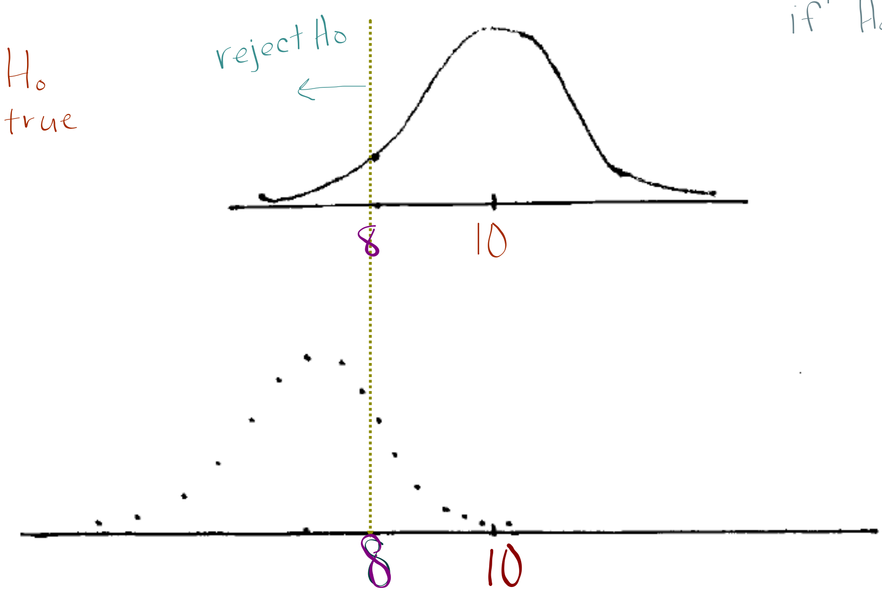


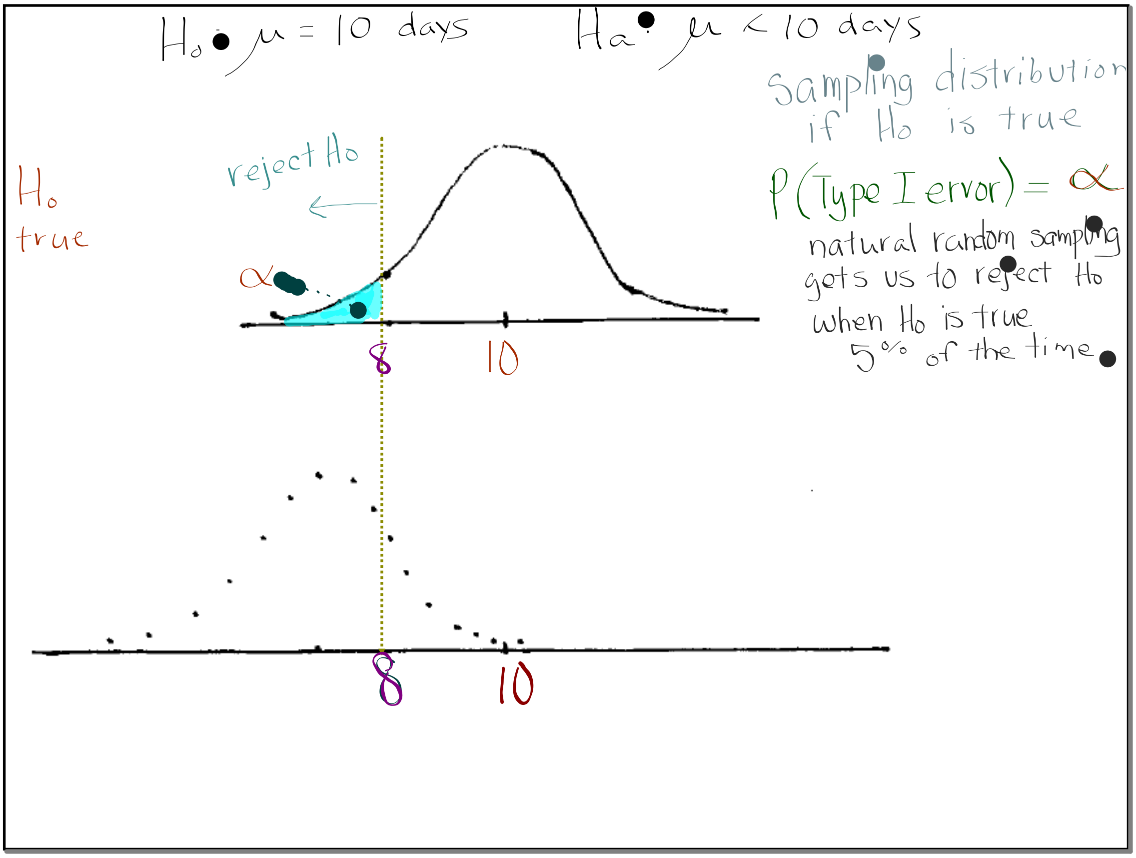


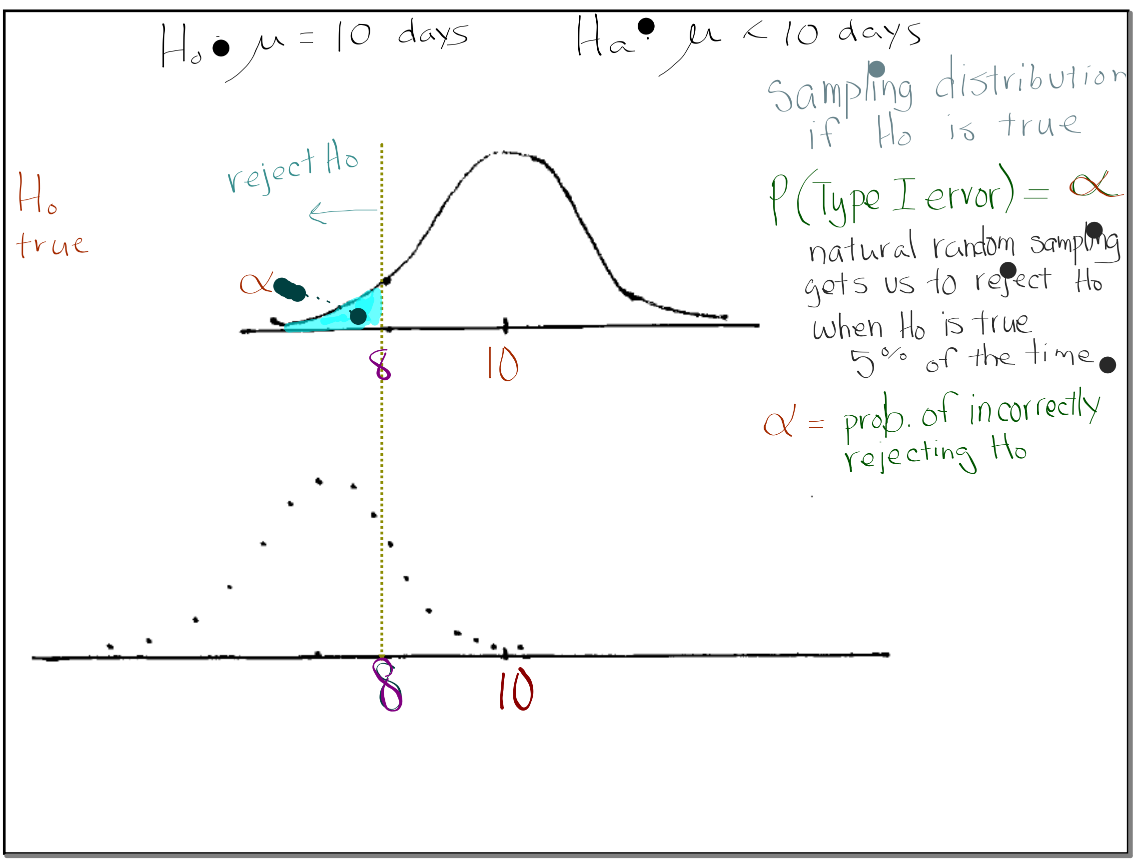
Suppose 8 days is the rejection value

$H_0: \mu = 10$ days $\quad H_a: \mu < 10$ days

Sampling distribution if H_0 is true




Ho: μ = 10 days
Ha: μ < 10 days
Sampling distribution if Ho is true
P(Type I error) = α
natural random sampling gets us to reject Ho when Ho is true 5% of the time.
Ho true
reject Ho
α
8
10
8
10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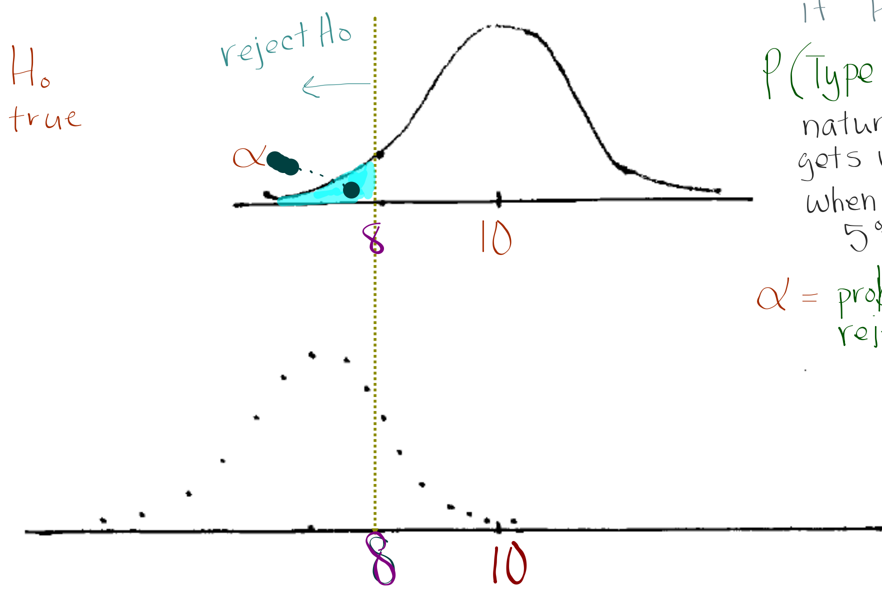


Sampling distribution if H_0 is true

$P(\text{Type I error}) = \alpha$

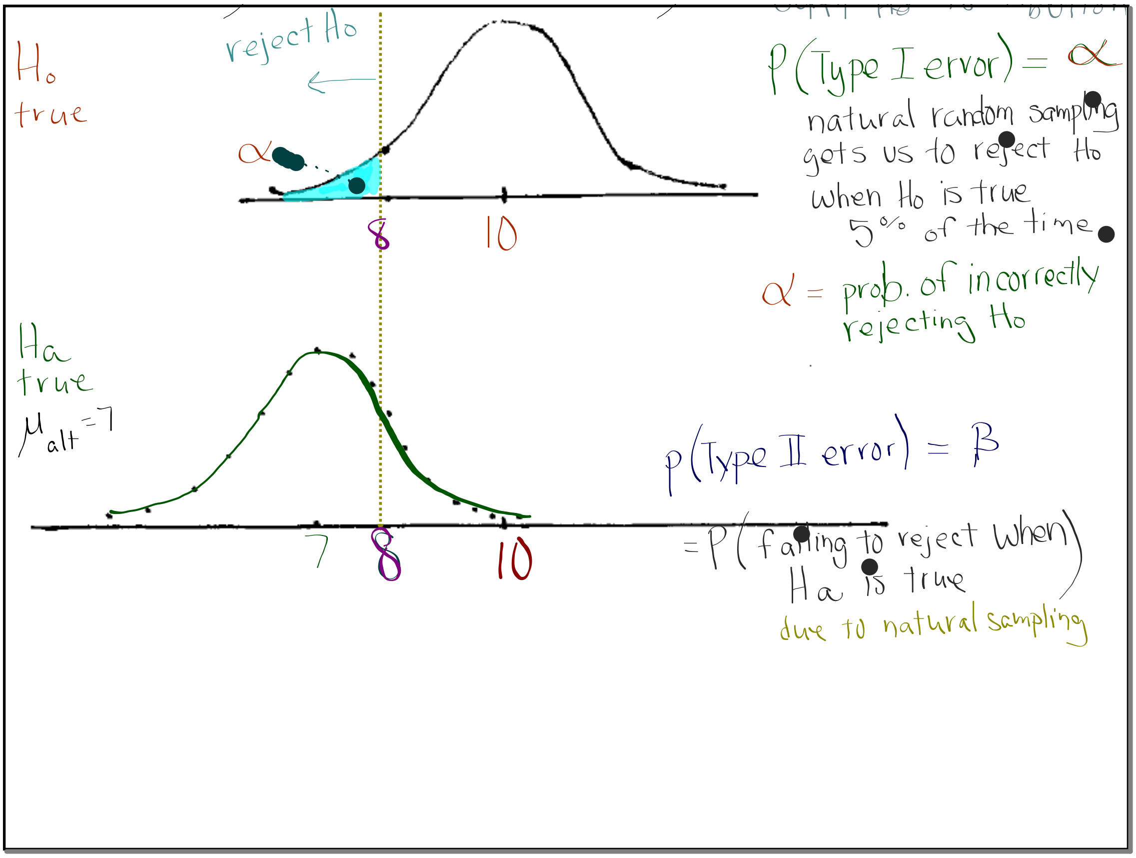
natural random sampling gets us to reject H_0 when H_0 is true 5% of the time.

α = prob. of incorrectly rejecting H_0

but

what if H_a is really true instead ?

We would expect the distribution of samples to be centered around a shorter time duration, say 7 days for example.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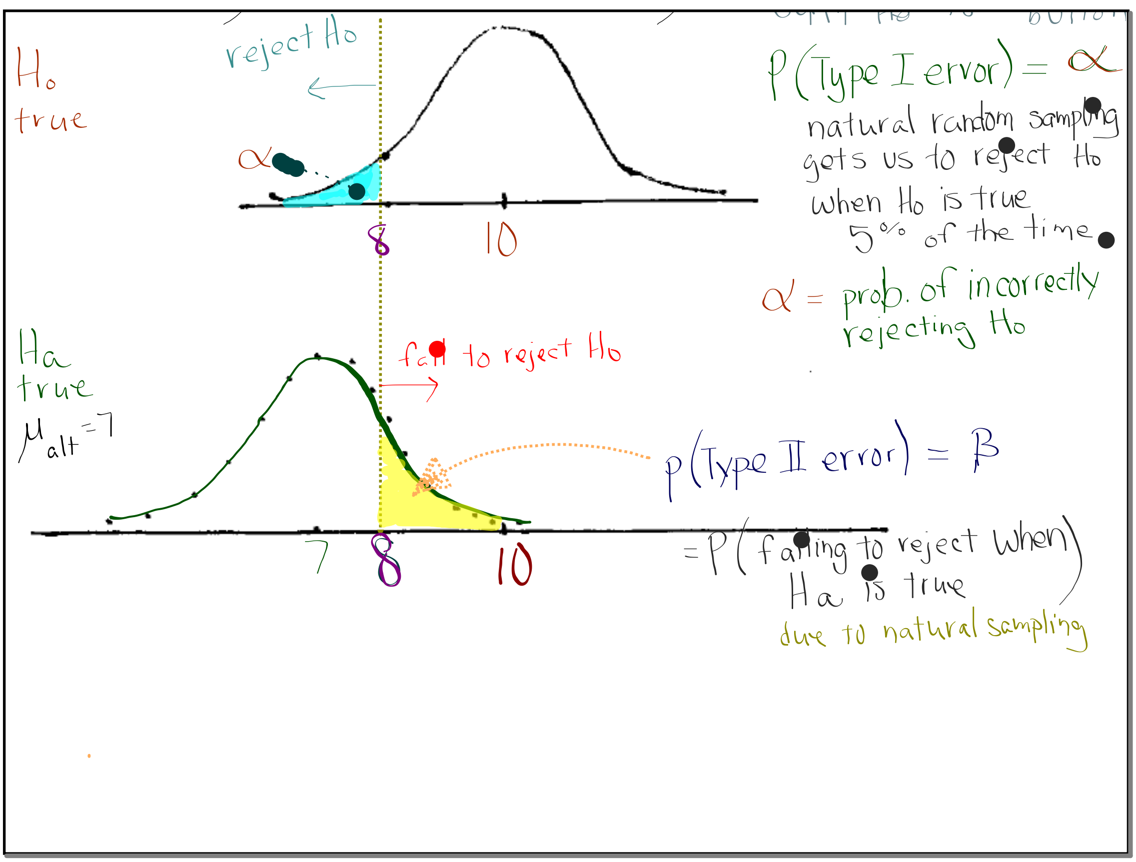
α = prob. of incorrectly rejecting Ho
Ha true
μalt = 7
7
8
10
p(Type II error) = β
= P(failing to reject when Ha is true)
due to natural sampling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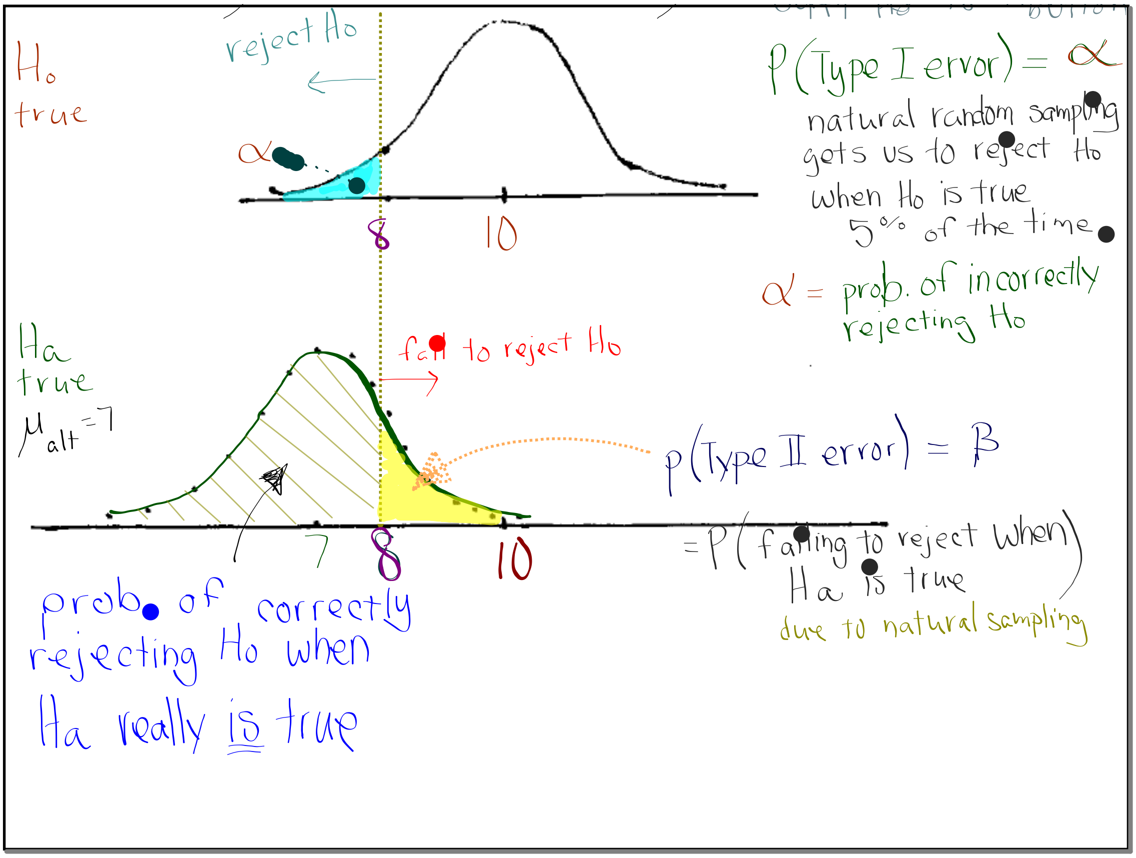
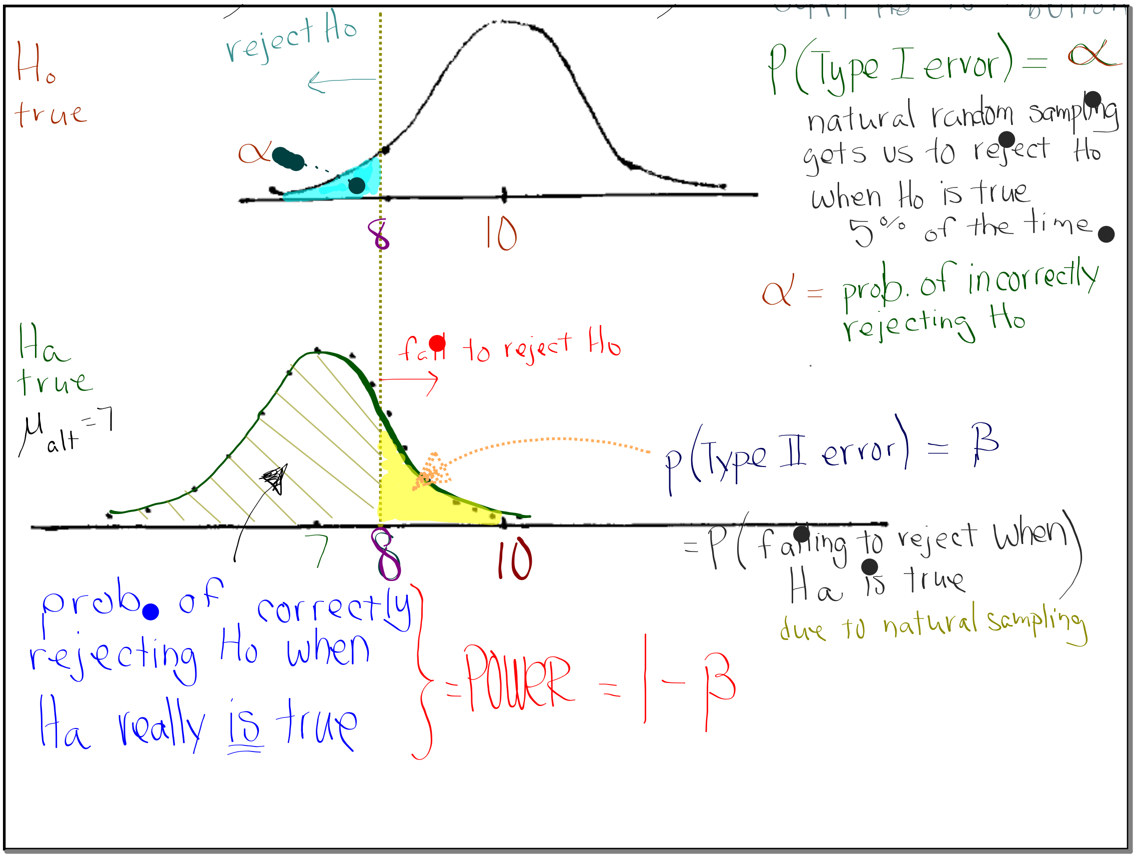


Power *is the probability of correctly rejecting* H_o *when* H_a *really is true.*



Power of a Significance Test

Important ideas:

Power of a Significance Test

Important ideas:

Interpretation

If the true mean ____ is ____, there is a ____ probability of finding convincing evividence for Ha: ____

Power of a Significance Test

Important ideas:

Interpretation

If the true mean cold length is $\mu = 10$ days, there is a .59 probability of finding convincing evividence for Ha: $\mu < 10$

Power of a Significance Test

Important ideas:

Interpretation

If the true mean cold length is $\mu = 10$ days, there is a .59 probability of finding convincing evividence for Ha: $\mu \neq 10$

TRUTH

Decision	Ho true	Ha true
reject Ho		
fail to reject Ho		

Power of a Significance Test

Important ideas:

Interpretation

If the true mean cold length is $\mu = 10$ days, there is a .59 probability of finding convincing evividence for Ha: $\mu \neq 10$

TRUTH

Decision	Ho true	Ha true
reject Ho		
fail to reject Ho	Accurate	

Power of a Significance Test

Important ideas:

Interpretation

If the true mean cold length is $\mu = 10$ days, there is a .59 probability of finding convincing evidence for $H_a: \mu \neq 10$

TRUTH

	H_0 true	Decision	H_a true
α	Type I error	reject H_0	
	Accurate	fail to reject H_0	

Power of a Significance Test

Important ideas:

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If the true mean cold length is $\mu = 10$ days, there is a .59 probability of finding convincing evidence for $H_a: \mu \neq 10$

TRUTH

	H_0 true	Decision	H_a true	
α	Type I error	reject H_0		
	Accurate	fail to reject H_0	Type II error	β

Power of a Significance Test

Important ideas:

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If the true mean cold length is $\mu = 10$ days, there is a .59 probability of finding convincing evidence for H_a: $\mu \neq 10$

TRUTH

Decision	H_0 true	H_a true
reject H_0	Type I error α	POWER (accurate)
fail to reject H_0	Accurate	Type II error β

Power of a Significance Test

Important ideas:

Interpretation

If the true mean cold length is $\mu = 10$ days, there is a .59 probability of finding convincing evidence for H_a: $\mu \neq 10$

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$= P(\text{reject } H_0 \mid H_a \text{ true})$

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Decision	H_o true	H_a true
reject H_o	Type I error α	POWER (accurate)
fail to reject H_o	Accurate	Type II error

$$\text{POWER} = 1 - \beta$$

Power is a probability...

that you are doing the right thing when H_o is not true

(and the right thing in that case is to reject H_o)

Power:

= P(avoiding Type II error)

If the alternative H_a is a good thing, then **Power is good.**

If the alternative IS true, we want to maximize the prob. of finding convincing evidence that it is true.

Increasing Power

By increasing the .....

By Increasing the ....

By increasing the difference between ......

Applet

Increasing Power

By increasing the ..... sample size

gives more info about the true parameter

By Increasing the .... α size $5\% \rightarrow 10\%$

makes it easier to reject H_0

By increasing the difference between ......

the null and and alternative parameter

on to:

Preventing ADD

Preventing ADHD

The Centers for Disease Control and Prevention claims that 11% of American children, ages 4–17, have attention deficit/hyperactivity disorder (ADHD). A company claims that it has developed a new vitamin tablet that will lower a child's risk for ADHD. Researchers will administer the vitamin tablet to 200 volunteer children under the age of 4 (with parental consent). The subjects will be tracked through childhood, and the researchers will record the proportion of the subjects who develop ADHD. The researchers will perform a test at the $\alpha = 0.05$ significance level of

$$H_0 : p = 0.11$$

$$H_\alpha : p < 0.11$$

where p = the true proportion of all children like those in the study who would develop ADHD when given the new vitamin tablet. The new vitamin tablet is expensive to produce, so researchers would like to be convinced that it really does reduce the risk of ADHD. The power of the test to detect that $p = 0.05$ is 0.937.

1. **Interpret this value in context.**

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If the true proportion of all children like those in the study who would develop ADHD when given the new vitamin is $p = .11$, there's a .937 probab. that the company will find convincing evidence for $H_a: p < 0.11$

2. Find the probability of a type I error and the probability of a Type II error for the test.

1. Interpret this value in context.

If the true proportion of all children like those in the study who would develop ADHD when given the new vitamin is $p = .11$, there's a .937 probab. that the company will find convincing evidence for $H_a: p < 0.11$

2. Find the probability of a type I error and the probability of a Type II error for the test.

Type I error $= \alpha = 0.05$

Type II error $= 1 - \text{POWER} = 1 - .937 = 0.063$

3. Determine whether each of the following changes would increase or decrease the power of the test. Explain your answers.

(a) Use $\alpha = 0.10$ instead of $\alpha = 0.05$

Increase Power

Using a larger significance level makes it easier to reject H_0 when H_a is true

(b) If the true proportion is $p = 0.08$ instead of $p = 0.05$

(c) Use $n = 500$ instead of $n = 200$

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Increase P

It is harder to detect a smaller difference between the null and alternative parameter value

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(c) Use $n = 500$ instead of $n = 200$

Increase Power

larger sample size gives more info about the true proportion p.

LCQ 9.2

9.3....81, 85, 87, 93, 95, 97, 102-108

and read/study pp.595-604