# Michigan's next water crisis is PFAS and you may already be affected 

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> Pick Up the
> harm UP

Warm Up

AP Exam Review Question




Given the above two histograms, which of the following statements is incorrect?
(A) Both sets have the'same mean.
(B) Both sets have the same median.
(C) Both sets have the same range.
$\rightarrow$ (D) Set $A$ has a greater variance than does set $B$.
(E) Each set has approximately 12 elements ( 6 under and 6 over the median).

Answer: (E) Both sets are symmetric about 30, so they have the same mean and median. Both sets have the range $50-10=40$. Set $A$ has a higher percentage of values further from the mean than does set $B$, so set $A$ has a greater variance. Histograms give relative frequencies, not actual numbers.

## A short video

$\checkmark$ INTERPRET a Type I error and a Type II error in context. GIVE a consequence of each error in a given setting.

The Wolverine Worldwide (a shoe company in Rockford) improperly disposed of chemicals (PFAS), which have leaked into the ground water. The state's drinking water limit of 70 parts per trillion (ppt) is considered safe, while anything above 70 ppt is considered dangerous Officials believes the water in Rockford may be unsafe. They take a random sample of 200 households in Rockford. They find the average lead level of the sample is 70.5 ppt .

1. State appropriate hypotheses for performing a significance test using words and symbols.
2. After conducting a significance test, a $P$-value of 0.045 is found. Interpret this value.

3. Based on the P-value, should Rockford keep the current water or switch to bottled water? Explain.

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1. State appropriate hypotheses for performing a significance test using words and symbols.

$$
H_{0}: \mu=70 \text { ppt. ; The water is safe. } \quad \alpha=0.05
$$

$$
H_{a}: \mu>70 \text { pot. : The water is unsafe. }
$$

2. After conducting a significance test, a P -value of 0.045 is found. Interpret this value. Assuming that the water is safe $(\mu=70)$ there is a 0.045 probability of getting a sample mean of 70.5 ppt . or more purely by chance.
3. Based on the P-value, should Rockford keep the current water or switch to bottled water? Explain.
4. Let's suppose this decision is wrong. What would be a consequence of this error?
5. Given the water is safe, how often will this error occur?
6. Based on the P-value, should Rockford keep the current water or switch to bottled water? Explain.

## They should switch to bottled water since we have

 convincing evidence for the alternative hypothesis.4. Let's suppose this decision is wrong. What would be a consequence of this error?
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6. Based on the P-value, should Rockford keep the current water or switch to bottled water? Explain. They should switch to bottled water since we have convincing evidence for the alternative hypothesis.
7. Let's suppose this decision is wrong. What would be a consequence of this error? They would waste money and resources

8. Given the water is safe, how often will this error occur?
$5 \%$ of the time we get statistically significant results purely by chance.


9. Now suppose the P -value was 0.14 . Should the town keep the current water or switch to bottled water?

10. Let's suppose this decision is wrong. What would be a consequence of this error?
11. Are the consequences in question \#4 or question \#7 more serious? Explain.
12. Now suppose the P-value was 0.14 . Should the town keep the current water or switch to bottled water? They should keep the current water since they don't have convincing evidence against the null.
13. Let's suppose this decision is wrong. What would be a consequence of this error?

14. Are the consequences in question \#4 or question \#7 more serious? Explain.
15. Now suppose the P-value was 0.14 . Should the town keep the current water or switch to bottled water?

They should keep the current water since they don't have convincing evidence against the null.
7. Let's suppose this decision is wrong. What would be a consequence of this error?

People would drink unsafe water and could get sick / possibly die.

 People would drink unsafe water and could get sick / possibly die.


8. Are the consequences in question \#4 or question \#7 more serious? Explain.
\#7. People getting sick is much worse than wasting money.

Type 1 and Type 2 Errors
When we draw conclusions from a significance test we hope our conclusion will be correct. But sometimes it will be wrong. There are two types of errors we can make.

| Typorant ideas: |
| :--- |
| Type 2 error: |
|  |


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$$
\begin{aligned}
& \text { Type I } \\
& \text { error } \\
& \text { f Ho is true } \\
& \left\{\begin{array}{l}
\text { Type I } \\
\text { occurs of } \\
\text { 5 the time }
\end{array}\right.
\end{aligned}
$$

6. Now suppose the P-value was 0.14 . Should the town keep the current water or switch to bottled water? They should Keep the current water since they dort have convincing evidence against the null.
7. Let's suppose this decision is wrong. What would be a consequence of this error?
8. Are the consequences in question \#4 or question \#7 more serious? Explain.

$$
\left.\left.\int_{\text {in }}\right\} \begin{array}{c}
\text { Type II } \\
\text { error } \\
H_{a} \text { is there }
\end{array}\right\}
$$ could get sick/possibly die.

\#7. People getting sick is much worse than wasting money.

If a type I error has more serious consequences, consider using a smaller $\alpha$ level

If a Type $I I$ error has more ${ }^{\text {serous }}$ consequences, consider using a larger $\alpha$


Truth about the population

Conclusion
based on sample

|  | Reject $H_{0}$ | Type I error |
| ---: | ---: | :---: |
| Correct <br> conclusion |  |  |
| Fail to reject $H_{0}$ | Correct <br> conclusion | Type II error |
|  |  |  |

If $\boldsymbol{H}_{\mathbf{0}}$ is true:

- Our conclusion is correct if we don't find convincing evidence that $H_{a}$ is true.
- We make a type I error if we find convincing evidence that $H_{a}$ is true.

If $\boldsymbol{H}_{\mathbf{0}}$ is false:

- Our conclusion is correct if we find convincing evidence that $H_{a}$ is true.
- We make a type II error if we don't find convincing evidence that $\overline{H_{a} \text { is }}$ true.

They are conditional probabilities

$$
\begin{aligned}
& P(\text { Type } 1 \text { error })=P\left(\text { reject } H_{0} \mid H_{0} \text { is true }\right) \\
& P(\text { Type } \| \text { error })=P\left(\left.\begin{array}{l}
\text { fail to } \\
\text { fact Ho }
\end{array} \right\rvert\, H_{\text {a is }} \text { true }\right)
\end{aligned}
$$

## Check Your Understanding

The manager of a fast-food restaurant wants to reduce the proportion of drivethru customers who have to wait longer than 2 minutes to receive their food after placing an order. Based on store records, the proportion of customers who had to wait longer than 2 minutes was $p=0.63$. To reduce this proportion, the manager assigns an additional employee to drive-thru orders. During the next month, the manager collects a random sample of 250 drive-thru times and finds that $\hat{p}=\frac{144}{250}=0.576$.
The manager then performs a test of the following hypotheses at the $\alpha=0.10$ significance level:

$$
\begin{aligned}
& H_{0}: p=0.63 \\
& H_{a}: p<0.63
\end{aligned}
$$

where $p=$ the true proportion of drive-thru customers who have to wait longer than 2 minutes to receive their food.

1. Describe a Type I error and a Type II error in this setting.
2. Describe a Type I error and a Type II error in this setting.

TYPE I:

## TYPE II:

2. Which type of error is more serious in this case? Justify your answer.
3. Describe a Type I error and a Type II error in this setting.

TYPE I: $63 \%$ of the customers wait longer than 2 min , but the manager thinks there is less than that.
TYPE II: Less than $63^{\prime \prime}$ wait but the manager
2. Which type of error is more serious in this case? Justify your answer.

Type I because the manager believes the extra employee reduces the proportion of customers who wait but it does nat? Anyone feel different about this?
3. Based on your answer to Question 2, do you agree with the company's choice of $\alpha=0.10 ?$ Why or why not?
4. The $P$-value of the manager's test is 0.0385 . Interpret the $P$-value.
3. Based on your answer to Question 2, do you agree with the company's choice of $\alpha=0.10$ ? Why or why not?
perhaps not. If the null is true, $\alpha=0.10$ will result in a Type I error $10 \%$ of the time just by chance. They should use a smaller p-value.
4. The $P$-value of the manager's test is 0.0385 . Interpret the $P$-value.

Assuming
3. Based on your answer to Question 2, do you agree with the company's choice of $\alpha=0.10$ ? Why or why not?
Perhaps not. If the null is true, $\alpha=0.10$ will result in a Type I error $10 \%$ of the time just by chance. They should use a smaller p-value.
4. The $P$-value of the manager's test is 0.0385 . Interpret the $P$-value.

Assuming the proportion of customers who have to wait is $p=0.63$, there is a 0.0385 prob. of getting a sample prop of 0.5 76 or less parley by chance.

We can decrease the probability of making a Type I error in a significance test by using a smaller significance level.

But there is a trade-off between $P$ (Type I error) and $P$ (Type II error): as one increases, the other decreases.

If we make it more difficult to reject $H_{0}$ by decreasing $\alpha$, we increase the probability that we will not find convincing evidence for $H_{a}$ when it is true.

HW \#15 on last night's assignment
Because the $P$-value of .1265 is greater
nouuct anu merpiet a solo connaence interval tor the true mean weight of cereal in the boxes filled by s machine. Show details as done in class.
tate: parameter: true mean $M$ weight of cereal in boxes filled. confidence level:90\% by this pence machine.
lan: one sample $t$ interval for $M$.
conditions: Careful
$\square$ Random: "SRE of 30 boxes"
(1) $10 \%$ : $n=30 \leqslant 0 \%$ of all cereal boxes

- Normal: $n \geq 30$ GLT

00: General formula for $C I_{n}$ : $C I_{n}$ : $P E \pm M O E$

$$
\begin{aligned}
& \text { specific formula for } C I_{x}: C I_{*}=P E=t * \cdot S_{x} \because t^{*}=1.699 \\
& \text { CI, } 17.92 \pm 1.699 \cdot 0.2 \quad \frac{1}{\sqrt{n}} \begin{array}{c}
\text { arch } 0.05 \\
\text { db: } 29
\end{array} \\
& C I_{N}=17.92 \pm 0.062^{\sqrt{30}} \\
& C I_{0}=(17.858,17.982)
\end{aligned}
$$

onclude: we are $90 \%$ confident that the interval from $17.8580 z$ to $17.9820 z$ captures the true mean weight $M$ of the cereal if the boxes filled by this machine.

You get good
at what you practice


# 9.1.... 21-29 (odds), 30-32 and study pp.560-562 

