

Today: Section 7.3 Day 2

Wed: Sampling Distribution of  $\mu_{\bar{x}} - \mu_{\bar{z}}$  ← from part of 10.2

Thur Cumulative Review

Fri: Review for Ch. 7 Test (includes small parts of 10.1 and 10.2)

Mon: TEST on Ch. 7

We'll start today with a video of Pardis Sabeti

00: to 07:29

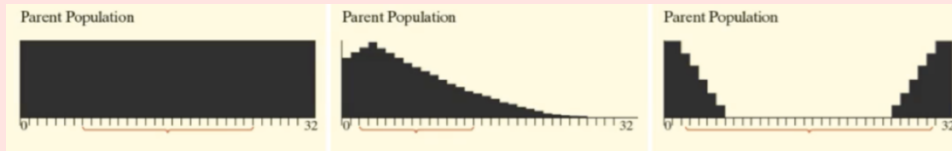
**Dr. Sabeti is a professor at the Center for Systems Biology and Department of Organismic and Evolutionary Biology at Harvard University and a professor in the Department of Immunology and Infectious Disease at the Harvard School of Public Health.**



Most Population Distributions are not Normal.

So, then the question becomes:

**"What is the shape of the sampling distribution of the sample means,  $\bar{x}$ , from a non-Normal population."**



Get a laptop. Log in. Open up a browser.

You will need to open up an applet:

Google: **online statbook sampling**

**Sampling Distributions - OnlineStatBook**

[onlinestatbook.com/stat\\_sim/sampling\\_dist/](http://onlinestatbook.com/stat_sim/sampling_dist/) ▼

This simulation lets you explore various aspects of **sampling** distributions. When the s begins, a histogram of a normal distribution is displayed at the ...



VS



The ACT test is scored with whole numbers from 0 to 36. We will use the website [www.tinyurl.com/EKstats66](http://www.tinyurl.com/EKstats66) to take samples of ACT scores from SHS and Rockford HS.

Click "Begin" and you will see the population distribution of *fake* ACT scores from SHS.

1. Describe the shape, center, and variability of the *population distribution* of ACT scores for SHS.

Approx.

$\mu =$

$\sigma =$

2. Click "Animated" to take a sample of 5 ACT scores. Look at the grey 5 grey boxes. Estimate and List the 5 the scores here: \_\_\_\_\_ Estimate their sample Mean (blue box): \_\_\_\_\_



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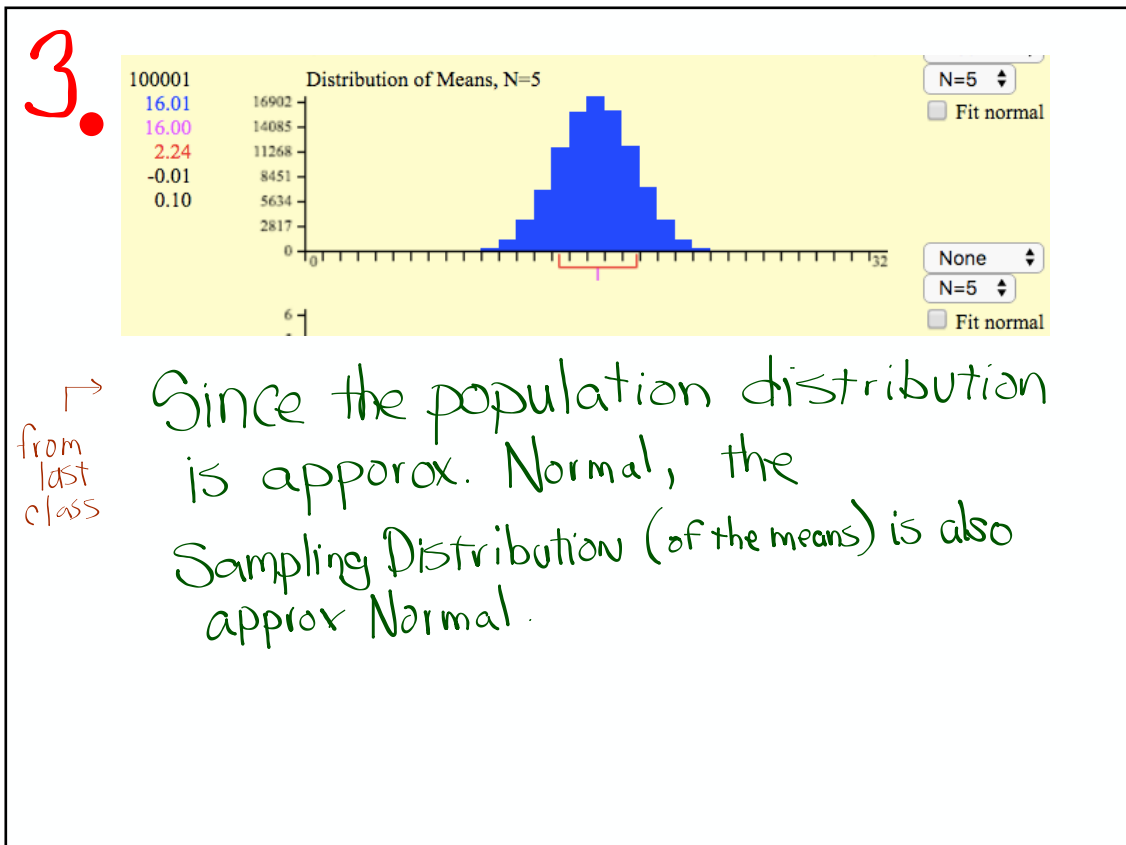
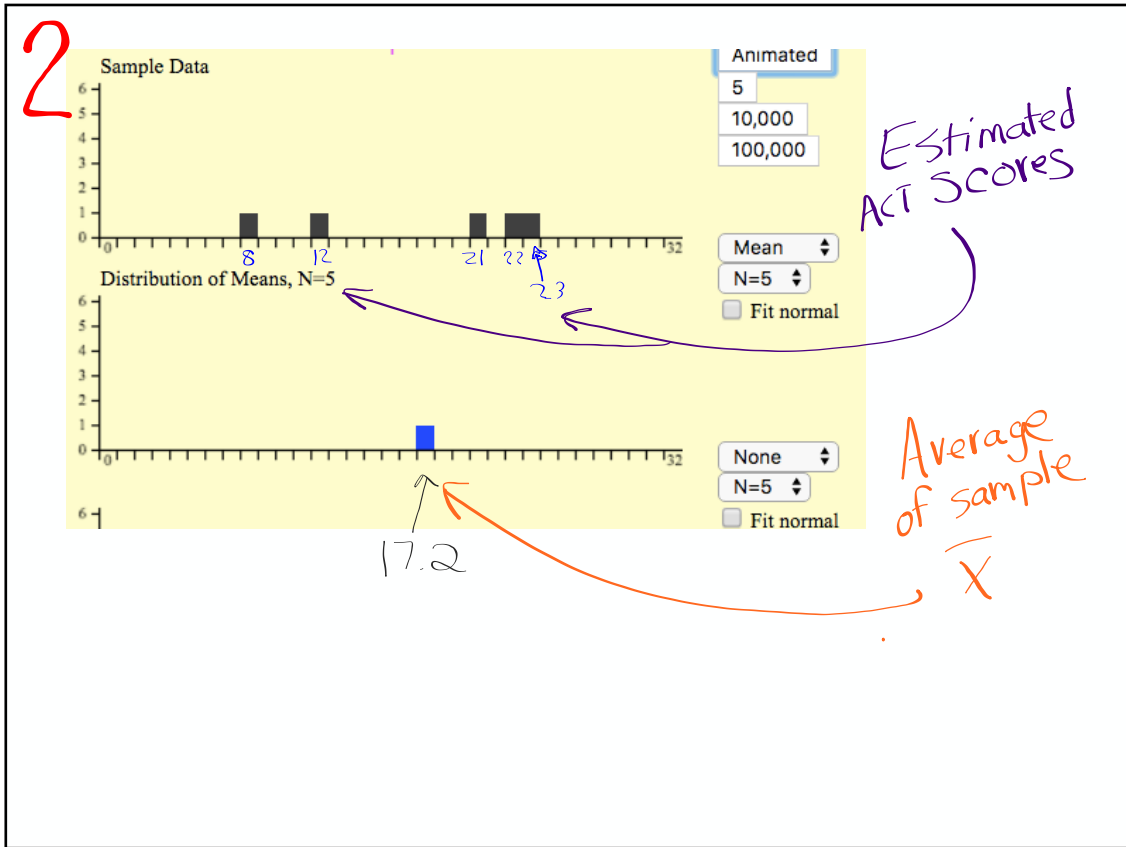
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Approx. Normal

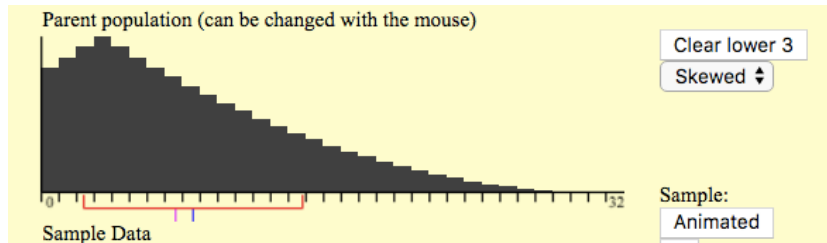
$\mu = 16$

$\sigma = 5$

2. Click "Animated" to take a sample of 5 ACT scores. Look at the grey 5 grey boxes. Estimate and List the 5 the scores here: \_\_\_\_\_ Estimate their sample Mean (blue box): \_\_\_\_\_



4.



- Skewed Right
- More students had lower ACT scores and fewer did well

5.

Mean ▾

N=2 ▾

☐ Fit normal

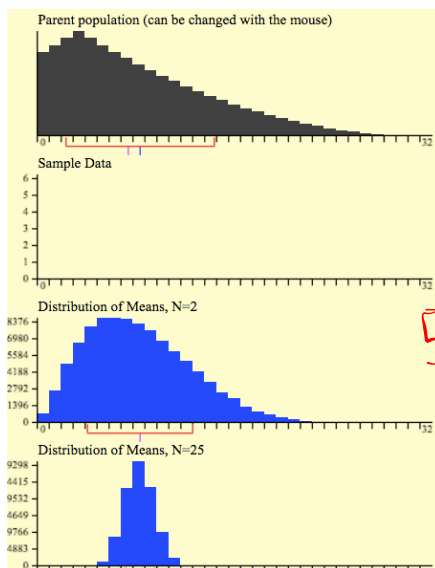
Mean ▾

N=5 ▾

☐ Fit normal

Change both of the bottom two dropdown menus to "Mean". The first one should be "N=2" and the second one should be "N=25". The click "10,000" to take 10,000 samples.

5. Describe the shape of the sampling distribution of  $\bar{x}$  when  $N = 2$ .



Skewed right, but  
not as strongly  
skewed as the  
population.

5.

6.

Approx. Normal  
Large Sample Size ↑

$n \geq ?$

## The Central Limit Theorem

Draw an SRS of size  $n$  from any population with mean  $\mu$  and finite standard deviation  $\sigma$ . The **central limit theorem (CLT)** says that when  $n$  is large, the sampling distribution of the sample mean  $\bar{x}$  is approximately Normal.

### Shape of the Sampling Distribution of the Sample Mean $\bar{x}$

- If the population distribution is Normal, the sampling distribution of  $\bar{x}$  will also be Normal, no matter what the sample size  $n$  is

## The Central Limit Theorem

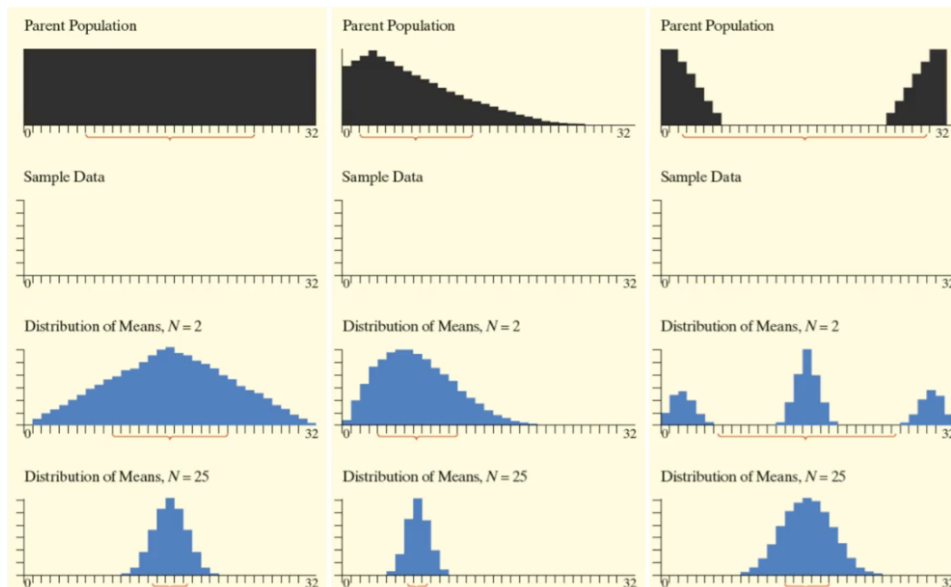
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### Shape of the Sampling Distribution of the Sample Mean $\bar{x}$

- If the population distribution is Normal, the sampling distribution of  $\bar{x}$  will also be Normal, no matter what the sample size  $n$  is
- If the population distribution is not Normal, the sampling distribution of  $\bar{x}$  will be approximately Normal when the sample size is large ( $n \geq 30$  in most cases). If the sample size is small and the population distribution is not Normal, the sampling distribution of  $\bar{x}$  will retain some characteristics of the population distribution (e.g., skewness).

## Now go back to the APP and try to create an **abnormal** Population Distribution

1. Select Custom (instead of skewed)
2. Click on a point on the population graph to insert a bar of that height OR click on a point on the horizontal axis, and drag up to define a bar. You can also shorten bars.
3. Make a distribution that looks as strange as you can.
4. Now go back and take 100,000 samples.
5. What do you notice about the sampling distributions?





## The Central Limit Theorem

Important ideas:

## The Central Limit Theorem

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CENTRAL LIMIT  
Theorem

The Sampling Distribution  
of the mean ( $\bar{x}$ ) will be  
approx. Normal if the sample  
size is large ( $n \geq 30$ )

**CLT** is only about the shape of the distribution of the sample mean.

So ..... don't refer to it as an explanation of how the variability of a sampling distribution decreases as the sample size goes up.

### The Central Limit Theorem

Important ideas:

**CENTRAL LIMIT Theorem**

The Sampling Distribution of the mean ( $\bar{x}$ ) will be approx. Normal if the sample size is large ( $n \geq 30$ )

**Probability**

If approx. Normal, then Z-score of sampling distr.

$$Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

↑  
not for Sampling Distrib. of proportions

# Auto Care-Center

**Auto Care Center:** Keith is the manager of an auto-care center. Based on service records of 3500 customers from the past year, the time (in hours), that a technician requires to complete a standard oil change and inspection follows a right-skewed distribution with  $\mu = 30$  minutes and  $\sigma = 20$  minutes. For a promotion, Keith randomly selects 40 current customers and offers them a free oil change and inspection if they redeem the offer during the next month. Keith budgets an average of 35 minutes per customer for a technician to complete the work. Will this be enough?

- (a) Describe the shape of the sampling distribution of  $\bar{x}$  for samples of 40 randomly selected customers. Justify your answer.

approx. Normal because  $40 \geq 30$  & CLT  
CLT  $40 \geq 30$

- (b) Find the mean and standard deviation of the sampling distribution of  $\bar{x}$ . Be sure to check the 10% condition. (this is a good time to point out that we are talking about sampling distribution of the mean, not a population --- formulas are not the same!)

$$\mu_{\bar{x}} = \mu \quad \mu_{\bar{x}} = 30 \text{ min}$$

$$\sigma_{\bar{x}} = \frac{20}{\sqrt{40}} = \frac{20}{6.324} = 3.16 \text{ min}$$

$$40 \geq 0.10 \checkmark$$

$$n < \frac{1}{10} \text{ of } N$$

$$40 < \frac{1}{10}(3500) \checkmark$$

$$40 < 350 \checkmark$$

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Approx. Normal by Central Limit Theorem  
since  $40 \geq 30$

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$$\mu_{\bar{x}} =$$

$$\sigma_{\bar{x}} =$$

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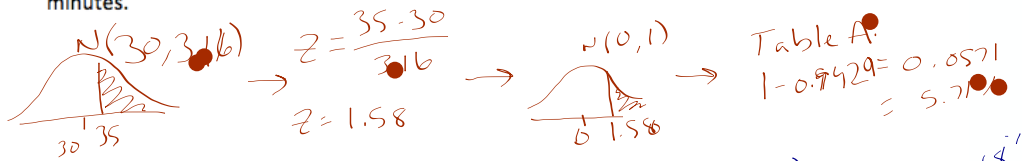
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$$\mu_{\bar{x}} = \mu = 30 \text{ min.}$$

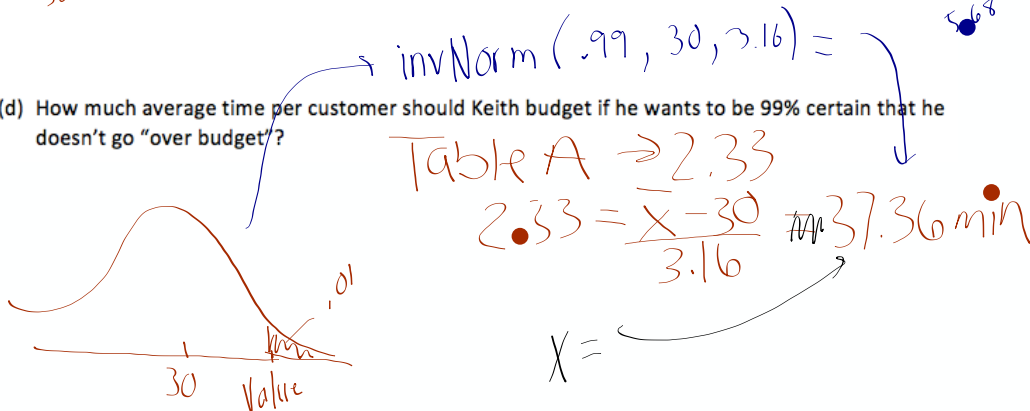
$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{20}{\sqrt{40}} = 3.16$$

10% condition  
 $40 < \frac{1}{10}(3500)$

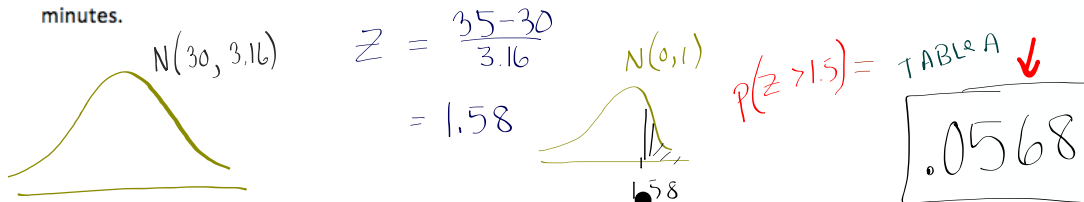
- (c) Calculate the probability that the average time it takes to complete the work exceeds 35 minutes.



- (d) How much average time per customer should Keith budget if he wants to be 99% certain that he doesn't go "over budget"?

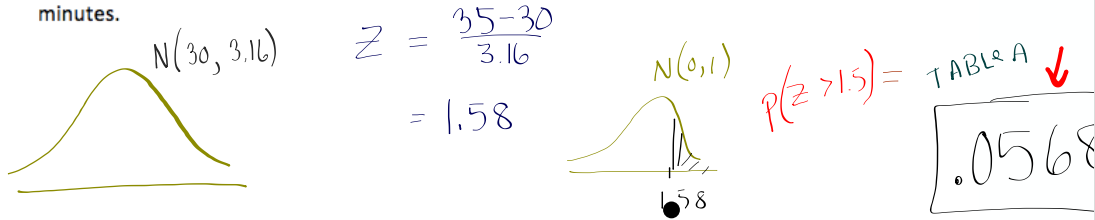


- (c) Calculate the probability that the average time it takes to complete the work exceeds 35 minutes.

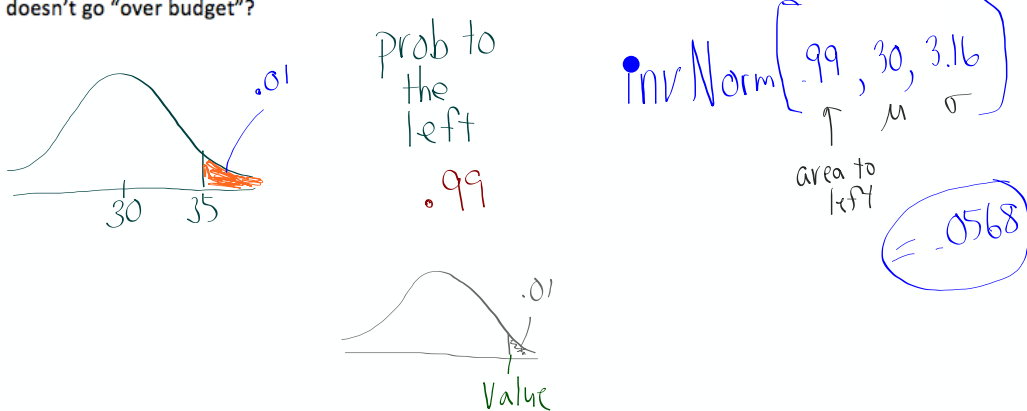


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Pardis Sabeti

07:29 to....

*7.3.....63-73 (odds) , and 74-76*

*study pp. 474-478*