Warm Up 6.3 Day 3

In a survey of 500 U.S. teenagers aged 14 to 18, subjects were asked a variety of questions about personal finance. One question asked was whether teens had a debit card. Suppose that exactly 12% of teens aged 14 to 18 have debit cards.

Let X = the number of teens in a random sample of size 500 who have a debit card.

1. Explain why **X** can be modeled by a binomial distribution even though the sample was selected without replacement.

2. Since a binomial distribution can be used, estimate the probability *(using binomial probability)* that exactly 60 teens in the sample have debit cards.

3. Use a binomial distribution to estimate the probability that 50 or fewer teens in the sample have debit cards.

NOT NECESSARILY ON AP EXAM

4. Justify why X can be approximated by a Normal distribution.

5. Use a Normal distribution to estimate the probability that 50 or fewer teens in the sample have debit cards.

Lesson 6.3: Day 4: GREED

We're going to play **Greed**. Each round you must decide if you want to sit or stand. If you sit, you keep all earned points but are no longer playing. If you stand, you must play the round. You earn 1 point for each round you make it through. Mr. Cedarlund is going to roll a die. If the die lands on a number from 1 to 5, the people standing move on to the next round and earn a point. If the die lands on a 6 the people standing lose all their points.

1. How many points did you earn?

2. Let ***X* = the number of rounds played until a 6 occurs.** Is this a binomial setting?

3. Use probability rules to calculate the probability for each of the following (knowing that it is not a binomial situation) Show work.

   a. P(X = 1)

   b. P(X = 2)

   c. P(X = 3)

   d. P(X = 4)

   e. P(X = k)

4. Write the probability that a 6 is rolled *within the first 4 rolls* in terms of *X* and find the probability. Show your work.

<u>Now Using technology</u>

5. How many rolls would you **predict** it to take until a 6 is rolled?

6. Prediction of what shape would the distribution of *X* have?

Geometric Distributions

Important ideas:

Calculator Commands:

$geometpdf\ (p, k)$ computes $P(Y = k)$

$geometcdf\ (p, k)$ computes $P(Y \leq k)$ $= P(Y = 1) + P(Y = 2) + \cdots \ldots . P(Y = k)$

Check Your Understanding

Marti decides to keep placing a $1 bet on number 15 in consecutive spins of a roulette wheel until she wins. On any spin, there's a 1-in-38 chance that the ball will land in the 15 slot. **Let T = the number of spins it takes until Marti wins.**

1. Show that T is a geometric random variable.

2. Find P(T = 3). Interpret this result.

3. How many spins do you expect it to take for Marti to win?

4. Would you be surprised if Marti won in 3 or fewer spins? Compute an appropriate probability to support your answer.

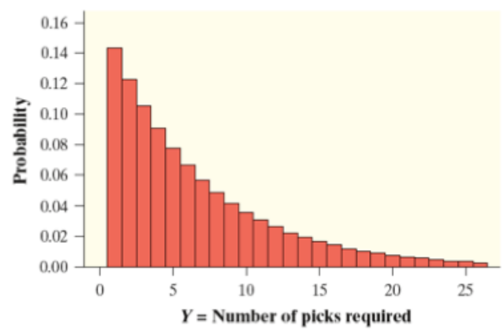
Describing a Geometric Distribution

A new topic in AP Statistics starting 2019_2020. This is not in your textbook, FYI

The table shows part of the probability distribution of Y = the number of picks it takes to match the lucky day. We can't show the entire distribution because the number of trials it takes to get the first success could be a very large number.

Value y_i	1	2	3	4	5	6	7	8	9	...
Probability p_i	0.143	0.122	0.105	0.090	0.077	0.066	0.057	0.049	0.042	

Below is a <u>histogram</u> of the probability distribution for values of Y from 1 to 26.



Let's describe what we see.

Shape: *Skewed to the right*. <u>Every</u> geometric distribution has this shape. That's because the most likely value of a geometric random variable is 1. The probability of each successive value decreases by a factor of $(1 - p)$.

Center: The mean (expected value) of Y is $\mu_Y = 7$. If the class played the Lucky Day game many times, they would receive an average of 7 homework problems. It's no coincidence that $p = 1/7$ and $\mu_Y = 7$. With probability of success 1/7 on each trial, we'd expect it to take an average of 7 trials to get the first success. That is, $\mu_Y = \frac{1}{\frac{1}{7}} = 7$.

Variability: The standard deviation of Y is $\sigma_Y = 6.48$. If the class played the Lucky Day game many times, the number of homework problems they receive would typically vary by about 6.5 problems from the mean of 7. That could mean a lot of homework! There is a simple formula for the standard deviation of a geometric random variable, but it isn't easy to explain. For the Lucky Day game,

$$\sigma_Y = \frac{\sqrt{1 - \frac{1}{7}}}{\frac{1}{7}} = 6.48$$

We can generalize the results for the mean and standard deviation of a geometric random variable. We interpret

MEAN (EXPECTED VALUE) AND STANDARD DEVIATION OF A GEOMETRIC RANDOM VARIABLE

If Y is a geometric random variable with probability of success p on each trial, then its mean (expected value) is $\mu_Y = E(Y) = \frac{1}{p}$ and its standard deviation is

$$\sigma_Y = \frac{\sqrt{1-p}}{p}$$

PROBLEM: A local fast-food restaurant is running a "Draw a three, get it free" lunch promotion. After each customer orders, a touchscreen display shows the message "Press here to win a free lunch." A computer program then simulates one card being drawn from a standard deck. If the chosen card is a 3, the customer's order is free. Otherwise, the customer must pay the bill. Let X = the number of customers it takes to get the first free order on a given day.

(a) Calculate and interpret the mean of X.

(b) Calculate and interpret the standard deviation of X.

Be sure you ask for the answer this question in the next class!