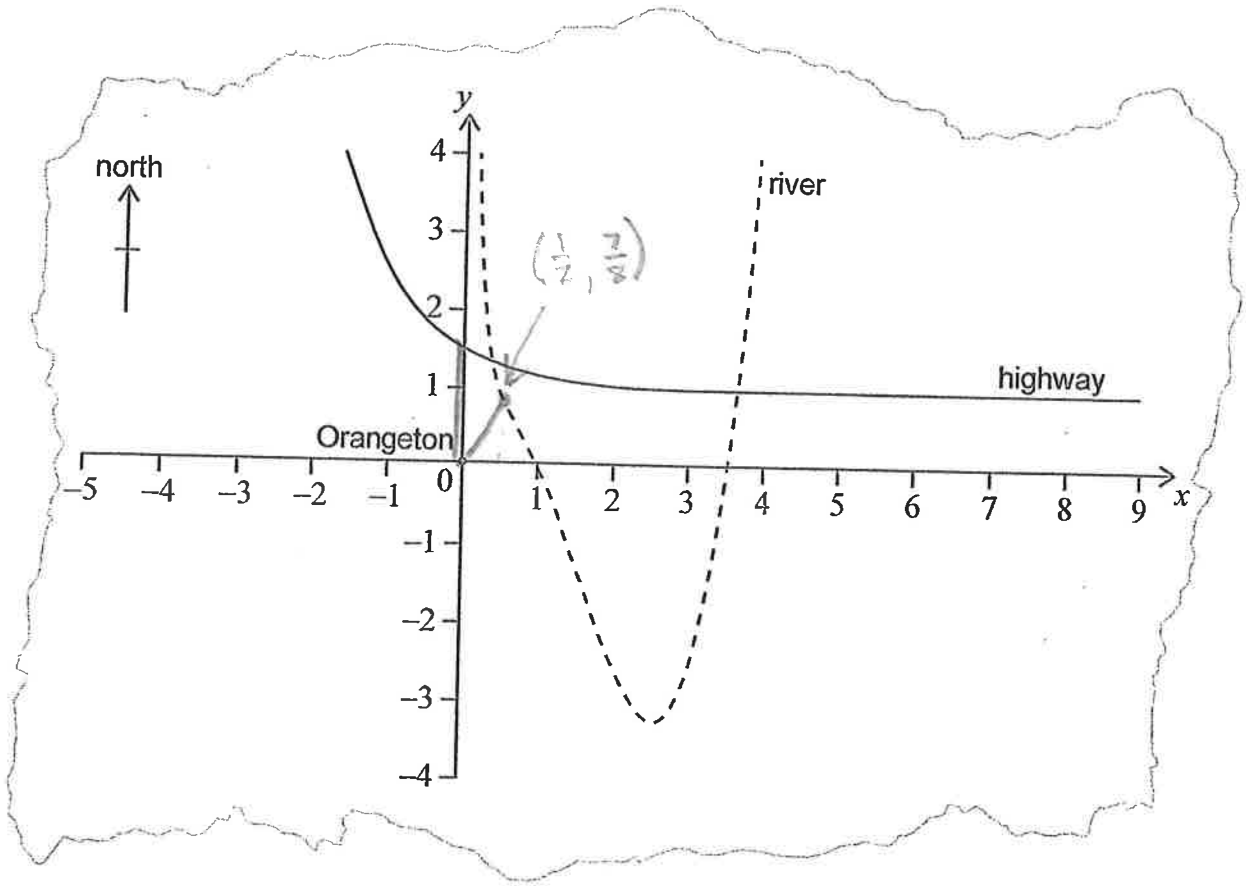
Nov 15 After Test Assignment Name Key

4. [Maximum mark: 15]

Consider the function $f(x) = x^3 - 5x^2 + 6x - 3 + \frac{1}{x}$, $x > 0$.

(a) Find the value of $f(x)$ when $x = \frac{1}{2}$. $f\left(\frac{1}{2}\right) = \frac{7}{8}$ or $.875$ [2]

The function $f(x) = x^3 - 5x^2 + 6x - 3 + \frac{1}{x}$, $x > 0$, models the path of a river, as shown on the following map, where both axes represent distance and are measured in kilometres. On the same map, the location of a highway is defined by the function $g(x) = 0.5(3)^{-x} + 1$.



The origin, $O(0, 0)$, is the location of the centre of a town called Orangeton.

A straight footpath, P, is built to connect the centre of Orangeton to the river at the point where $x = \frac{1}{2}$.

(b) (i) Find the function, $P(x)$, that would define this footpath on the map.

gradient $= \frac{0 - 7/8}{0 - 1/2} = \frac{7}{4}$

so $y = \frac{7}{4}x$

or $y = 1.75x$

(ii) State the domain of P. [5]

$0 < x < \frac{1}{2}$

(This question continues on the following page)

(Question 4 continued)

Bridges are located where the highway crosses the river. (0.360, 1.34) and (3.63, 1.01)

(c) Find the coordinates of the bridges relative to the centre of Orangeton. Use GDC [4]

A straight road is built from the centre of Orangeton, due north, to connect the town to the highway.

(d) Find the distance from the centre of Orangeton to the point at which the road meets the highway. [2]

This straight road crosses the highway and then carries on due north.

(e) State whether the straight road will ever cross the river. Justify your answer. [2]

(d) Find y-intercept of road, $g(x) = 0.5(3)^{-x} + 1$

$g(0) = 0.5\left(\frac{1}{3}\right)^0 + 1$

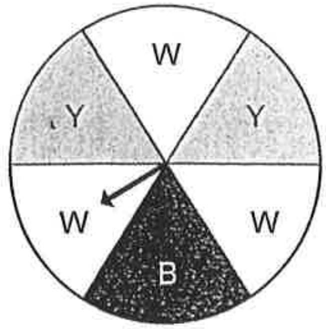
$= 1.5$

so, the distance is 1.5 km

(e) domain of river, $f(x)$, is given as $x > 0$ but the equation of the road is $x = 0$ so the road will not cross the river.

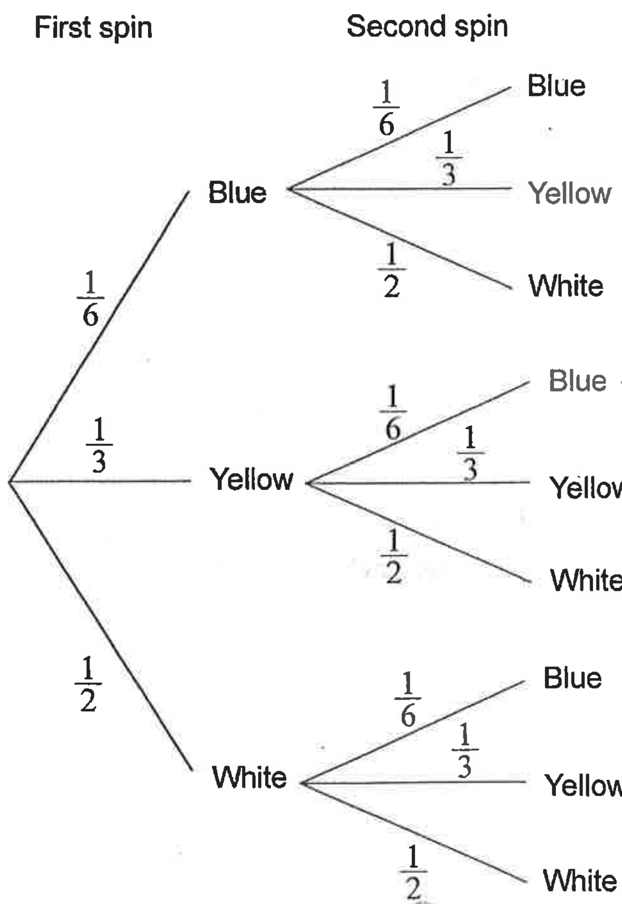
one possible answer

12. The diagram shows a circular horizontal board divided into six equal sectors. The sectors are labelled white (W), yellow (Y) and blue (B).



A pointer is pinned to the centre of the board. The pointer is to be spun and when it stops the colour of the sector on which the pointer stops is recorded. The pointer is equally likely to stop on any of the six sectors.

Eva will spin the pointer twice. The following tree diagram shows all the possible outcomes.



$\frac{1}{6} \cdot \frac{1}{3} = \frac{1}{18}$

$\frac{1}{3} \cdot \frac{1}{6} = \frac{1}{18}$

$\frac{1}{3} \cdot \frac{1}{3} = \frac{1}{9}$

$\frac{1}{3} \cdot \frac{1}{2} = \frac{1}{6}$

$\frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}$

b) $\frac{1}{18} + \frac{1}{18} + \frac{1}{9} + \frac{1}{6} + \frac{1}{6}$

$= \frac{5}{9}$

or 0.556

or 55.6%

c) $\dfrac{\frac{1}{18}}{\frac{1}{6}} = \frac{1}{3}$

(a) Find the probability that both spins are yellow. $\frac{1}{9}$ **[2]**

(b) Find the probability that at least one of the spins is yellow. 5/9 **[3]**

(c) Write down the probability that the second spin is yellow, given that the first spin is blue. **[1]**

$= 1/3$

(This question continues on the following page)

6. [Maximum mark: 14]

The function $f(x)=\frac{1}{3}x^3+\frac{1}{2}x^2+kx+5$ has a local maximum and a local minimum. The local maximum is at $x=-3$.

(a) Show that $k=-6$. [5]

(b) Find the coordinates of the local **minimum**. [2]

(c) Write down the interval where the gradient of the graph of $f(x)$ is negative. [2]

(d) Determine the equation of the normal at $x=-2$ in the form of $y=mx+c$. [5]

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a) $f'(x)=\frac{1}{3}(3)x^2+\frac{1}{2}(2)x^1+K = x^2+x+K$

MUST show $f'(x)=0$ and $x=-3$

$x^2+x+K=0$ ← MUST show

$(-3)^2+(-3)+K=0$ ←

$9-3+K=0$

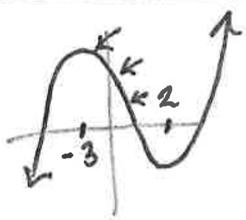
$K=-6$

b) $x^2+x-6=0$

↳ solve to get $x=-3$ and $\underline{x=2}$ ← location of minimum

$f(2)=-2.33$

Local min $(2,-2.33)$

c) 

gradient is downward sloping (−) in the interval $-3<x<2$

d) POINT OF Tangency $(-2, \frac{49}{3})$
↑
$f(-2)$

$\boxed{f'(-2)=-4}$

Possible answers $\boxed{y-\frac{49}{3}=\frac{1}{4}(x+2)}$

CONVERT

$\boxed{y=\frac{1}{4}x+\frac{101}{6}}$

or $\boxed{y=.25x+16.8}$