be sure to have your Formula packet out on your desk today

The solutions for yesterday's HW have been posted for you to check tonight. If there is time, we will go over them toward the end of the class.

Today:

New Unit on

Sequences & Series

Here's a casual definition:

A sequence is a list of numbers (or other things) that changes according to some sort of pattern.

There are finite sequences that just stop after a certain number of terms.

Like this guy:

$$-3, 1, 5, 9, 13, 17, 21$$

And there are infinite sequences that keep on going forever and ever.

Like:

$$0, 2, 4, 6, 8, 10, 12, \ldots$$

These three dots means that it keeps going.

Pick up the
Notes packet

Here's a casual definition:

A sequence is a list of numbers (or other things) that changes according to some sort of pattern.

B

- **Sequence** (A second definition)

A list of numbers, called *terms*, written in a specific order. Each term has second number associated with it that relates to its position in the sequence.

it's all about the symbols

<u>Sequence</u> <u>(A second definition)</u>

A list of numbers, called <u>*terms*</u>, written in a specific order. Each term has second number associated with it that relates to its position in the sequence.

With numbers, we usually assign each spot with a special symbol:

$$a_1, a_2, a_3, a_4, a_5, \ldots, a_n, \ldots$$

Each of these is called a "term."

This is called the n^{th} term.

n^{th} term

$a_1, a_2, a_3, a_4, \ldots, a_n, \ldots$

C

The n^{th} term is given by a formula. We can use this formula to build the sequence.

a_n is known as the U_n

explicit formula

Let's build the sequence whose n^{th} term is given by

$$a_n = 2^n - 3n$$

$a_1 =$

If we let $n=1$, we'll get the first term of the sequence:

$n=1 \rightarrow \quad a_1 = 2^1 - 3(1) = -1$

If we let $n=2$, we'll get the second term:

$$a_2 = 2^2 - 3(2) = -2$$

If we let $n=3$, we'll get the third term:

$$a_3 = 2^3 - 3(3) = -1$$

and so on...

$$a_4 = 2^4 - 3(4) = 4$$

$$a_5 = 2^5 - 3(5) = 17$$

$$a_6 = 2^6 - 3(6) = 46$$

$$\vdots$$

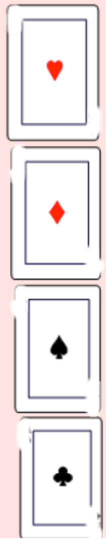
So, our sequence is

$$-1, -2, -1, 4, 17, 46, \ldots$$

For this class, you will be responsible for two types of sequences for the most part.

a) Arithmetic Sequences
b) Geometric Sequences

Arithmetic Sequences



2, 5, 8, 11, ...

2,6,10,14,18,...

15,11,7,3,...

19,16,13,10,...

d

Geometic Sequences

$r = 2$ 5, 10, 20, 40, ...

$r = .25$ 80, 20, 5, $\frac{5}{4}$, $\frac{5}{16}$, ...

$r = \frac{1}{4}$

$r = 2$

$r = \frac{1}{100}$ 100, 1, .01, .0001, ...

$r = .01$

Aim today

Create and use an explicit formula for
ARITHMETIC SEQUENCES

Find the Sum of an ARITHMETIC SEQUENCE

Finding the Explicit Formula
(nth term)

for Arithmetic Sequences

U_n

a_n

$g_n =$

5, 15, 25, 35, ...

What is the common difference ? 10

Therefore, d = 10

Starting from 5, how many differences do we need to get to the 5th term?
6th term?
7th term?
50th term?

$$a_n = 5 + 10(n-1)$$

example Find the explicit formula, a_n (P)

6, 10, 14, 18, ...

1 2 3

$$a_n = 6 + 4(n-1)$$

Given an arithmetic sequence

$a_1, a_2, a_3, \ldots, a_n, \ldots$

that has a difference of d,

the n^{th} term is

$$\boxed{a_n = a_1 + (n-1)d}$$

or as shown in the Formula Packet

$$U_n = U_1 + (n-1)d$$

1.7

~~2.5~~	The n^{th} term of an arithmetic sequence	$u_n = u_1 + (n-1)d$
	The sum of n terms of an arithmetic sequence	$S_n = \frac{n}{2}(2u_1 + (n-1)d) = \frac{n}{2}(u_1 + u_n)$

$$U_n = U_1 + (n-1)d$$

from Algebra $t(n) =$

$t_1 =$

5

Find the 85th term of the sequence
3, 8, 13, 18, ...

$$U_n = 3 + 5(n-1)$$

$$U_{85} = 3 + 5(84)$$

$$U_{85} = 423$$

example from a different angle

The 32nd term of a ____________sequence is 349.
The first term is 8. What is the common difference ?

it depends

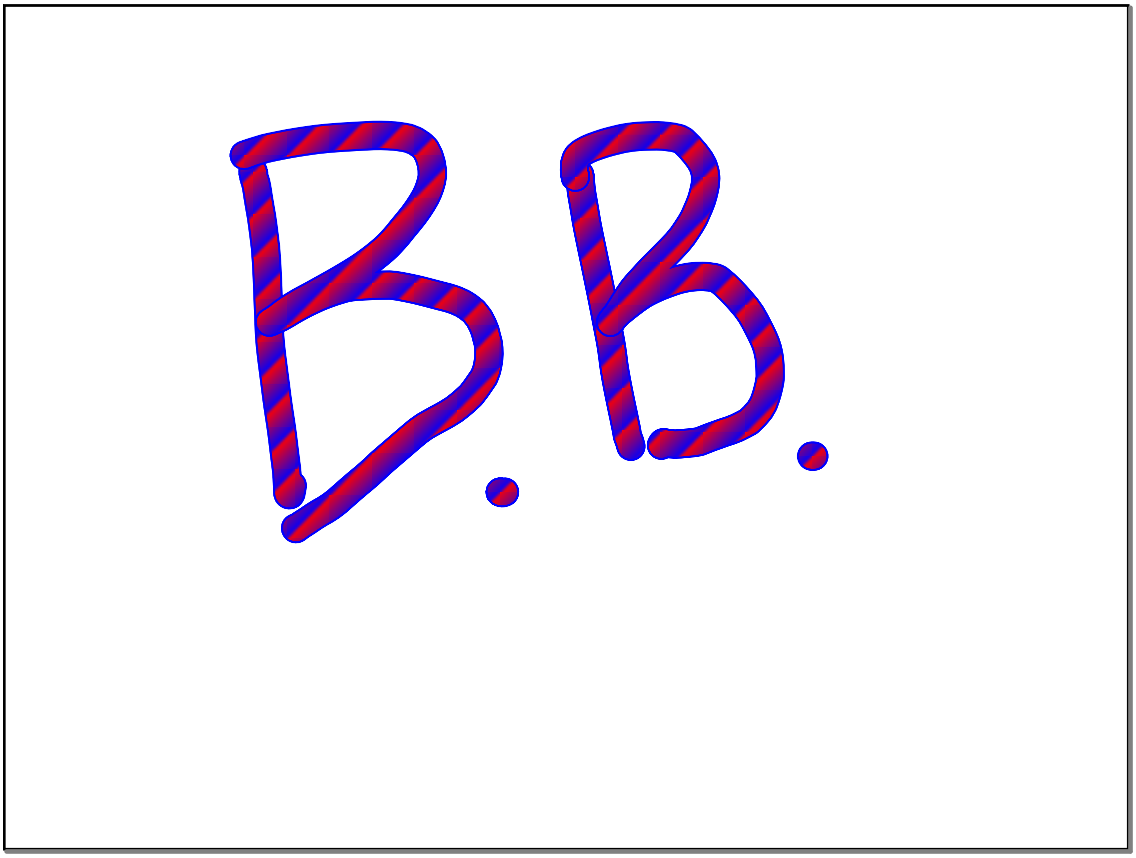
arithmetic ?

geometric ?

The 32nd term of an **ARITHMETIC** sequence is 349. The first term is 8. What is the common difference ?

$$U_n = U_1 + d(n-1)$$

$$349 = 8 + d(32-1)$$


B.B.


Series

A series is the sum of a sequence.

Here's a sequence:

1, 2, 3, 4, 5

Here's the corresponding series:

1 + 2 + 3 + 4 + 5

A true story about Carl Friedrich Gauss

Here was the day's problem:

Add the integers from 1 to 100.

They got out their slate boards and chalk and started hammering away!

$1+2=3$ $3+3=6$ $6+4=10$

$10+5=15$ $15+6=21$

$21+7=28$ $28+8=36$ $36+9=45$

$1+2+3+4+\dots+50+51+\dots+97+98+99+100$

There's a pattern here!

Check this out:

$1+2+3+4+\dots+50+51+\dots+97+98+99+100$

101

101

101

101

101

There are 50 pairs of 101...

That's $50(101)=5050$

Pick Up Notes
on SERIES

W Find the sum of the integers from 1 ~~to 40~~

11 to 40

11, 12, 13 . . . . , 38, 39, 40

51

51

51

Generalize

Find the sum of the terms in the sequence from $\mathbf{a_1}$ to $\mathbf{a_n}$ using the same method if there are $\boldsymbol{n}$ terms.

$a_1, \cdot \cdot \cdot \cdot \; \cdot \cdot \cdot \, a_n$

1, 2, 3 . . . . 98, 99, 100

$$S_n = \frac{n}{2} \cdot (a_1 + a_n)$$

$$S_{100} = \frac{100}{2}(1 + 100)$$

Arithmetic Series

To find the sum of the first n terms:

$$a_1 + a_2 + a_3 + \ldots + a_n$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

*

IB Formula Packet:

10, 11, 12, ...

S_{7000}

$S_n = \frac{n}{2}(u_1 + u_n)$

$U_n = u_1 + d(n-1)$

$S_n = \frac{n}{2}\left(u_1 + u_1 + d(n-1)\right)$

$S_n = \frac{n}{2}\left[2u_1 + d(n-1)\right]$

The cool thing about his formula is that it works on an ODD number of terms

3 + 13 + 23 + 33 + 43

X

remember → **Find the 85th term of the sequence 3, 8, 13, 18, ...**

Instead, find the sum of all 85

Find the 85th term first and then find the sum

OR → just find the sum directly

Y

a) Determine the number of terms in the sequence
$24, 23\frac{1}{4}, 22\frac{1}{2}, \ldots, -36$

b) Then find the sum.

c) Then find only the 35^{th} term.

$24 + 23\frac{1}{4} + 22\frac{1}{2} + \ldots + {}^{-}36$

Find the Sum of the Series

a) Determine the number of terms in the sequence

$24, 23\frac{1}{4}, 22\frac{1}{2}, \ldots, \ldots {}^{-}36$

b) Then find the sum.

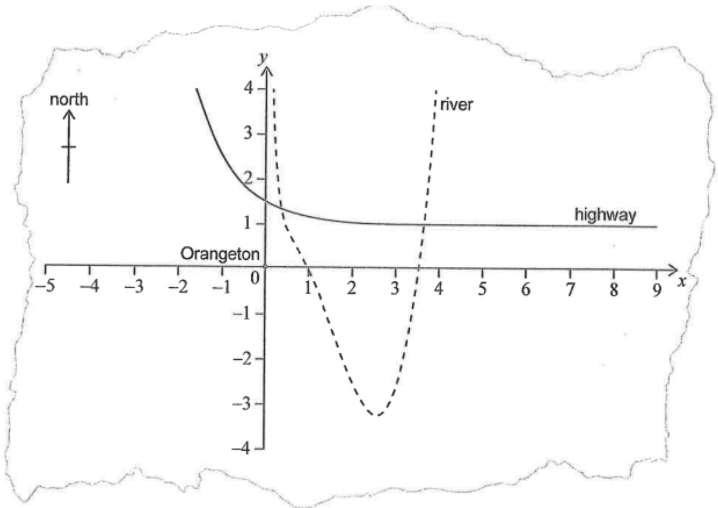
c) Then find only the 35th term.

4. [Maximum mark: 15]

Consider the function $f(x)=x^3-5x^2+6x-3+\frac{1}{x}$, $x>0$.

(a) Find the value of $f(x)$ when $x=\frac{1}{2}$.

The function $f(x)=x^3-5x^2+6x-3+\frac{1}{x}$, $x>0$, models the path of a river, as shown on the following map, where both axes represent distance and are measured in kilometres. On the same map, the location of a highway is defined by the function $g(x)=0.5(3)^{-x}+1$.

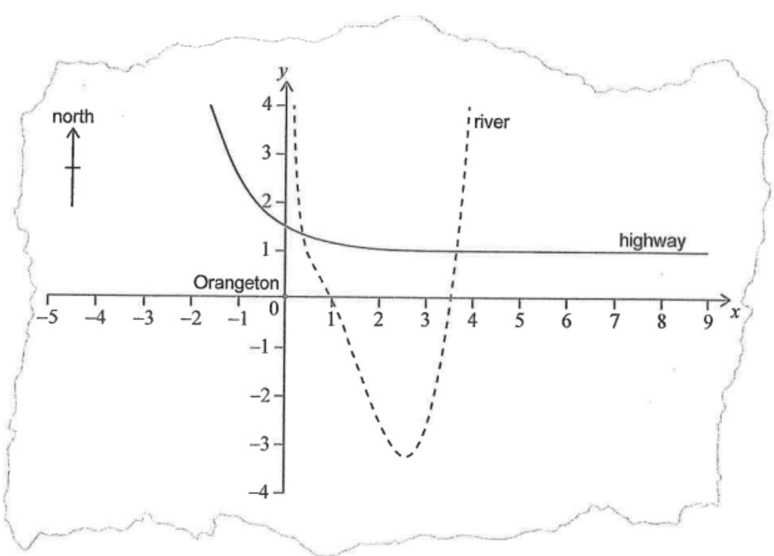


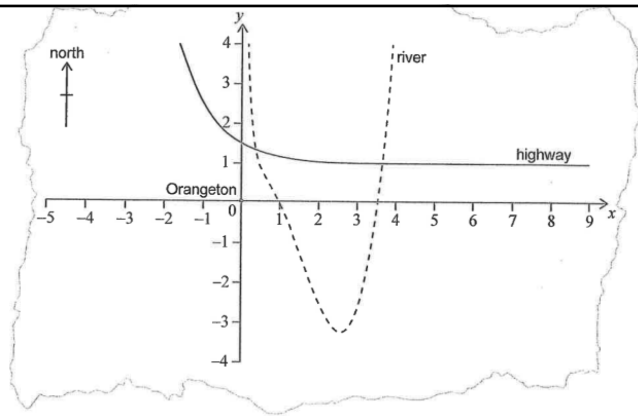
The origin, $\mathrm{O}(0, 0)$, is the location of the centre of a town called Orangeton.

A straight footpath, P, is built to connect the centre of Orangeton to the river at the point where $x=\frac{1}{2}$.

(b) (i) Find the function, $P(x)$, that would define this footpath on the map.

(ii) State the domain of P.





Bridges are located where the highway crosses the river.

(c) Find the coordinates of the bridges relative to the centre of Orangeton. [4]

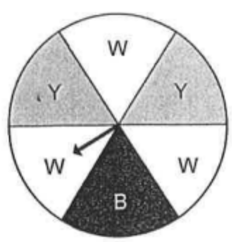
A straight road is built from the centre of Orangeton, due north, to connect the town to the highway.

(d) Find the distance from the centre of Orangeton to the point at which the road meets the highway. [2]

This straight road crosses the highway and then carries on due north.

(e) State whether the straight road will ever cross the river. Justify your answer. [2]

12. The diagram shows a circular horizontal board divided into six equal sectors. The sectors are labelled white (W), yellow (Y) and blue (B).

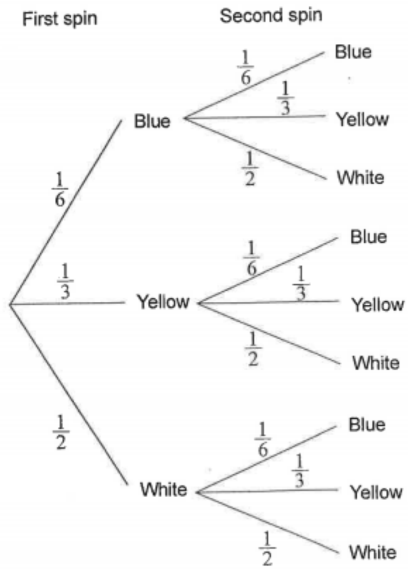


A pointer is pinned to the centre of the board. The pointer is to be spun and when it stops the colour of the sector on which the pointer stops is recorded. The pointer is equally likely to stop on any of the six sectors.

g

November 14, 2019

Eva will spin the pointer twice. The following tree diagram shows all the possible outcomes.



(a) Find the probability that both spins are yellow. [2

(b) Find the probability that at least one of the spins is yellow. [3

(c) Write down the probability that the second spin is yellow, given that the first spin is blue. [1

6. [Maximum mark: 14]

The function $f(x)=\frac{1}{3}x^3+\frac{1}{2}x^2+kx+5$ has a local maximum and a local minimum. The local maximum is at $x=-3$.

(a) Show that $k=-6$. [5]

(b) Find the coordinates of the local **minimum**. [2]

(c) Write down the interval where the gradient of the graph of $f(x)$ is negative. [2]

(d) Determine the equation of the normal at $x=-2$ in the form of $y=mx+c$. [5]

Assignment

Worksheet

Use good notation

The solutions for yesterday's HW have been posted for you to check tonight.