Pick Up the Warm Up

read the answers that are already there to see if they make sense.

Then we'll go over #4 and #5 together.

Warm Up 6.3 Day 3

In a survey of 500 U.S. teenagers aged 14 to 18, subjects were asked a variety of questions about personal finance. One question asked was whether teens had a debit card. Suppose that exactly 12% of teens aged 14 to 18 have debit cards.

Let X = the number of teens in a random sample of size 500 who have a debit card.

1. Explain why **X** can be modeled by a binomial distribution even though the sample was selected without replacement.

When taking a random sample of size n from a population of size N, we can use a binomial distribution to model the count of successes in the sample as long as $n < 0.10N$. We refer to this as the **10% condition**.

1. Explain why X can be modeled by a binomial distribution even though the sample was selected without replacement.

10% Condition As long as $n < 0.10N$ we can use a binomial distribution

500 < 10% of Population of teenagers, so ok.

2. Since a binomial distribution can be used, estimate the probability *(using binomial probability)* that exactly 60 teens in the sample have debit cards.

3. Use a binomial distribution to estimate the probability that 50 or fewer teens in the sample have debit cards.

2. Since a binomial distribution can be used, estimate the probability *(using binomial probability)* that exactly 60 teens in the sample have debit cards.

$P(X=60) = {}_{500}C_{60}\,(.12)^{60}(.88)^{440} = .055$

or binompdf(n: 500, p: .12, K: 60) = same

3. Use a binomial distribution to estimate the probability that 50 or fewer teens in the sample have debit cards.

$P(X \le 50) =$ binomcdf(n: 500, p: .12, value: 50)

up to 50

$= 0.093$

same as $P(0) + P(1) + P(2) + \ldots\ldots P(50)$

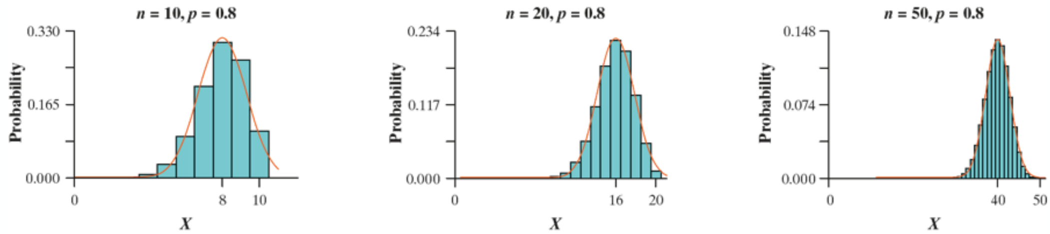
NOT ON AP EXAM

4. Justify why X can be approximated by a Normal distribution.

5. Use a Normal distribution to estimate the probability that 50 or fewer teens in the sample have debit cards.

The Normal Approximation to Binomial Distributions*

As the number of observations n becomes larger, the binomial distribution gets close to a Normal distribution.



$np = 8$

$n(1-p) = 2$

$10(.2)$

$np = 16$

$n(1-p) = 4$

$np = 40$

$n(1-p) = 10$

$50(.2)$

Large Counts Condition

Suppose that a count X of successes has the binomial distribution with n trials and success probability p. The **Large Counts condition** says that the probability distribution of X is approximately Normal if

$$np \geq 10 \text{ and } n(1-p) \geq 10$$

That is, the expected numbers (counts) of successes and failures are both at least 10.

3. Justify why X can be approximated by a Normal distribution.

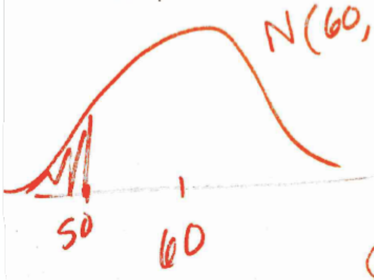
$500 \times .12 = 60 \geq 10$ ✓

$500 \times .88 = 440 \geq 10$ ✓

Large counts condition was met.

4. Use a Normal distribution to estimate the probability that 50 or fewer teens in the sample have debit cards.

$N(60, 7.27)$



$$z = \frac{x - \mu}{\sigma}$$

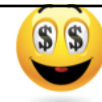
$$z = \frac{50 - 60}{7.27}$$

$$= -1.38$$

Area = .084

A game called Greed

Lesson 6.3: Day 4: GREED



We're going to play **Greed**. Each round you must decide if you want to sit or stand. If you sit, you keep all earned points but are no longer playing. If you stand, you must play the round. You earn 1 point for each round you make it through. Mr. Cedarlund is going to roll a die. If the die lands on a number from 1 to 5, the people standing move on to the next round and earn a point. If the die lands on a 6 the people standing lose <u>all</u> their points.

Practice Game

Now a real game

with some added incentive

greed

grēd/

noun

noun: **greed**

intense and selfish desire for something, especially wealth, power, food, or homework points.

For every point you end up with at the end of the game, I will add an extra point to your homework score for the chapter.

This game could end quickly, last a long time, or somewhere in between.

Remember.
Once you sit, I will write down your total at that point.

We keep going as long as at least one person is standing AND a six has not shown up.

2. Let ***X* = the number of rounds played until a 6 occurs.** Is this a binomial setting?

B

I

N

S

2. Let X = **the number of rounds played until a 6 occurs.** Is this a binomial setting?

B Success → Roll a 6
Failure → Other than a 6

I Rolls are independ.

N $n = ?$ not a set number of trials

S $p = 1/6$

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S $p = 1/6$

it's a Geometric Distrib

4.7.2

3. Use probability rules to calculate the probability for each of the following (knowing that it is not a binomial situation) Show work.

a. P(*X* = 1) $= \frac{1}{6} = .167$

b. P(*X* = 2)

c. P(*X* = 3)

d. P(*X* = 4)

e. P(*X* = k)

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a. P(*X* = 1) $= \frac{1}{6} = .167$

b. P(*X* = 2) $= \frac{5}{6} \times \frac{1}{6} = .139$

not a 6 ↑ a six

c. P(*X* = 3)

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b. $P(X = 2)$ $= \frac{5}{6} \times \frac{1}{6} = .139$

not a 6 ... a six

c. $P(X = 3)$ $= \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} = .116$

not ... a six

d. $P(X = 4)$

e. P(X = k)

•

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not ... a six

geomet pdf( )

d. $P(X = 4)$ $\frac{5}{6} \cdot \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} = .096$

geomet pdf$(\cancel{4}, \frac{1}{6})$

$(\frac{1}{6}, 4)$

e. P(X = k) 15

$\left(\frac{5}{6}\right)^{k-1} \frac{1}{6}$

•

$\left(\frac{5}{6}\right)^{15-1} \left(\frac{1}{6}\right)$

3. Use probability rules to calculate the probability for each of the following (knowing that it is not a binomial situation) Show work.

a. P(*X* = 1) $= \frac{1}{6} = .167$

b. P(*X* = 2) $= \frac{5}{6} \times \frac{1}{6} = .139$

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d. P(*X* = 4) $\frac{5}{6} \cdot \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} = .096$

e. P(*X* = k) $\left(\frac{5}{6}\right)^{k-1}\left(\frac{1}{6}\right)$

4. Write the probability that a 6 is rolled *within the first 4 rolls* in terms of *X* and find the probability. Show your work.

$P(X \le 4) = P(X=1) + P(X=2) + P(X=3) + P(X=4) = .518$

$\left(\frac{5}{6}\right)^0\left(\frac{1}{6}\right)^1 \quad \left(\frac{5}{6}\right)^1\left(\frac{1}{6}\right)^1 \quad \left(\frac{5}{6}\right)^2\left(\frac{1}{6}\right)^1 \quad \left(\frac{5}{6}\right)^3\left(\frac{1}{6}\right)^1$

Now Using technology

4. Write the probability that a 6 is rolled *within the first 4 rolls* in terms of X and find the probability. Show your work.

$P(X \le 4) = P(X=1) + P(X=2) + P(X=3) + P(X=4)$

Now Using technology

for individuals

ie. $P(X=3) = \text{geometpdf}\left(\frac{1}{6}, 3\right) =$

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$P(X \le 4) = P(X=1) + P(X=2) + P(X=3) + P(X=4)$

$\underbrace{.167 + .139 + .116 + .096}$ $= .518$

Now Using technology

for all

$P(X \le 4) = \text{geometcdf}\left(\frac{1}{6}, 4\right) = .518$

Starting from 1

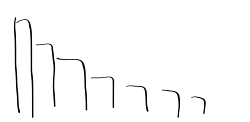
5. How many rolls would you **predict** it to take until a 6 is rolled? ~~(you won't be asked this on an AP Exam or a test in this class, FYI)~~

6. What shape would the distribution of *X* have? ~~(you won't be asked this either)~~

5. How many rolls would you **predict** it to take until a 6 is rolled? ~~(you won't be asked this on an AP Exam or a test in this class, FYI)~~

mean $\frac{1}{p} = \frac{1}{\frac{1}{6}} = 6$ rolls

6. What shape would the distribution of *X* have? ~~(you won't be asked this either)~~



Always skewed right

Geometric Distributions

Important ideas:

B

I

T

S

Calculator Commands:

$geometpdf\ (p, k)$ computes $P(Y = k)$

$geometcdf\ (p, k)$ computes $P(Y \leq k) = P(Y = 1) + P(Y = 2) + \cdots \ldots . P(Y = k)$

Geometric Distributions

Important ideas:

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I independent

T #trials until a success

S same probability

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$P(Y=k) = (1-p)^{k-1}(p)$

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$P(Y=k) = (1-p)^{k-1}(p)$

mean $= \mu = \frac{1}{p}$

std. dev $\sigma = \frac{\sqrt{1-p}}{p}$

shape : Always skewed right

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Geom. distrib. starts at 1, not 0

Check Your Understanding

Marti decides to keep placing a $1 bet on number 15 in consecutive spins of a roulette wheel until she wins. On any spin, there's a 1-in-38 chance that the ball will land in the 15 slot. **Let T = the number of spins it takes until Marti wins.**

1. Show that T is a geometric random variable.

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T number of TRIALS until Marti wins

S same prob. $p = \frac{1}{38}$

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1. Show that T is a geometric random variable.

B success → win
failure → lose

context

I spins are independent

T number of TRIALS until Marti wins

S same prob. $p = \frac{1}{38}$

2. Find P(T = 3). Interpret this result.

3. How many spins do you expect it to take for Marti to win? (not on AP exam)

always show

$(1-\frac{1}{38})^2(\frac{1}{38})$

2. Find P(T = 3). Interpret this result.

$P(T=3) = \frac{37}{38} \cdot \frac{37}{38} \cdot \frac{1}{38} = .025$

or geometpdf (p: $\frac{1}{38}$, k: 3)

There is a 2.5% probability that Marti wins for the first time on the 3rd spin.

3. How many spins do you expect it to take for Marti to win? (not on ~~AP~~ exam)

$\text{mean} = \frac{1}{\frac{1}{38}} = 38$

4. Would you be surprised if Marti won in 3 or fewer spins? Compute an appropriate probability to support your answer.

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$P(T \leq 3) =$

4. Would you be surprised if Marti won in 3 or fewer spins? Compute an appropriate probability to support your answer.

$$P(T \le 3) = P(T=1) + P(T=2) + P(T=3)$$

$$= \text{geometcdf}\left(p: \tfrac{1}{38}, k: 3\right)$$

$$= .077$$

Not too surprised. winning in 3 or fewer spins has a prob of 7.7% which is not that rare.

6.3.....107, 109, 111, 113-116

Study..... pp. 422-426

and complete the final page of today's notes including the last question.

also, consider doing ICQ 6.2 and the Ch. 6 FRAPPY! ahead of time

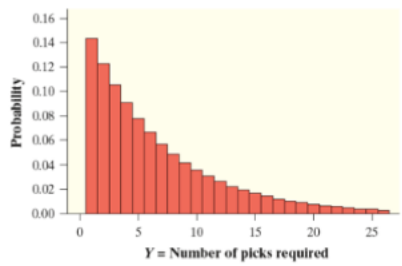
Describing a Geometric Distribution

A new topic in AP Statistics starting 2019_2020. This is not in your textbook, FYI

The table shows part of the probability distribution of Y = the number of picks it takes to match the lucky day. We can't show the entire distribution because the number of trials it takes to get the first success could be a very large number.

Value y_i	1	2	3	4	5	6	7	8	9	...
Probability p_i	0.143	0.122	0.105	0.090	0.077	0.066	0.057	0.049	0.042	

Below is a <u>histogram</u> of the probability distribution for values of Y from 1 to 26.



Let's describe what we see.

Shape: *Skewed to the right*. <u>Every</u> geometric distribution has this shape. That's because the most likely value of a geometric random variable is 1. The probability of each successive value decreases by a factor of $(1-p)$.

Center: The mean (expected value) of Y is $\mu_Y = 7$. If the class played the Lucky Day game many times, they would receive an average of 7 homework problems. It's no coincidence that $p = 1/7$ and $\mu_Y = 7$. With probability of success 1/7 on each trial, we'd expect it to take an average of 7 trials to get the first success. That is, $\mu_Y = \frac{1}{\frac{1}{7}} = 7$.

Variability: The standard deviation of Y is $\sigma_Y = 6.48$. If the class played the Lucky Day game many times, the number of homework problems they receive would typically vary by about 6.5 problems from the mean of 7. That could mean a lot of homework! There is a simple formula for the standard deviation of a geometric random variable, but it isn't easy to explain. For the Lucky Day game,

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$$\sigma_Y = \frac{\sqrt{1 - \frac{1}{7}}}{\frac{1}{7}} = 6.48$$

MEAN (EXPECTED VALUE) AND STANDARD DEVIATION OF A GEOMETRIC RANDOM VARIABLE

If Y is a geometric random variable with probability of success p on each trial, then its mean (expected value) is $\mu_Y = E(Y) = \frac{1}{p}$ and its standard deviation is

$$\sigma_Y = \frac{\sqrt{1-p}}{p}$$

PROBLEM: A local fast-food restaurant is running a "Draw a three, get it free" lunch promotion. After each customer orders, a touchscreen display shows the message "Press here to win a free lunch." A computer program then simulates one card being drawn from a standard deck. If the chosen card is a 3, the customer's order is free. Otherwise, the customer must pay the bill. Let X = the number of customers it takes to get the first free order on a given day.

(a) Calculate and interpret the mean of X.

$$\mu_X = \frac{1}{4/52} = 13$$

If the restaurant runs this lunch promotion on many days, the average number of customers it takes to get the first free order is about 13

(b) Calculate and interpret the standard deviation of X.

$$\sigma_X = \frac{\sqrt{1 - 4/52}}{4/52} = 12.49$$

If the lunch promotion runs for many many days, the Number of customers it takes to get the first free would typically vary by about 12.5 days from the mean of 13 days.