https://www.youtube.com/watch?v=uNS1QvDzCVw&feature=youtu.be

Start video right at the start of class

How many ways can you arrange a deck of cards

Today's lesson will be a bit tight for time.

Let's just say that we'll be moving swiftly. This includes going through a long warm up together (and quickly)

A video at the start of class will assist us on the first question.

How many ways can you arrange a deck of cards?

Answer: $52!$

$52 \cdot 51 \cdot 50 \cdot 49 \cdot \cdots \cdot 1$

$n! = n \cdot (n-1) \cdot (n-2) \cdot \ldots 1$

Next question:

5 friends are hanging out.

How many ways can you choose 2 people to go get food?

Translation How many "combinations" of two people can be chosen from 5?

Binomial Coefficient

The number of ways to arrange k successes among n trials

$$\binom{n}{k} = \frac{n!}{k!\,(n-k)!}$$

$$\binom{5}{2} = \frac{5!}{2!\,(3!)} = \frac{5 \cdot \cancel{4}^{2} \cdot \cancel{3 \cdot 2 \cdot 1}}{1\cancel{2} \cdot 1 \cdot \cancel{3 \cdot 2 \cdot 1}} = 10$$

$\binom{7}{3} =$

Other notation : $\binom{7}{3}$ or ${}_7C_3$

Calculator ☺

Pop Quiz

good news I's multiple choice

bad news : 10 questions

good news : There's only 5 possible answers
A B C D E

bad news : You don't get to see the questions

Answer all 10 in $1\frac{1}{2}$ minutes

you have the same prob. of success on each question.

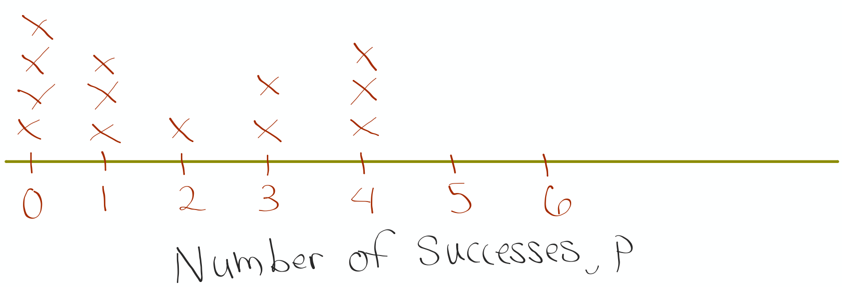
it's simple you're either right or wrong

Now Correct

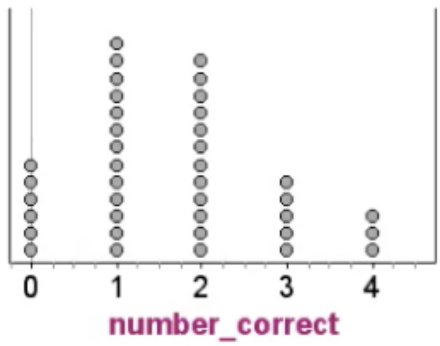
1 B 2 D 3 A 4 B 5 A

6 C 7 E 8 A 9 E 10 B

Record No. of Successes, P.



What is the center?



This is an example of a binomial Setting
(base on the information provided)

- *Each question had two outcomes (correct or not correct).*
- *The probability of getting an answer correct is 1/5.*
- *The probability remained 1/5 for each questions (questions are independent).*
- *There were ten questions. (n was fixed.)*

Binomial Settings and Binomial Random Variables

(pages 403-406)

Aim #1

DETERMINE whether the conditions for a binomial setting are met.

A binomial setting arises when we perform n independent trials of the same chance process and count the number of times that a particular outcome (called a "success") occurs.

The four conditions for a binomial setting are

•***B**inary*? The possible outcomes of each trial can be classified as "success" or "failure."

•***I**ndependent*? Trials must be independent. That is, knowing the outcome of one trial must not tell us anything about the outcome of any other trial.

•***N**umber*? The number of trials *n* of the chance process must be fixed in advance.

•***S**ame probability*? There is the same probability of success *p* on each trial.

The four conditions for a binomial setting are

•***B**inary*? The possible outcomes of each trial can be classified as "success" or "failure."

•***I**ndependent*? Trials must be independent. That is, knowing the outcome of one trial must not tell us anything about the outcome of any other trial.

•***N**umber*? The number of trials *n* of the chance process must be fixed in advance.

•***S**ame probability*? There is the same probability of success *p* on each trial.

B
I
N
S

The four conditions for a binomial setting are

- **Binary?** The possible outcomes of each trial can be classified as "success" or "failure."
- **Independent?** Trials must be independent. That is, knowing the outcome of one trial must not tell us anything about the outcome of any other trial.
- **Number?** The number of trials *n* of the chance process must be fixed in advance.
- **Same probability?** There is the same probability of success *p* on each trial.

About this

We'll try to avoid: "one trial doesn't affect another"

why: we don't want to think that the independence condition is violated only when there is a cause and effect condition.

Instead: Knowing the outcome of one trial tells us nothing about the outcome of another.

"Success" does not always mean something awesome happened

For example:

Success could be defined as a faulty part

a person being diabetic

NOTE:

The term trials can be used interchangeably with the term observations.

Dice, cars, hoops
example

Dice, cars, and hoops

Determine whether the random variables below have a binomial distribution. Justify your answer.

(a) Roll a fair die 10 times and let X = the number of 6s.

B

I

N

S

Dice, cars, and hoops

Determine whether the random variables below have a binomial distribution. Justify your answer.

(a) Roll a fair die 10 times and let X = the number of 6s.

B yes success = 6 , failure = not a 6

I Yes • knowing outcome of past rolls tells you nothing about future rolls

N Yes . # trials fixed at n = 10

S Yes p is alway $\frac{1}{6}$

This is a binomial setting. The number of 6s, *X*, is a binomial random variable with *n* = 10 and *p* = 1/6.

(b) Shoot a basketball 20 times from various distances on the court. Let Y = number of shots made.

B

I

N

S

(b) Shoot a basketball 20 times from various distances on the court. Let Y = number of shots made.

B Yes success = makes shot failure = miss

I Yes It is reasonable to assume that knowing the outcome of one shot does not change the prob. of next shot

N Yes $n = 20$ trials

S No The prob. of success changes because the shots are taken from various distances 30% 5%

So this is not a binomial random variable

(c) Observe the next 100 cars that go by and let C = color of each car.

B

I

N

S

(c) Observe the next 100 cars that go by and let C = color of each car.

B NO. there are more than two colors.

I

N

S success has NOT been defined so we cannot determine if p is same.

AP Tip

Free response questions about the binomial distribution are one of lowest scoring questions on average.

Why?

Test takers do not recognize that a binomial setting is present.

Aim #2

CALCULATE and INTERPRET probabilities involving **binomial** distributions.

Lesson 6.3: Day 1B: Is it smart to foul at the end of the game?

In the 2005 Conference USA men's basketball tournament, Memphis trailed Louisville by two points. At the buzzer, Memphis's Darius Washington attempted a 3-pointer; he missed but was fouled, and went to the line for three free throws. Each made free throw is worth 1 point. Was it smart to foul?

1. What are all the possible ways the shots could fall (e.g. make-miss-miss, etc.)?

✓✓✓ ✗✗✓ ✗✓✗ ✓✗✗ ✗✗✗ ✓✓✗ ✓✗✓ ✗✓✓

2. Darius Washington was a 72% free-throw shooter. Find the probability that Memphis will win, lose or go to overtime. When you have found the probabilities put them in the table in #3.

Win	Lose	Overtime

Lesson 6.3: Day 1B: Is it smart to foul at the end of the game?

In the 2005 Conference USA men's basketball tournament, Memphis trailed Louisville by two points. At the buzzer, Memphis's Darius Washington attempted a 3-pointer; he missed but was fouled, and went to the line for three free throws. Each made free throw is worth 1 point. Was it smart to foul?

1. What are all the possible ways the shots could fall (e.g. make-miss-miss, etc.)?

✓✓✓ ✓✓✗

2. Darius Washington was a 72% free-throw shooter. Find the probability that Memphis will win, lose or go to overtime. When you have found the probabilities put them in the table in #3.

Win	Lose	Overtime
3 makes (.72)(.72)(.72) = .373	0 makes and 3 misses 1 make and 2 misses	2 makes and 1 miss

Lesson 6.3: Day 1B: Is it smart to foul at the end of the game?

In the 2005 Conference USA men's basketball tournament, Memphis trailed Louisville by two points. At the buzzer, Memphis's Darius Washington attempted a 3-pointer; he missed but was fouled, and went to the line for three free throws. Each made free throw is worth 1 point. Was it smart to foul?

1. What are all the possible ways the shots could fall (e.g. make-miss-miss, etc.)?

✓✓✓ ✓✓✗ ✓✓✗ ✗✗✗

✓✗✓ ✓✗✓

✗✓✓ ✗✓✓

2. Darius Washington was a 72% free-throw shooter. Find the probability that Memphis will win, lose or go to overtime. When you have found the probabilities put them in the table in #3.

Win	Lose	Overtime
3 makes (.72)(.72)(.72) = .373	0 makes and 3 misses (.28)(.28)(.28) = .022 1 make and 2 misses	2 makes and 1 miss

Lesson 6.3: Day 1B: Is it smart to foul at the end of the game?

In the 2005 Conference USA men's basketball tournament, Memphis trailed Louisville by two points. At the buzzer, Memphis's Darius Washington attempted a 3-pointer; he missed but was fouled, and went to the line for three free throws. Each made free throw is worth 1 point. Was it smart to foul?

1. What are all the possible ways the shots could fall (e.g. make-miss-miss, etc.)?

✓✓✓ ✓✓✗ ✓✗✓ ✗✓✓ ✓✓✗ ✓✗✓ ✗✓✓ ✗✗✗

2. Darius Washington was a 72% free-throw shooter. Find the probability that Memphis will win, lose or go to overtime. When you have found the probabilities put them in the table in #3.

Win	Lose	Overtime
3 makes (.72)(.72)(.72) = .373	0 makes and 3 misses (.28)(.28)(.28) = .022 1 make and 2 misses (.72)(.28)(.28) x 3 ways = .056 x 3 ways = .169	2 makes and 1 miss

Lesson 6.3: Day 1B: Is it smart to foul at the end of the game?

In the 2005 Conference USA men's basketball tournament, Memphis trailed Louisville by two points. At the buzzer, Memphis's Darius Washington attempted a 3-pointer; he missed but was fouled, and went to the line for three free throws. Each made free throw is worth 1 point. Was it smart to foul?

1. What are all the possible ways the shots could fall (e.g. make-miss-miss, etc.)?

✓✓✓ ✓✓✗ ✓✗✓ ✗✓✓ ✓✓✗ ✓✗✓ ✗✓✓ ✗✗✗

2. Darius Washington was a 72% free-throw shooter. Find the probability that Memphis will win, lose or go to overtime. When you have found the probabilities put them in the table in #3.

Win	Lose	Overtime
3 makes (.72)(.72)(.72) = .373	0 makes and 3 misses (.28)(.28)(.28) = .022 1 make and 2 misses (.72)(.28)(.28) x 3 ways = .056 x 3 ways = .169	2 makes and 1 miss (.72)(.72)(.28) x 3 ways = .145 x 3 ways = .435

3. Prior to watching each shot, calculate the probability that Memphis wins the game in regulation, loses the game in regulation, or sends the game into overtime.

Louisville	Memphis	Shots Remain.	Probability Memphis Win	Probability Memphis Lose	Probability Overtime
75	73	3			
75		2			
75		1			

4. Washington is a 40% 3-point shooter. Do you think Louisville was smart to foul? Why or why not?

3. Prior to watching each shot, calculate the probability that Memphis wins the game in regulation, loses the game in regulation, or sends the game into overtime.

Louisville	Memphis	Shots Remain.	Probability Memphis Win	Probability Memphis Lose	Probability Overtime
75	73	3	$= .373$	$.022 + .169 = 0.191$	$= 0.435$
75	74	2			
75		1			

4. Washington is a 40% 3-point shooter. Do you think Louisville was smart to foul? Why or why not?

3. Prior to watching each shot, calculate the probability that Memphis wins the game in regulation, loses the game in regulation, or sends the game into overtime.

		Shots Remain.	Probability Memphis Win	Probability Memphis Lose	Probability Overtime
75	73		= .373	.022+.169 =0.191	= 0.435
75	74		make make	miss miss	make miss or miss make
75	74				

4. Washington is a 40% 3-point shooter. Do you think Louisville was smart to foul? Why or why not?

3. Prior to watching each shot, calculate the probability that Memphis wins the game in regulation, loses the game in regulation, or sends the game into overtime.

		Shots Remain.	Probability Memphis Win	Probability Memphis Lose	Probability Overtime
75	73		= .373	.022+.169 =0.191	= 0.435
75	74		make make $(.72)^2 = .5184$	miss miss	make miss or miss make
75	74				

4. Washington is a 40% 3-point shooter. Do you think Louisville was smart to foul? Why or why not?

3. Prior to watching each shot, calculate the probability that Memphis wins the game in regulation, loses the game in regulation, or sends the game into overtime.

Louisville	Memphis	Shots Remain.	Probability Memphis Win	Probability Memphis Lose	Probability Overtime
75	73	3	$=.373$	$.022+.169$ $=0.191$	$=0.435$
75	74	2	make make $(.72)^2=.5184$	miss miss $(.28)^2=.0789$	make miss or miss make $2(.72)(.28)$
75	74	1			

4. Washington is a 40% 3-point shooter. Do you think Louisville was smart to foul? Why or why not?

3. Prior to watching each shot, calculate the probability that Memphis wins the game in regulation, loses the game in regulation, or sends the game into overtime.

Louisville	Memphis	Shots Remain.	Probability Memphis Win	Probability Memphis Lose	Probability Overtime
75	73	3	$=.373$	$.022+.169$ $=0.191$	$=0.435$
75	74	2	make make $(.72)^2=.5184$	miss miss $(.28)^2=.0789$	make miss or miss make $2(.72)(.28)$
75	74	1		miss	make

4. Washington is a 40% 3-point shooter. Do you think Louisville was smart to foul? Why or why not?

3. Prior to watching each shot, calculate the probability that Memphis wins the game in regulation, loses the game in regulation, or sends the game into overtime.

LOUISVILLE	M	Shots Remain.	Probability Memphis Win	Probability Memphis Lose	Probability Overtime
75	73		$= .373$	$.022 + .169 = 0.191$	$= 0.435$
75	74		make make $(.72)^2 = .5184$	miss miss $(.28)^2 = .0789$	make miss or miss make $2(.72)(.28)$
75	74		0	miss	make

4. Washington is a 40% 3-point shooter. Do you think Louisville was smart to foul? Why or why not?

3. Prior to watching each shot, calculate the probability that Memphis wins the game in regulation, loses the game in regulation, or sends the game into overtime.

LOUISVILLE	M	Shots Remain.	Probability Memphis Win	Probability Memphis Lose	Probability Overtime
75	73		$= .373$	$.022 + .169 = 0.191$	$= 0.435$
75	74		make make $(.72)^2 = .5184$	miss miss $(.28)^2 = .0789$	make miss or miss make $2(.72)(.28)$
75	74		0	miss 0.28	make

4. Washington is a 40% 3-point shooter. Do you think Louisville was smart to foul? Why or why not?

3. Prior to watching each shot, calculate the probability that Memphis wins the game in regulation, loses the game in regulation, or sends the game into overtime.

		Shots Remain.	Probability Memphis Win	Probability Memphis Lose	Probability Overtime
75	73		= .373	.022+.169 =0.191	= 0.435
75	74		make make $(.72)^2 = .5184$	miss miss $(.28)^2 = .0789$	make miss or miss make $2(.72)(.28)$
75	74		0	miss 0.28	make .72

4. Washington is a 40% 3-point shooter. Do you think Louisville was smart to foul? Why or why not?

MEMPHIS, March 12 - LOUISVILLE 75, MEMPHIS 74

The Memphis freshman Darius Washington slumped to the court, covering his head in anguish over his two missed free throws.

Nobody -- his coach, his teammates, even the player who fouled him -- could console him.

Washington missed two of three free throws with no time left on the clock Saturday, allowing sixth-ranked Louisville to escape with a 75-74 victory and the Conference USA championship.

Blood Type

Each child of a particular set of parents has probability 0.25 of having type O blood. Suppose these parents have 5 children.

What's the probability that exactly one of the five children has type O blood?

$P(X = 1) = P(\text{exactly 1 child with type O blood})$

$= 5(0.25)^1(0.75)^4 = 0.3955$

Number of ways to get 1 child out of 5 with type O blood

1 child with type O blood

4 children don't have type O blood

$\binom{5}{1}$

$P(X = 1) = P(\text{exactly 1 child with type O blood})$

$= 5(0.25)^1(0.75)^4 = 0.3955$

Number of ways to get 1 child out of 5 with type O blood

1 child with type O blood

4 children don't have type O blood

$\binom{5}{1}$

$1+4$
$=5$

${}_5C_1$

$P(X=k) = \binom{\text{\# ways to get}}{k \text{ successes in } n \text{ trials}} (\text{success probability})^k (\text{failure probability})^{n-k}$

Binomial Probability Formula

Suppose that X is a binomial random variable with *n* trials and probability *p* of success on each trial. The probability of getting exactly *k* successes in *n* trials (k = 0, 1, 2, ..., *n*) is

$$P(x = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

where

$$\binom{n}{k} = \frac{n!}{k!\,(n-k)!}$$

If X has a binomial distribution with parameters n and p, then:

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

but formula for binomial coeffient is not shown.

$$_nC_K = \frac{n!}{K!(n-K)!}$$

Red Light
Green Light

Red Light-Green Light

Pedro drives the same route to work on Monday through Friday. His route includes one traffic light. According to the local traffic department, there is a 55% chance that the light will be red on a randomly selected work day. Suppose we choose 10 of Pedro's work days at random and let Y = the number of times that the light is red.

a. Explain why Y is a binomial random variable.

b. Find the probability that the light is red on exactly 7 days.

Red Light-Green Light

Pedro drives the same route to work on Monday through Friday. His route includes one traffic light. According to the local traffic department, there is a 55% chance that the light will be red on a randomly selected work day. Suppose we choose 10 of Pedro's work days at random and let Y = the number of times that the light is red.

a. Explain why Y is a binomial random variable.

B I

N S

b. Find the probability that the light is red on exactly 7 days.

Red Light-Green Light

Pedro drives the same route to work on Monday through Friday. His route includes one traffic light. According to the local traffic department, there is a 55% chance that the light will be red on a randomly selected work day. Suppose we choose 10 of Pedro's work days at random and let Y = the number of times that the light is red.

a. Explain why Y is a binomial random variable.

B success → Red Light
failure → Not Red I

N S

b. Find the probability that the light is red on exactly 7 days.

Red Light-Green Light

Pedro drives the same route to work on Monday through Friday. His route includes one traffic light. According to the local traffic department, there is a 55% chance that the light will be red on a randomly selected work day. Suppose we choose 10 of Pedro's work days at random and let Y = the number of times that the light is red.

a. Explain why Y is a binomial random variable.

B success → Red Light
failure → Not Red

I Each day Independent

N

S

b. Find the probability that the light is red on exactly 7 days.

Red Light-Green Light

Pedro drives the same route to work on Monday through Friday. His route includes one traffic light. According to the local traffic department, there is a 55% chance that the light will be red on a randomly selected work day. Suppose we choose 10 of Pedro's work days at random and let Y = the number of times that the light is red.

a. Explain why Y is a binomial random variable.

B success → Red Light
failure → Not Red

I Each day Independent

N Set # trials
$n = 10$

S

b. Find the probability that the light is red on exactly 7 days.

Red Light-Green Light

Pedro drives the same route to work on Monday through Friday. His route includes one traffic light. According to the local traffic department, there is a 55% chance that the light will be red on a randomly selected work day. Suppose we choose 10 of Pedro's work days at random and let Y = the number of times that the light is red.

a. Explain why Y is a binomial random variable.

B success → Red Light
failure → Not Red

I Each day Independent

N Set # trials
$n = 10$

S $p = .55$ same

b. Find the probability that the light is red on exactly 7 days.

Red Light-Green Light

Pedro drives the same route to work on Monday through Friday. His route includes one traffic light. According to the local traffic department, there is a 55% chance that the light will be red on a randomly selected work day. Suppose we choose 10 of Pedro's work days at random and let Y = the number of times that the light is red.

a. Explain why Y is a binomial random variable.

B success → Red Light
failure → Not Red

I Each day Independent

N Set # trials
$n = 10$

S $p = .55$ same

$\binom{10}{7}$

b. Find the probability that the light is red on exactly 7 days.

$$P(Y=7) = {}_{10}C_7 \bullet (.55)^7 \bullet (.45)^3$$

total # trials; # of successes; P success; P failure

Red Light-Green Light

Pedro drives the same route to work on Monday through Friday. His route includes one traffic light. According to the local traffic department, there is a 55% chance that the light will be red on a randomly selected work day. Suppose we choose 10 of Pedro's work days at random and let Y = the number of times that the light is red.

a. Explain why Y is a binomial random variable.

B success → Red Light
failure → Not Red

I Each day Independent

N Set # trials
$n = 10$

S $p = .55$
same

b. Find the probability that the light is red on exactly 7 days.

$$P(Y=7) = \underbrace{_{10}}_{\text{total \# trials}}\underbrace{C_7}_{\text{\# of Successes}} \cdot \underbrace{(.55)^7}_{p \text{ Success}} \cdot \underbrace{(.45)^3}_{p \text{ failure}} = .166$$

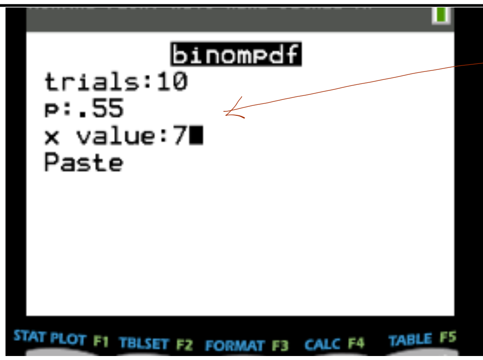
Using technology

binom pdf
↑ Probability

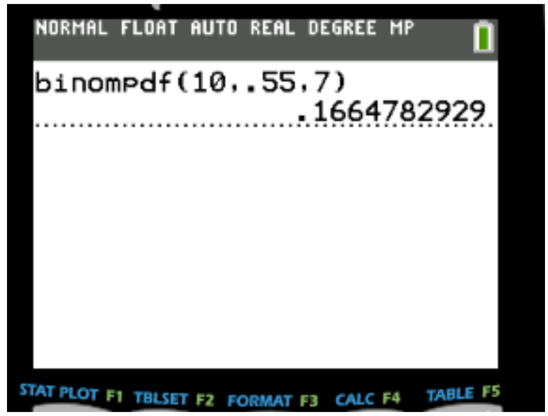
binom pdf (n, p, k) computes $P(X=k)$

last example: $n=10$
$p=.55$
$k=7$

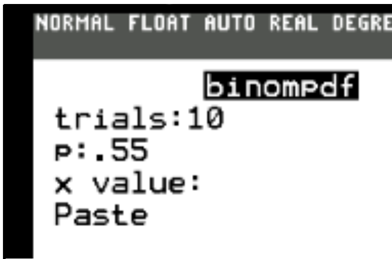
binompdf$(10, .55, 7) =$

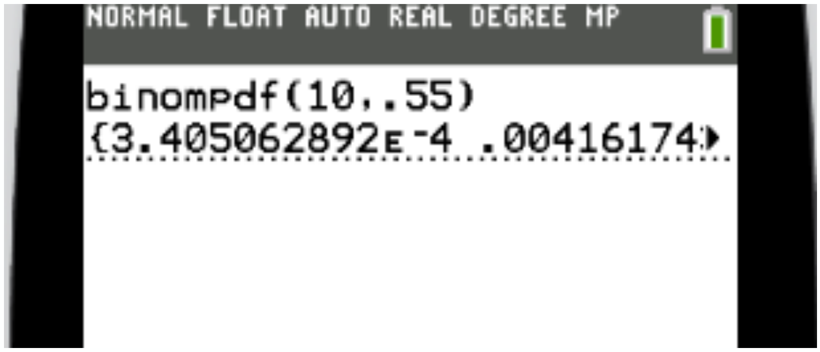


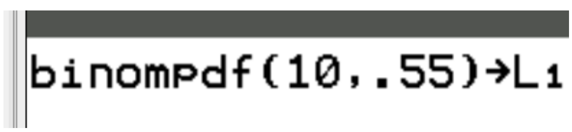
There is a way older 84's and 83's can display prompts.

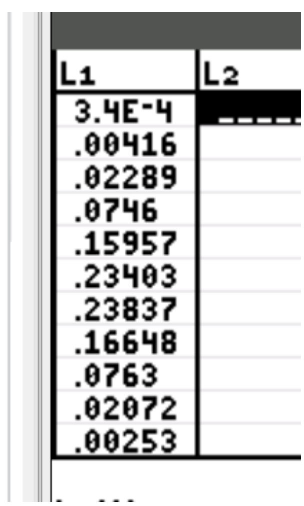


Watch what happens if you leave out K
?


NORMAL FLOAT AUTO REAL DEGRE
binompdf
trials:10
p:.55
x value:
Paste


NORMAL FLOAT AUTO REAL DEGREE MP
binompdf(10,.55)
{3.405062892E-4 .00416174


binompdf(10,.55)→L1


L1
L2
3.4E-4
.00416
.02289
.0746
.15957
.23403
.23837
.16648
.0763
.02072
.00253

BLOOD TYPE FOLLOW UP

Blood Type follow up

The preceding example tells us that each child of a particular set of parents has probability 0.25 of having type O blood. Suppose these parents have 5 children. Should the parents be surprised if more than 3 of their children have type O blood? Calculate an appropriate probability to support your answer.

0 1 2 3 4 5

~~0~~ ~~1~~ ~~2~~ ~~3~~ 4 5

$$P(X>3) = P(X=4) + P(X=5)$$

$$= \binom{5}{4}(.25)^4(.75)^1 + \binom{5}{5}(.25)^5(.75)^0$$

$$= 0.01465 + .00098$$

$$= 0.01563$$

Because there is only about a 1.6% chance of having more than 3 children with Type O blood, the parents should definitely be surprised if it happens.

How to Find Binomial Probabilities

Step 1: state the distribution and the values of interest. Specify a binomial distribution with the number of trials *n*, success probability *p*, and the values of the variable clearly identified.

Step 2: Perform calculations—show your work! Do one of the following:

(i) Use the binomial probability formula to fi nd the desired probability; or

(ii) Use the binompdf or binomcdf command and label each of the inputs.

Be sure to answer the question that was asked.

Aussie Instant Lottery

Aussie Instant lottery

The Australian Official Lottery has scratch-off instant lottery tickets that can be purchased for $1. The probability of winning a prize is 1 in 4.

(a) Mr. Urban is feeling lucky one day and decides to purchase 100 of the scratch-off instant lottery tickets. Find the probability that fewer than 20 tickets are winners.

(b) In fact, Mr. Urban won a prize for only 19 of the tickets. Does this result give convincing evidence that the probability of winning is less than 1 in 4?

a) Let Y = the number of tickets that win a prize

Y has a binomial distribution with $n = 100$ and $p = \frac{1}{4}$

$P(Y < 20)$

$= P(Y \leq 19)$

$= P(0) + P(1) + P(2) + \cdots P(19)$

$=$ binomcdf (trials: 100 p: $\frac{1}{4}$ value: 19)

$= 0.09953$

b) If the prob. of winning is 1 in 4, there is a 0.09953 prob. that fewer than 20 of his 100 tickets win a prize. (this is Plausible)

Because it is plausible that Mr. Urban would win on 19 tickets purely by chance, we do <u>not</u> have convincing evidence that the prob of winning is less than 1 in 4.

6.3......77, 79, 80, 81, 83, 85, 89, <u>118</u>

Study pp. 402-412