Reminder
A $z$-score tells us the number of standard deviations above or below the mean that a value falls in a distribution.

"Cu
well be looking at
your Ch. 5 TEST

# DESCRIBE the effect of: adding or subtracting a constant or multiplying or dividing by a constant 

## on the probability distribution of a random variable.



1. Copy the data collected from yesterday's lesson below.

| $X$ | 1 | 5 | 7 | 10 | 15 | 25 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Probability |  |  |  |  |  |  |

Mean: $\qquad$ Standard Deviation: $\qquad$
$ף$
Use TI-direct


## Lesson 6.2: Day 1: Time for a Raise

Mr. Cedarlund's employees have been working very hard and it's time he gives them a raise. He is trying to decide if he should give everyone a $\$ 10$ raise (add $\$ 10$ per hour) or double everyone's wage (multiply by 2).

1. Copy the data collected from yesterday's lesson below.

| $X$ | 1 | 5 | 7 | 10 | 15 | 25 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Probability |  |  |  |  |  |  |

Mean: $\qquad$ Standard Deviation: $\qquad$
2. To make a decision about what raise should be given, complete the tables below and calculate the new mean and standard deviation using your calculator.
a. Option 1: Add $\mathbf{\$ 1 0}$ per hour to all employees

| $X-$ Old <br> Wage | 1 | +10 | 5 | 10 | 15 | 25 |
| :--- | :---: | :---: | :--- | :--- | :--- | :--- |
| Y -New <br> Wage |  | 110 | 5 | 17 | 20 | 2 |
| Probability | $1 / 14$ | $3 / 14$ | $1 / 14$ | $2 / 14$ | $0 / 14$ | $1 / 14$ |

New Mean $(\mu+10)$


New Standard Deviation:
How did adding a constant affect the mean and standard deviation?

$$
\begin{aligned}
E(x)= & \sum \times 0 \\
& \text { Dean increased by }{ }^{*} \$ 0
\end{aligned}
$$

and std deviation remained the
2. To make a decision about what raise should be given, complete the tables below and calculate the new mean and standard deviation using your calculator.
a. Option 1: Add $\$ 10$ per hour to all employees


Same as previous

New Mean $(\mu+10)$ : $\qquad$ \# 15.78 2

New Standard Deviation? table

How did adding a constant affect the mean and standard deviation?


$$
\sigma=\sqrt{\sum(x-\mu)^{2} \cdot p}
$$ doubled!!!

$$
\mu=\sum x \cdot P=2\left(\frac{1}{14}\right)+10\left(\frac{3}{14}\right) \cdot \cdot=1 \cdot 02
$$

Adding or subtracting a constant: Think about a histogram for some random variable. If we added 12 to each value, this would simply slide the histogram 12 units to the right,
 - but it would not change the variability or shape.
b. Option 2: Double the wage of all employees

| $X$ - Old <br> Wage | 1 | 5 | 7 | 10 | 15 | 25 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Z -New <br> Wage |  |  |  |  |  |  |
| Probability |  |  |  |  |  |  |

New Mean( $2 \mu$ ): $\qquad$ Standard Deviation: $\qquad$
How did multiplying by a constant affect the mean and standard deviation?
b. Option 2: Double the wage of all employees

| $X-$ Old <br> Wage | 1 | 5 | 7 | 10 | 15 | 25 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $Z-$ New <br> Wage | $\# 2$ | $\# 10$ | 14 | 20 | 30 | 50 |
| Probability | $2 / 17$ | $3 / 17$ | $7 / 17$ | $3 / 17$ | $0 / 17$ | $2 / 17$ |

New Mean( $2 \mu$ ): $\qquad$ Standard Deviation: $\qquad$
How did multiplying by a constant affect the mean and standard deviation?

Multiplying or dividing by a constant:
Think about a histogram for a rand. variable. that takes values between 1 and 8. If we multiplied each value by 10 the new histogram would go from 10 to 80


This would multiply the measures of center, location, and variability by 10 , but it would not change the shape.

These are the same results we got with transformation of summary statistics back in Ch .2

## Transforming Probability Distributions

Important ideas:

Transforming Probability Distributions
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Adding the same constant, C, to each value $\ldots$

## Transforming Probability Distributions

Important ideas:
Adding the same constant, c, to each value .0
Shape stays the same
adds $C$ to the center
variability stays the
same.

Transforming Probability Distributions

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Adding the same constant, C, to each value
Shape stays the same adds $C$ to the center variability stays the

Multiplying the same constant, $b$, to each value.
same.

Transforming Probability Distributions
Important ideas: Multiplying the same constant, $b$,

Adding the same constant, to each value. C, to each value

- Shape stays the same

Shape stays the same

- multiplies center by b adds $C$ to the center Variability gets multiplied by $b$ variability stays the same.

$$
\begin{aligned}
& S D=\pi \rightarrow b \sigma \\
& \text { STd der }=\sqrt{\operatorname{Var}} \\
& \operatorname{Var}=(b \sigma)^{2}=b^{2} \sigma^{2}
\end{aligned}
$$

Same with Normal Distributions


While the standard deviation is multiplied by $b$, the variance is multiplied by $b^{2}$.

## The Effect of Adding or Subtracting a Constant

Adding the same positive number $a$ to (subtracting a from) each value of a random variable:

- Adds $a$ to (subtracts $a$ from) measures of center and location (mean, median, quartiles, percentiles).
- Does not change measures of variability (range, IQR, standard deviation).
- Does not change the shape of the probability distribution.


## The Effect of Multiplying or Dividing by a Constant

Multiplying (or dividing) each value of a random variable by the same positive number $b$ :

- Multiplies (divides) measures of center and location (mean, median, quartiles, percentiles) by $b$.
- Multiplies (divides) measures of variability (range, IQR, standard deviation) by b.
- Does not change the shape of the distribution.


## Check Your Understanding \#1 -- Everyone gets a bonus

A large corporation has thousands of employees. The distribution of annual salaries for the employees is skewed to the right, with a mean of $\$ 68,000$ and a standard deviation of $\$ 18,000$. Because business has been good this year, the CEO of the company decides that every employee will receive a $\$ 5000$ bonus. Let $X$ be the current annual salary of a randomly selected employee before the bonus and $Y$ be the employee's salary after the bonus. Describe the shape, center, and variability of the probability distribution of $y$.


Center
Variability*

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Shape: Skewed Right
Center: $\mu_{Y}=\mu_{x}+5000=68,000+5,000=\$ 73,000$
Variability: $\sigma_{r}=\sigma_{x}=\$ 18,000$

## Check Your Understanding \#2

A large auto dealership keeps track of sales made during each hour of the day Let $X=$ the number of cars sold during the first hour of business on a randomly selected Friday. Based on previous records, the probability distribution of $X$ is as follows:

| Cars sold | 0 | 1 | 2 | 3 |
| :--- | :---: | :---: | :---: | :---: |
| Probability | 0.3 | 0.4 | 0.2 | 0.1 |

The random variable $X$ has mean $\mu_{\mathrm{x}}=1.1$ and standard deviation $\sigma_{\mathrm{x}}=0.943$.
Suppose the dealership's manager receives a $\$ 500$ bonus from the company for each car sold. Let $\mathbf{Y}=$ the bonus received from car sales during the first hour on a randomly selected Friday.

1. Sketch a graph of the probability distribution of $X$ and a separate graph of the probability distribution of Y. How do their shapes compare?

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$$
\text { DISTRIbution af } Y
$$




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$$
\text { DisTRibution af } Y_{1}
$$




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| :--- | :---: | :---: | :---: | :---: |
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1. Sketch a graph of the probability distribution of $X$ and a separate graph of the probability distribution of Y. How do their shapes compare?

DISTRIbution af $Y$


$\square$
2. Find the mean of $\mathbf{Y}$. $\mu_{Y}=1.1 \mu_{X}=1.1$ (900)

$$
\$ 5.50
$$

3. Calculate and interpret the standard deviation of $\mathbf{Y}$.

$$
\sigma_{y}=0.943 .500=421.70
$$

$$
\text { The bonus received typically varies lay out } \$ 471.20
$$

4. The manager spends $\$ 75$ to provide coffee and doughnuts to prospective customers each morning. So, the manager's net profit $T$ during the first hour on a randomly selected Friday is $\$ 75$ less than the bonus earned. Describe the shape, center, and variability of the probability distribution of $T$.

$$
\begin{aligned}
& \text { Shape. Skewed rant } \\
& \text { Center } \cdot M y=550-75=\$ 475= \\
& \text { variability: } 471.7
\end{aligned}
$$

2. Find the mean of $\mathbf{Y}$. $\mu_{Y}=1.1 \times 500=\$ 550$
3. Calculate and interpret the standard deviation of $\mathbf{Y}$.

$$
\sigma_{Y}=0.943 \times 500=\$ 471.50
$$

$\sigma_{Y}=0.943 \times 500=$
The bonuses typically vary by $\$ 471.50$
from the mean ( 3550 )
4. The manager spends $\$ 75$ to provide coffee and doughnuts to prospective customers each morning. So, the manager's net profit $T$ during the first hour on a randomly selected Friday is $\$ 75$ less than the bonus earned. Describe the shape, center, and variability of the probability distribution of $T$.
The shape will remain the same. The mean will be subtracted by 75 .

$$
(\mu=550-75=\$ 475)
$$

The SD does not change

$$
(\sigma=471.70)
$$

Employees selling refrigerators at an appliance store make money on commission based on how many refrigerators they sell. The number of refrigerators $R$ sold in a randomly selected hour has the following probability distribution:

| Number of <br> refrigerators | 0 | 1 | 2 | 3 | 4 | 5 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Probability | 0.22 | 0.31 | 0.12 | 0.25 | 0.08 | 0.02 |

Here is a histogram of the probability distribution along with the mean and standard deviation.


At this appliance store, the commission earned is $\mathbf{\$ 3 0}$ for each refrigerator sold. That is, if $C=$ total commission earned for a randomly selected hour, $C=30 R$.

| Number of <br> refrigerators | 0 | 1 | 2 | 3 | 4 | 5 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Probability | 0.22 | 0.31 | 0.12 | 0.25 | 0.08 | 0.02 |

Here is a histogram of the probability distribution along with the mean and standard deviation.


At this appliance store, the commission earned is $\mathbf{\$ 3 0}$ for each refrigerator sold. That is, if $C=$ total commission earned for a randomly selected hour, $C=30 R$.
(a) What shape does the probability distribution of C have?
(a) What shape does the probability distribution of C have?

- The same shape as the prob. distrib. of $R$ - slightly skewed right with two peaks
(b) Find the mean of C .

$$
\mu_{c}=30 \mu_{R}=30(1.72)=451.60
$$

(c) Calculate the standard deviation of C .

$$
\sigma_{c}=30 \sigma_{R}=30(1.36)=\$ 40.80
$$

## See your ch. 5 Test

6.2 ....37, 39, 41, 43, 47, 75
study pp. 381-387 and be sure to study the example on p. 387

