Prepare cards that show wages per hour (one per studen)

sample

wage	quant.
1	1
5	3
7	6
10	2
15	0
25	2

WARM UP

Write down the formula for standard deviation (way back from chapter 1)

– Look it up if you need to

$S_x =$

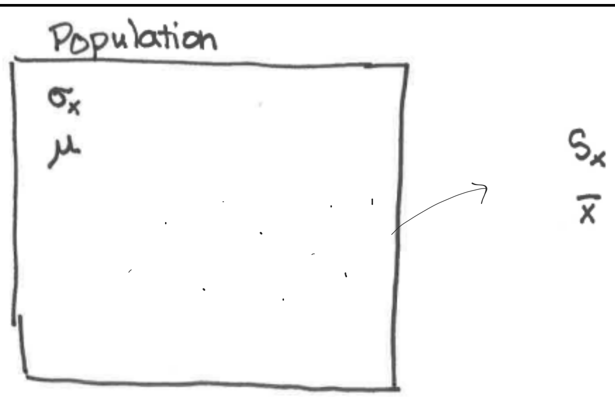
Variance $= S_x^2 =$

Warm Up

Write down the formula for standard deviation (way back from Chapter 1)
– Look it up if you need to

$$S_x = \sqrt{\frac{(x_1-\bar{x})^2 + (x_2-\bar{x})^2 + \cdots}{n-1}} = \sqrt{\frac{\sum (x_i-\bar{x})^2}{n-1}}$$

$$\text{Variance} = S_x^2 = \frac{\sum (x_i-\bar{x})^2}{n-1}$$



We'll use σ_x for distributions of random variables because we'll know the entire probability distribution.

Learing Targets

(1) CALCULATE and INTERPRET the standard deviation of a discrete random variable.

(2) USE the probability distribution of a continuous random variable (uniform or Normal) to CALCULATE the probability of an event.

Suppose you got a new job and each day your boss, Mr. Cedarlund, draws a slip of paper from a bag to determine your wage for the day.

Let the random variable:

X = daily wage ($ per hour).

X	1	5	7	10	15	25
	1	3	7	2	0	1

Lesson 6.1: Day 2: How much do you get paid?

MINIMUM WAGE

Suppose you got a new job and each day your boss draws a slip of paper from a bag to determine your wage for the day. Let the random variable *X* = daily wage ($ per hour).

1. What is your wage for the day? ____________ Add your data to the table on the board and complete the table below.

X	1	5	7	10	15	25
Probability	1/14	3/14	7/14	2/14	0/14	1/14

2. Calculate and **interpret** the expected value of *X*, perhaps with the help of technology.

$$\mu = 1\left(\frac{1}{14}\right)+5\left(\frac{3}{14}\right)+7\left(\frac{7}{14}\right)+10\left(\frac{2}{14}\right)+15\left(\frac{0}{14}\right)+25\left(\frac{1}{14}\right)$$

$=$

If we dra

Lesson 6.1: Day 2: How much do you get paid?

MINIMUM WAGE

Suppose you got a new job and each day your boss draws a slip of paper from a bag to determine your wage for the day. Let the random variable *X* = daily wage ($ per hour).

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Probability	1/14	3/14	7/14	2/14	0/14	1/14

2. Calculate and **interpret** the expected value of *X*, perhaps with the help of technology.

$\mu = 7.86$

If we draw many many wages, the average wage is about \$ 7.86

$\mu = 7.86$

$x-\mu$

Value	Distance from mean	(Distance from mean)^2	Weighted (Distance from the mean)^2
1	−6.86	47.0596	3.3614
5	−2.86	8.1796	1.7528
7	−.86	.7396	.3698
10	2.14	4.5796	.65423
15	7.14	50.98	0
25	17.14	293.78	20.984
		Total =	27.122
		SD =	5.21

$(x-\mu)^2 \cdot \frac{1}{14}$

$\mu =$

Variance

Standard deviation

Formula $\sqrt{\sum (x-\mu)^2 \cdot prob}$

4. Interpret the standard deviation.

4. Interpret the standard deviation.

The wages typically...

4. Interpret the standard deviation.

The wages typically vary by \$5.21 from the mean of \$7.86.

Probability and Continuous Random Variables

Important ideas:

Standard Deviation of a Discrete Rand. Variable

$$\sigma_x = \sqrt{\sum (x_i - \mu_x)^2 \cdot P(x_i)}$$

σ_x^2 = Variance

We'll return to #5

AP EXAM TIP

AP FORMULA Sheet → Formula for variance of of a discrete random variable is included.

(but not standard deviation)

So just remember that the standard deviation is just the square root of the variance

$$\sigma_x = \sqrt{\sigma_x^2}$$

It is possible, but slightly unlikely that the AP exam will require you to calculate the standard deviation "by hand", but you could be

4. Interpret the standard deviation.

The wages typically vary by ____
from the mean of $ ____.

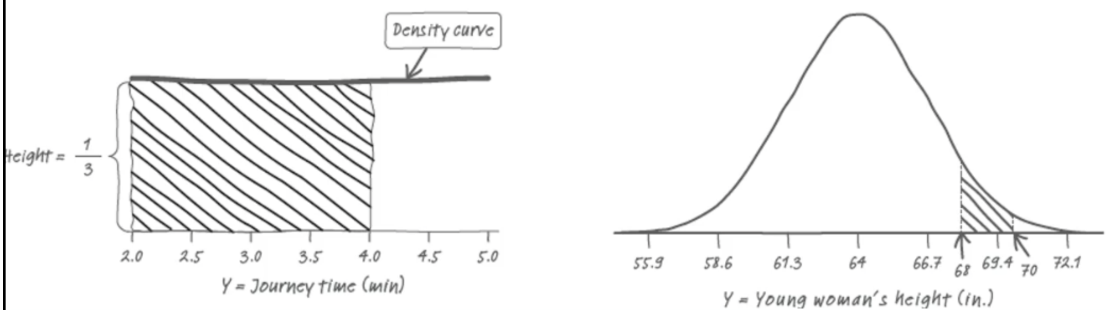
5. Mrs. Gallas decides she would rather assign wages so that employees could get any amount from $10 to $20 and all are equally likely. Draw a graph to represent this probability distribution.

Continuous Random Variables

(pages 371-374)

A continuous random variable can take any value in an interval on the number line.

We describe the probability distribution of a continuous random variable with a density curve, such as a Normal curve



continuous random variables can take on any value in an interval

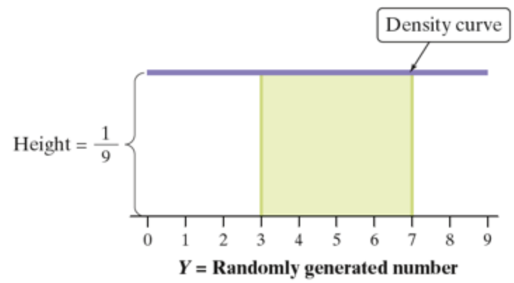
Continuous Random Variables

The possible values for a continuous random variable have no gaps. These variables can take values that are "all the decimals and all the decimals that are in between all the decimals".

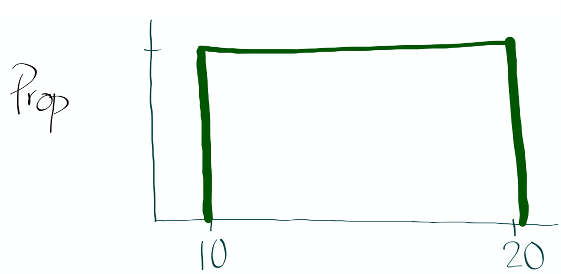
How to Find Probabilities for a Continuous Random Variable

The probability of any event involving a continuous random variable is the area under the density curve and directly above the values on the horizontal axis that make up the event.

FIGURE 6.2 The probability distribution of the continuous random variable Y = randomly generated number between 0 and 9. The shaded area represents $P(3 \leq Y \leq 7)$.



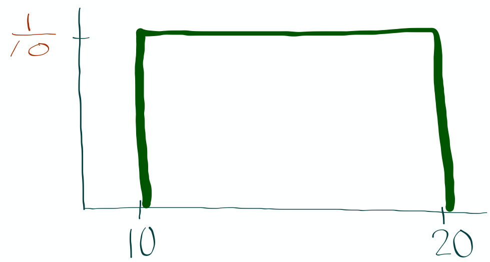
5. Mrs. Gallas decides she would rather assign wages so that employees could get any amount from \$10 to \$20 and all are equally likely. Draw a graph to represent this probability distribution.



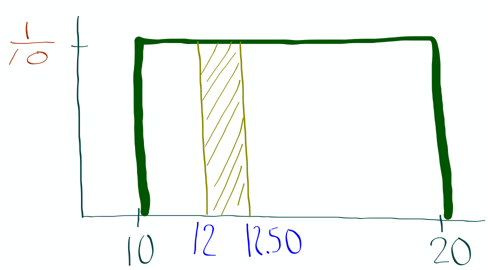
Needs to have an area of 1 since it's a density curve

So.....

5. Mrs. Gallas decides she would rather assign wages so that employees could get any amount from \$10 to \$20 and all are equally likely. Draw a graph to represent this probability distribution.



6. What is the probability that an employee makes between \$12 and \$12.50?



Width $12.50 - 12 = 0.5$

Height $1/10$

Prob.

$= (0.5)(\frac{1}{10})$

$= .05$

Probability and Continuous Random Variables

Important ideas:

Standard Deviation of a Discrete Rand. Variable

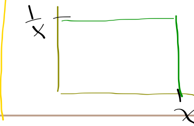
$$\sigma_x = \sum (x_i - \mu_x)^2 \cdot P(x_i)$$

σ^2 = Variance

Probability for a continuous rand. variable

Find area under curve

Uniform



Normal

$N(\mu, \sigma)$

$$Z = \frac{X - \mu}{\sigma}$$

Check Your Understanding

Check Your Understanding -- The heights of young women can be modeled by a Normal distribution with mean $\mu = 64$ inches and standard deviation $\sigma = 2.7$ inches. Suppose we choose a young woman at random and let Y = her height (in inches).

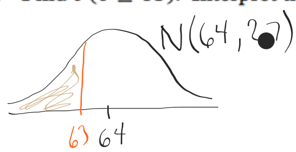
1. What type of variable is Y, discrete or continuous? Explain.

   continuous, all heights are possible

2. Interpret the standard deviation.

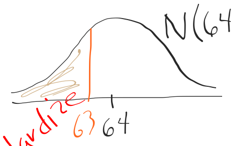
   The heights typically vary by 2.7 inches from the mean of 64 in.

3. Find $P(Y \leq 63)$. Interpret this value.



4. Find $P(68 \leq Y \leq 70)$. Interpret this value.

3. Find P(Y ≤ 63). Interpret this value.



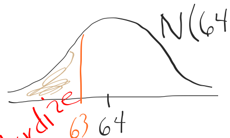
$N(64, 2.7)$

standardize

$Z = \frac{Y-\mu}{\sigma} =$

4. Find P(68 ≤ Y ≤ 70). Interpret this value.

3. Find P(Y ≤ 63). Interpret this value.



$N(64, 2.7)$

standardize

$Z = \frac{Y-\mu}{\sigma} = \frac{63-64}{2.7} = -.37$

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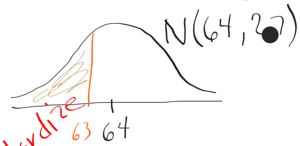

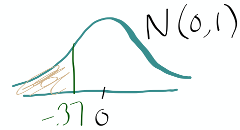

standardize

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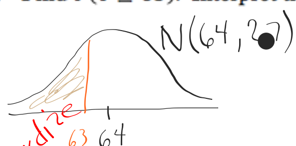


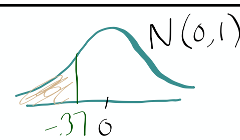


Table A

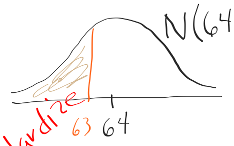
standardize

$$Z = \frac{Y-\mu}{\sigma} = \frac{63-64}{2.7} = -.37$$

4. Find P(68 ≤ Y ≤ 70). Interpret this value.

3. Find P(Y ≤ 63). Interpret this value.

$N(64, 2.7)$



standardize

$$Z = \frac{Y-\mu}{\sigma} = \frac{63-64}{2.7} = -.37$$

$N(0,1)$

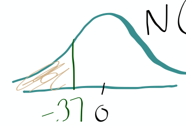
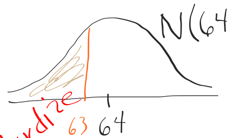


Table A → .3557

4. Find P(68 ≤ Y ≤ 70). Interpret this value.

3. Find P(Y ≤ 63). Interpret this value.

$N(64, 2.7)$



standardize

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$N(0,1)$

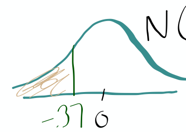


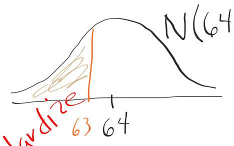
Table A → .3557

There is a prob of .3557 that a rand. selected female is less than or equal to 63 in.

4. Find P(68 ≤ Y ≤ 70). Interpret this value.

We'll use an Alternative to TABLE A but still standardize

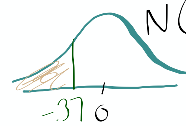
3. Find P(Y ≤ 63). Interpret this value.



N(64, 2.7)

standardize

$Z = \dfrac{Y-\mu}{\sigma} = \dfrac{63-64}{2.7} = -.37$



N(0,1)

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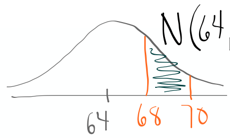

N(64, 2.7)

standardize

$Z =$

$Z =$

3. Find P(Y ≤ 63). Interpret this value.

N(64, 2.7)

standardize

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N(64, 2.7)

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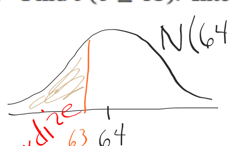
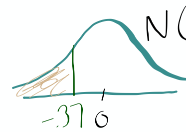
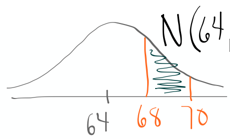


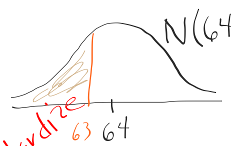




$Z = \dfrac{70-64}{2.7} = 2.22$

$Z = \dfrac{68-64}{2.7} = 1.48$

3. Find P(Y ≤ 63). Interpret this value.



standardize

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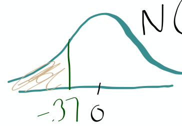

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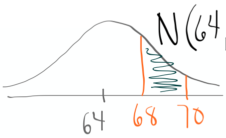

standardize

$$Z = \frac{70-64}{2.7} = 2.22$$

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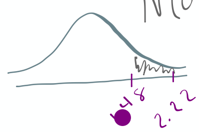

Normal cdf (Lower 1.48, Upper 2.22, mean 0, SD 1)

= 0.0562

standardize

$$Z = \frac{Y-\mu}{\sigma} = \frac{63-64}{2.7} = -.37$$

There is a prob of .3557 that a rand. selected female is less than or equal to 63 in.

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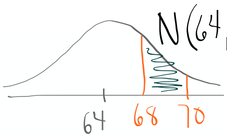

standardize

$$Z = \frac{70-64}{2.7} = 2.22$$

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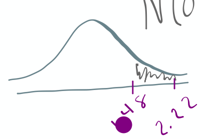

Normal cdf (Lower 1.48, Upper 2.22, mean 0, SD 1)

= 0.0562

There is a 0.0562 prob that a rand. selected female is between 68 to 70 in.

Prac
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Practice

1. Buffalo Wild Wings ran a promotion called the Blazin' Bonus, in which every $25 gift card purchased also received a "Bonus" gift card for $5, $15, $25, or $100. According to the company, here are the probabilities for each Bonus gift card. Let X be the amount of money that is won on the Bonus gift card. Recall from the previous example that $\mu_X = \$6.37$.

Value x_i	\$5	\$15	\$25	\$100
Probability p_i	0.890	0.098	0.010	0.002

Calculate and interpret the standard deviation of X. *(Remember you must show numerical values substituted into the appropriate formula. Once you start you can use ellipses (.....)*

$$\sigma_x^2 = (5-6.37)^2(.890) + (15-6.37)^2(.098) + \ldots\ldots = 29.97$$

$$\sigma_x = \sqrt{\sigma_x^2} = 5.47$$

The amount of money that is won on a randomly selected bonus card will typically vary from the mean ($6.37) by about $ 5.47

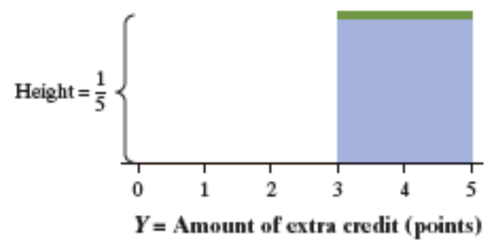
2. A certain AP® Statistics teacher is feeling generous one day and decides that each student deserves some extra credit. The teacher assigns each student a random extra credit value between 0 and 5 (decimals included) by using 5*rand on the calculator.

Let Y = amount of extra credit for a randomly selected student. The probability distribution of Y can be modeled by a uniform density curve on the interval from 0 to 5. *Find the probability that a randomly selected student will get more than 3 points of extra credit.*

2. A certain AP® Statistics teacher is feeling generous one day and decides that each student deserves some extra credit. The teacher assigns each student a random extra credit value between 0 and 5 (decimals included) by using 5*rand on the calculator.

Let Y = amount of extra credit for a randomly selected student. The probability distribution of Y can be modeled by a uniform density curve on the interval from 0 to 5. *Find the probability that a randomly selected student will get more than 3 points of extra credit.*

SOLUTION:



$$\text{Area} = \text{base} \times \text{height} = 2 \times 1/5 = 2/5$$

$$P(Y > 3) = 2/5 = 0.40$$

3. The weights of 3-year-old females closely follow a Normal distribution with a mean of m 5 30.7 pounds and a standard deviation of 3.6 pounds. Suppose we randomly choose a 3-year-old female and call her weight X. What is the probability that she weighs at least 30 pounds? 30.7

Hint: You must draw a diagram. Then practice by calculating the Z-score. Then Using Table A or appropriate technology that used the Z-score, with correct terminology written.

3. The weights of 3-year-old females closely follow a Normal distribution with a mean of m 5 ~~3~~0.7 pounds and a standard deviation of 3.6 pounds. Suppose we randomly choose a 3-year-old female and call her weight *X*. What is the probability that she weighs at least 30 pounds?

30.7

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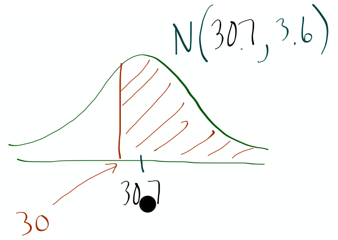

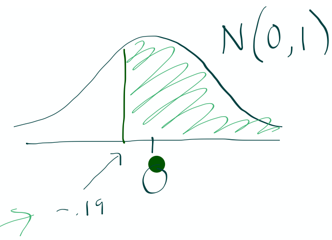

$$Z = \frac{30 - 30.7}{3.6} = -.19$$

$$\text{Normcdf}(\underbrace{-.19}_{\text{Lower}},\ \underbrace{1000}_{\text{upper}},\ \underbrace{0}_{\text{mean}},\ \underbrace{1}_{\text{SD}})$$

$= .5753$

See your test

6.1 .... 13, 19, 21, 23, 27, 29, 31-34

Teach yourself how to make a histogram of a discrete random variable by following the instrutions on page 370.

study pp. 368-374