

Turn in
Take Home LCQ

Period 1

Warm Up →

do the very last
problem on
yesterday's classwork

"Where are the
best Tacos?"

Where are the best tacos?

A survey of all students at a large high school revealed that, in the last month, 38% of them had dined at Taco Bell, 16% had dined at Chipotle, and 9% had dined at both. Suppose we select a student at random. What's the probability that the student has dined at Taco Bell or Chipotle in the last month?

a)

b) Now create a **Venn Diagram** to display the sample space in a different way.

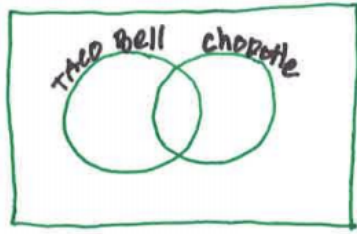
3. Where are the best tacos?

A survey of all students at a large high school revealed that, in the last month, 38% of them had dined at Taco Bell, 16% had dined at Chipotle, and 9% had dined at both. Suppose we select a student at random. What's the probability that the student has dined at Taco Bell or Chipotle in the last month?

General Addition Rule

$$\begin{aligned}
 &= P(\text{Taco Bell OR Chipotle}) \\
 &= P(\text{Taco Bell}) + P(\text{Chipotle}) - P(\text{Taco Bell and Chipotle}) \\
 &= .38 + .16 - .09 = .45
 \end{aligned}$$

Now create a Venn Diagram to display the sample space in a different way.



TODAY 5.3 day 1

CALCULATE and INTERPRET conditional probabilities.

DETERMINE if two events are independent.

USE the general multiplication rule to CALCULATE probabilities.

TODAY 5.3 day 1

CALCULATE and INTERPRET conditional probabilities.

DETERMINE if two events are independent.

USE the general multiplication rule to CALCULATE probabilities.

some of the hardest concepts in prob.

Day 2 of
Can You Taco Tongue and Evil Eyebrow?

5.3 Day 1



Are the events "Yes Taco Tongue" and "Yes Evil Eyebrow" **independent** events?

1. Find class data from the previous lesson and fill in the following two-way table.

	Yes Evil Eyebrow	No Evil Eyebrow	Total
Yes Taco Tongue			
No Taco Tongue			
Total			

Day 2 of
Can You Taco Tongue and Evil Eyebrow?

5.3 Day 1



Are the events "Yes Taco Tongue" and "Yes Evil Eyebrow" **independent** events?

1. Find class data from the previous lesson and fill in the following two-way table.

	Yes Evil Eyebrow	No Evil Eyebrow	Total
Yes Taco Tongue	4	5	9
No Taco Tongue	1	2	3
Total	5	7	12

	Yes Evil Eyebrow	No Evil Eyebrow	Total
Yes Taco Tongue			
No Taco Tongue			
Total			

Suppose we randomly choose a student from class.

- Find $P(\text{Yes Taco Tongue OR Yes Evil Eyebrow})$
- Given that the person selected is a Yes Evil Eyebrow, what is the probability that they are a Yes Taco Tongue? Write as a fraction, a decimal, and a percent.
- Given that the person selected is a No Evil Eyebrow, what is the probability that they are a Yes Taco Tongue? Write as a fraction, a decimal, and a percent.

Definition: Two events are **independent** if knowing whether or not one event has occurred does not change the probability that the other event will occur.

- Are the events "Yes Taco Tongue" and "Yes Evil Eyebrow" independent? Explain.

	Yes Evil Eyebrow	No Evil Eyebrow	Total
Yes Taco Tongue	4	5	9
No Taco Tongue	1	2	3
Total	5	7	12

Suppose we randomly choose a student from class.

- Find $P(\text{Yes Taco Tongue OR Yes Evil Eyebrow})$

$$\frac{9}{12} + \frac{5}{12} - \frac{4}{12} = \frac{10}{12}$$

- Given that the person selected is a Yes Evil Eyebrow, what is the probability that they are a Yes Taco Tongue? Write as a fraction, a decimal, and a percent.

$$\frac{4}{5} = .80 = 80\%$$

- Given that the person selected is a No Evil Eyebrow, what is the probability that they are a Yes Taco Tongue? Write as a fraction, a decimal, and a percent.

$$\frac{1}{7} = .143 = 14.3\%$$

Definition: Two events are **independent** if knowing whether or not one event has occurred does not change the probability that the other event will occur.

- Are the events "Yes Taco Tongue" and "Yes Evil Eyebrow" independent? Explain.

No! Knowing whether or not a student is Yes or No Evil Eyebrow changes the probability they are Yes Taco Tongue from 80% to 14.3%.

	Yes Evil Eyebrow	No Evil Eyebrow	Total
Yes Taco Tongue	4	5	9
No Taco Tongue	1	2	3
Total	5	7	12

Suppose we randomly choose a student from class.

2. Find $P(\text{Yes Taco Tongue OR Yes Evil Eyebrow})$

$$\frac{9}{12} + \frac{5}{12} - \frac{4}{12} = \frac{10}{12}$$

3. Given that the person selected is a Yes Evil Eyebrow, what is the probability that they are a Yes Taco Tongue? Write as a fraction, a decimal, and a percent.

$$\frac{4}{5} = .80 = 80\%$$

4. Given that the person selected is a No Evil Eyebrow, what is the probability that they are a Yes Taco Tongue? Write as a fraction, a decimal, and a percent.

$$\frac{5}{7} = .714 = 71.4\%$$

Conditional Probability

Definition: Two events are **independent** if knowing whether or not one event has occurred does not change the probability that the other event will occur.

5. Are the events "Yes Taco Tongue" and "Yes Evil Eyebrow" independent? Explain.

No! Knowing whether or not a student is Yes or No Evil Eyebrow changes the probability they are Yes Taco Tongue from 80% to 71.4%.

	Yes Evil Eyebrow	No Evil Eyebrow	Total
Yes Taco Tongue	4	5	9
No Taco Tongue	1	2	3
Total	5	7	12

Suppose we randomly choose a student from class.

2. Find $P(\text{Yes Taco Tongue OR Yes Evil Eyebrow})$

$$\frac{9}{12} + \frac{5}{12} - \frac{4}{12} = \frac{10}{12}$$

3. Given that the person selected is a Yes Evil Eyebrow, what is the probability that they are a Yes Taco Tongue? Write as a fraction, a decimal, and a percent.

$$\frac{4}{5} = .80 = 80\% \quad P(\text{Yes Taco Tongue} \mid \text{Yes Evil Eyebrow})$$

4. Given that the person selected is a No Evil Eyebrow, what is the probability that they are a Yes Taco Tongue? Write as a fraction, a decimal, and a percent.

$$\frac{5}{7} = .714 = 71.4\% \quad P(\text{Yes Taco Tongue} \mid \text{No Evil Eyebrow})$$

Conditional Probability

Definition: Two events are **independent** if knowing whether or not one event has occurred does not change the probability that the other event will occur.

5. Are the events "Yes Taco Tongue" and "Yes Evil Eyebrow" independent? Explain.

No! Knowing whether or not a student is Yes or No Evil Eyebrow changes the probability they are Yes Taco Tongue from 80% to 71.4%.

given that

Consider the data for all Seniors at EKHS.

	Yes Evil Eyebrow	No Evil Eyebrow	Total
Yes Taco Tongue	180	300	480
No Taco Tongue	20	100	120
Total	200	400	600

6. Find each of the following using the data in the table. Write as a fraction, a decimal, and a percent.

- $P(\text{Yes Taco Tongue}) =$
- $P(\text{Yes Taco Tongue} \mid \text{Yes Evil Eyebrow}) =$
- $P(\text{Yes Taco Tongue} \mid \text{No Evil Eyebrow}) =$
- Are "Yes Taco Tongue" and "Yes Evil Eyebrow" independent?

Consider the data for all Seniors at EKHS.

	Yes Evil Eyebrow	No Evil Eyebrow	Total
Yes Taco Tongue	180	300	480
No Taco Tongue	20	100	120
Total	200	400	600

6. Find each of the following using the data in the table. Write as a fraction, a decimal, and a percent.

- $P(\text{Yes Taco Tongue}) = \frac{480}{600} = 0.8 = 80\%$
- $P(\text{Yes Taco Tongue} \mid \text{Yes Evil Eyebrow}) = \frac{180}{200} = 0.9 = 90\%$
- $P(\text{Yes Taco Tongue} \mid \text{No Evil Eyebrow}) = \frac{300}{400} = 0.75 = 75\%$
- Are "Yes Taco Tongue" and "Yes Evil Eyebrow" independent?

No. \rightarrow dependent events

	Yes Evil Eyebrow	No Evil Eyebrow	Total
Yes Taco Tongue	180	300	480
No Taco Tongue	20	100	120
Total	200	400	600

7. Fill in the table as if the events were **INDEPENDENT**.

	Yes Evil Eyebrow	No Evil Eyebrow	Total
Yes Taco Tongue			480
No Taco Tongue			120
Total	200	400	600

8. Find each of the following using the **INDEPENDENT** table. Write as a fraction, a decimal, and a percent.

a. $P(\text{Yes Taco Tongue}) =$

b. $P(\text{Yes Taco Tongue} \mid \text{Yes Evil Eyebrow}) =$

c. $P(\text{Yes Taco Tongue} \mid \text{No Evil Eyebrow}) =$

	Yes Evil Eyebrow	No Evil Eyebrow	Total
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	Yes Evil Eyebrow	No Evil Eyebrow	Total
Yes Taco Tongue	180	300	480
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Total	200	400	600

What percent of all seniors in this group Yes Taco Tongue?

7. Fill in the table as if the events were INDEPENDENT.

	Yes Evil Eyebrow	No Evil Eyebrow	Total
Yes Taco Tongue			480
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Yes Taco Tongue	180	300	480
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Total	200	400	600

What percent of all seniors in this group Yes Taco Tongue?
 $\frac{480}{600} = 80\%$

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	Yes Evil Eyebrow	No Evil Eyebrow	Total
Yes Taco Tongue			480
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No Taco Tongue			120
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80% of 600

8. Find each of the following using the **INDEPENDENT** table. Write as a fraction, a decimal, and a percent.

- a. $P(\text{Yes Taco Tongue}) =$
- b. $P(\text{Yes Taco Tongue} \mid \text{Yes Evil Eyebrow}) =$
- c. $P(\text{Yes Taco Tongue} \mid \text{No Evil Eyebrow}) =$

So if Taco Tongue and Evil Eyebrow are independent, what percent of YES EVIL Eyebrow should be Yes Taco Tongue?

	Yes Evil Eyebrow	No Evil Eyebrow	Total
Yes Taco Tongue	180	300	480
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What is 80% of 200?

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Yes Taco Tongue	180	300	480
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Total	200	400	600

7. Fill in the table as if the events were **INDEPENDENT**.

What is 80% of 200?

	Yes Evil Eyebrow	No Evil Eyebrow	Total
Yes Taco Tongue	160		480
No Taco Tongue			120
Total	200	400	600

8. Find each of the following using the **INDEPENDENT** table. Write as a fraction, a decimal, and a percent.

- $P(\text{Yes Taco Tongue}) =$
- $P(\text{Yes Taco Tongue} \mid \text{Yes Evil Eyebrow}) =$
- $P(\text{Yes Taco Tongue} \mid \text{No Evil Eyebrow}) =$

So if Taco Tongue and Evil Eyebrow are independent what percent of YES EVIL Eyebrow should be Yes Taco Tongue?

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Yes Taco Tongue	180	300	480
No Taco Tongue	20	100	120
Total	200	400	600

7. Fill in the table as if the events were INDEPENDENT.

80% of 200 (pointing to 160)

80% of 400 (pointing to 320)

15% of 600 (pointing to 90)

	Yes Evil Eyebrow	No Evil Eyebrow	Total
Yes Taco Tongue	160	320	480
No Taco Tongue	40	80	120
Total	200	400	600

8. Find each of the following using the **INDEPENDENT** table. Write as a fraction, a decimal, and a percent.

a. $P(\text{Yes Taco Tongue}) =$

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	Yes Evil Eyebrow	No Evil Eyebrow	Total
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80% of 200 (pointing to 160)

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	Yes Evil Eyebrow	No Evil Eyebrow	Total
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8. Find each of the following using the **INDEPENDENT** table. Write as a fraction, a decimal, and a percent.

a. $P(\text{Yes Taco Tongue}) = \frac{480}{600} = .8 = 80\%$

b. $P(\text{Yes Taco Tongue} \mid \text{Yes Evil Eyebrow}) = \frac{160}{200} = 0.8 = 80\%$

c. $P(\text{Yes Taco Tongue} \mid \text{No Evil Eyebrow}) = \frac{320}{400} = 0.8 = 80\%$

9. What do you notice about your answers in #6 and #8?

10. **Generalize:** Complete the following statement using a formula.

Let $A \rightarrow \text{Yes Taco Tongue}$ and $B \rightarrow \text{Yes Evil Eyebrow}$

If events A and B are **INDEPENDENT** then...

9. What do you notice about your answers in #6 and #8?

When events are not indep., all 3 prob. are different.
When events are independent, all 3 prob. are equal.

10. **Generalize:** Complete the following statement using a formula.

Let $A \rightarrow$ Yes Taco Tongue and $B \rightarrow$ Yes Evil Eyebrow

If events A and B are INDEPENDENT then...

$$P(A|B) = P(A|B^c)$$

Conditional Probability and Independence

Big Ideas:

Conditional Probability

Independent Events

Conditional Probability and Independence

Big Ideas:

Conditional Probability

$P(A|B)$ is the
"probability of"
A, given B

Independent Events

Conditional Probability and Independence

Big Ideas:

Conditional Probability

$P(A|B)$ is the
"probability of"
A, given B

Independent Events

Knowing whether or not one event has occurred does not change the probability that the other event will occur.

$$P(A) = P(A|B) = P(A|B^c)$$

AP Exam Tip

Formula Sheet

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$



but try to use
context as much
as possible.

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Probability of both

Prob. of the "Given" that

(reduced Sample Space)

Check Your Understanding:

Yellowstone National Park surveyed a random sample of 1526 winter visitors to the park. They asked each person whether he or she owned, rented, or had never used a snowmobile. Respondents were also asked whether they belonged to an environmental organization (like the Sierra Club). The [two way table](#) summarizes the survey responses.

		Environmental club		
		No	Yes	Total
Snowmobile experience	Never used	445	212	657
	Renter	497	77	574
	Owner	279	16	295
	Total	1221	305	1526

Suppose we randomly select one of the survey respondents. Define events E: environmental club member, S: snowmobile owner, and N: never used.

1. Find $P(N | E)$. Interpret this value in context.

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Suppose we randomly select one of the survey respondents. Define events E: environmental club member, S: snowmobile owner, and N: never used.

1. Find $P(N | E)$. Interpret this value in context.

The probability that someone has never used a snowmobile given they are in the environmental club.

$$= \frac{212}{305}$$

$$= 69.5\%$$

		Environmental club		
		No	Yes	Total
Snowmobile experience	Never used	445	212	657
	Renter	497	77	574
	Owner	279	16	295
	Total	1221	305	1526

2. Given that the chosen person is not a snowmobile owner, what's the probability that she or he is an environmental club member? Write your answer as a probability statement using correct symbols for the events.

3. Are the events "Snowmobile owner" and "Environmental club member" independent? Explain.

		Environmental club		
		No	Yes	Total
Snowmobile experience	Never used	445	212	657
	Renter	497	77	574
	Owner	279	16	295
	Total	1221	305	1526

2. Given that the chosen person is not a snowmobile owner, what's the probability that she or he is an environmental club member? Write your answer as a probability statement using correct symbols for the events.

$$P(E | S^c) = \frac{212 + 77}{657 + 574} = \frac{289}{1231} = 0.23$$

3. Are the events "Snowmobile owner" and "Environmental club member" independent? Explain.

$$P(\text{snowmobile}) = \frac{295}{1526} = 0.19$$

$$P(\text{snowmobile} | \text{Environ}) = \frac{16}{305} = 0.05$$

$$P(\text{snowmobile} | \text{Enviro}^c) = \frac{279}{1221} = 0.23$$

No.
 $P(S) \neq P(S|E) \neq P(S|E^c)$
 If you are in the environmental club you are less likely to be a snowmobile owner.

Tip

many students lose credit
on AP exams for not
showing enough work (detail)
on probability questions.

Brain
Break

and then ●●●...●●●

Extra Extra Practice

≡ this material can potentially more difficult

5.3 61-71 (odds), 77, 79

study pp. 330-338