

Quickly Check your HW. Then do the following Warm Up

### Warm Up

Solve  $30(2.1)^n = 6000$

•

$$30(2.1)^n = 6000$$

divide by 30

$$(2.1)^n = 200$$

↓

$$\log(2.1^n) = \log(200)$$

$$n \cdot \log(2.1) = \log(200)$$

$$n =$$

$$30(2.1)^n = 6000$$

divide by 30

$$(2.1)^n = 200$$

Log Form

$$\log_{2.1}(200) = n$$

$$n = \frac{\log(200)}{\log(2.1)}$$

$$n = 7.14$$

$$\log(2.1^n) = \log(200)$$

$$n \cdot \log(2.1) = \log(200)$$

$$n =$$

## #6 Solution does not match

deposit \$50 on first BD.

50, 75, 100, ...

①      ②      ③

a) How much \$ on 16<sup>th</sup> birthday?

$$U_n = U_1 + d(n-1)$$

$$U_{16} = 50 + 25(16-1)$$

$$= \$425$$

↓

b) How much would he have deposited in total on 16<sup>th</sup> Birthday?

Translation •

Find the Sum of the first 16 terms

$$S_n = \frac{n}{2} [2u_1 + d(n-1)]$$

$$= \frac{16}{2} [2(50) + 25(16-1)]$$

$$= \$3,800$$

## Today

Finding the Sum of Geometric Sequences  
(and of course the  $n^{\text{th}}$  term form those  
same sequences)

Example

2, 10, 50, 250, .....

U

A sequence is geometric if each term can be obtained from the previous term by **multiplying** by the same number.

This number is the constant ratio,  $r$

$$\begin{array}{ccccccc}
 & u_1 & u_2 & u_3 & & & \\
 & 2, & 10, & 50, & 250, & \dots & \dots & \dots & u_n & u_{n+1} \\
 \nearrow & & \nearrow & \nearrow & & & & & & \\
 u_1 & & u_2 & u_3 & & & & & & 
 \end{array}$$

$$\begin{aligned}
 \text{Common ratio} = r &= \frac{u_2}{u_1} \\
 &= \frac{u_3}{u_2} \\
 &= \frac{u_4}{u_3}
 \end{aligned}$$

$$\begin{aligned}
 r &= \frac{u_{n+1}}{u_n} \\
 r &= \frac{u_n}{u_{n-1}}
 \end{aligned}$$



What is the common ratio of

135, 90, 60, 40, ...

If the terms are getting smaller, then the common ratio must be less than 1

Note

**You are expected to show work,  
using good notation in this unit.**

**Just fiddling around with  
a calculator won't work out well  
for you.**

Is the following sequence geometric?

W

$0.5, 1, 2, 4, 8, 16, \dots$

①      ②      ③

If so, what is common ratio?  $r = 2$

How many applications of 2 do you need to get from the first term to the 3rd term?

$U_n = 0.5(2)^{n-1}$  to the 4th term?

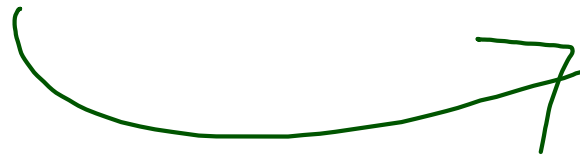
to the 85th term?

to the nth term?

if Geometric.....

Your best friend

will be



## Explicit formula for Geometric Sequences



$$=$$

$n = \# \text{ of terms}$   
 any term of interest  
 first term  
 common ratio

## Explicit formula for Geometric Sequences



$$u_n = u_1 r^{n-1}$$

$n = \# \text{ of terms}$   
 $n-1$   
 any term of interest  
 first term  
 common ratio

Let's try it!

What's the **23<sup>rd</sup>** term of this sequence?

$$\frac{1}{9}, -\frac{1}{3}, 1, -3, 9, \dots$$

Find the ratio:

$$r = \frac{u_2}{u_1} = \frac{-\frac{1}{3}}{\frac{1}{9}} = -3$$

$$\frac{1}{-\frac{1}{3}} =$$

$$\frac{-3}{1} =$$

$$u_{23} = \left(\frac{1}{9}\right)^{23-1} (-3)$$

$$= 3,486,784,401$$

$$3,490,000,000$$

$$\text{or } 3.49 \times 10^9$$

Z

The first term of a geometric sequence is 4 and the last term is 26,244. If there are 9 terms in the sequence, what is the common ratio?

$$26244 = 4(r)^{9-1}$$

$$26244 = 4r^8$$

$$r^8 = \frac{26244}{4}$$

$$r = \sqrt[8]{\frac{26244}{4}}$$

$$r = \pm 3$$

$$u_n = u_1 + d(n-1)$$

$$u_n = u_1(r)^{n-1}$$



The first term of a geometric sequence is 4 and the last term is 26,244. If there are 9 terms in the sequence, what is the common ratio?

$$(-3)^8$$

Alternative wording for the same



find  $r$  if  $u_1 = 4$  and  $u_9 = 26,244$

Best friend

$$u_n = u_1 (r)^{n-1}$$

Note

If something grows at 15%, then the common ratio would be  $(1+.15)$  or 1.15

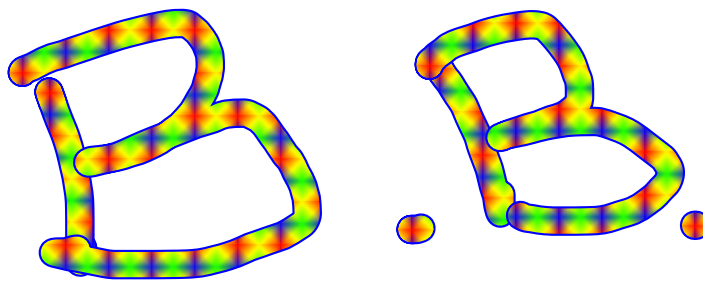
100% + 15%  
115%

$$U_n = U_1 \cdot r^{n-1}$$

↖ ? ↗

$$y = ab^x$$

↑ 0 term



# Geometric Series

the sum of a geometric sequence

Add the  
first 10 terms  
of:

$$4, 2, 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8} \dots$$

$$S_{10} = 4 + 2 + 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} \dots$$

Luckily, there's a formula!

To find the sum of the first  $n$   
terms of a geometric sequence:

See your  
friendly  
Formula  
Packet

Luckily, there's a formula!

To find the sum of the first  $n$  terms of a geometric sequence:

$$S_n = \frac{u_1(1-r^n)}{1-r}$$

for finite sequences

$$S_{10} = 4 + 2 + 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} \dots$$

$$S_{10} =$$

$$= \frac{4(1 - (\frac{1}{2})^{10})}{1 - \frac{1}{2}}$$

$$= 8(1 - (\frac{1}{2})^{10})$$

$$= \frac{1023}{128}$$

$$S_n = \frac{u_1(1-r^n)}{1-r}$$

← reverse  
← reverse

$$S_n = \frac{u_1(r^n - 1)}{r - 1}$$

$$+ (8-8)$$

$$- (8-9)$$

## Formula Packet

The $n^{\text{th}}$ term of an arithmetic sequence	$u_n = u_1 + (n-1)d$
The sum of $n$ terms of an arithmetic sequence	$S_n = \frac{n}{2}(2u_1 + (n-1)d) = \frac{n}{2}(u_1 + u_n)$
The $n^{\text{th}}$ term of a geometric sequence	$u_n = u_1 r^{n-1}$ ✓
The sum of $n$ terms of a geometric sequence	$S_n = \frac{u_1(r^n - 1)}{r - 1} = \frac{u_1(1 - r^n)}{1 - r}, r \neq 1$

$$S_n = \frac{u_1(r^n - 1)}{r - 1}$$

$$S_n = \frac{u_1(1 - r^n)}{1 - r}$$

$$S_n = \frac{u_1(1 - r^n)}{1 - r}$$

$$S_n = \frac{u_1(r^n - 1)}{r - 1}$$

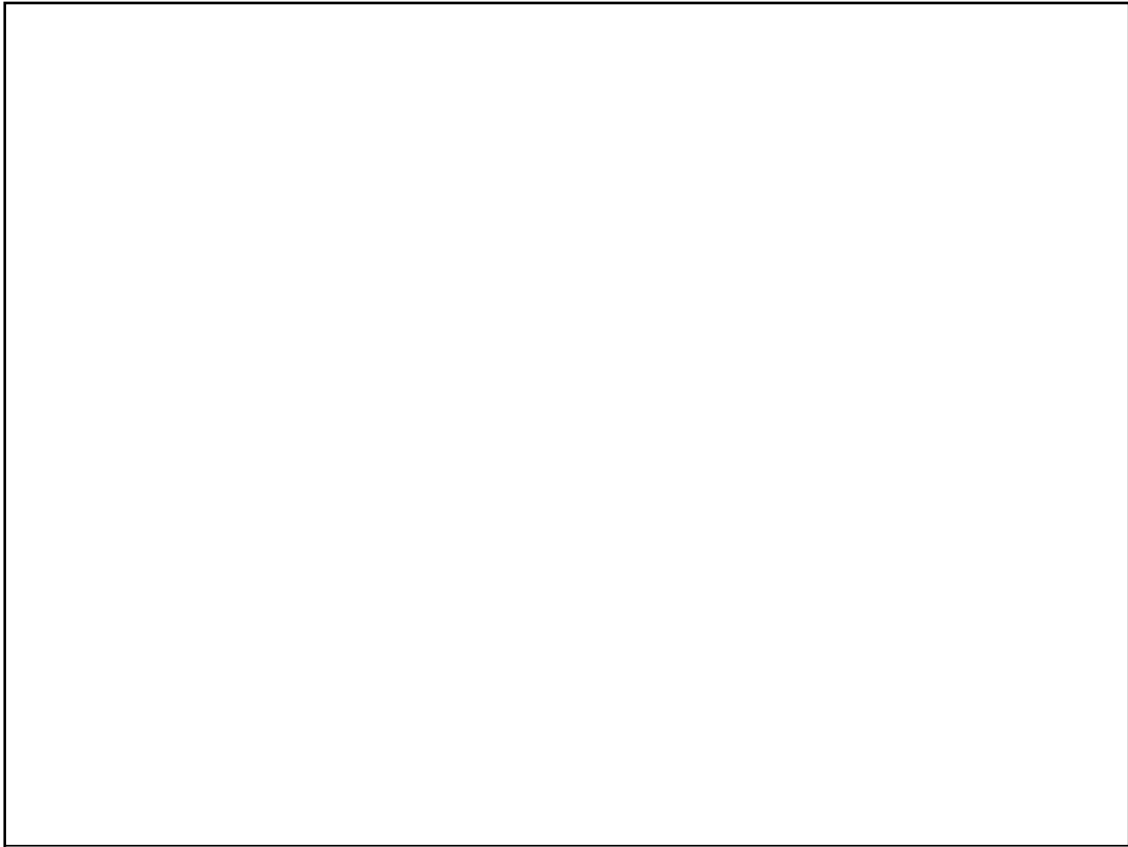
$$4 \left( \frac{1 - \left(\frac{-3}{4}\right)^{20}}{1 - \left(\frac{-3}{4}\right)} \right)$$

$$4 \left( \frac{\left(\frac{-3}{4}\right)^{20} - 1}{\left(\frac{-3}{4}\right) - 1} \right)$$

See your  
test

Assignment #2

Worksheet



Why does  $S_n = \frac{g_1(1-r^n)}{1-r}$  work?

$$S_n = g_1 + g_1 r + g_1 r^2 + \dots + g_1 r^{n-1}$$

Typical Geometric  
Series

↑  
last (or Nth)  
term

multiply by  $r$



$$S_n = g_1 + g_1 r + g_1 r^2 + \dots + g_1 r^{n-1}$$

Typical Geometric Series

↑  
last (or Nth) term

multiply by r

$$rS_n = g_1 r + g_1 r^2 + g_1 r^3 + \dots + g_1 r^{n-1} + g_1 r^n$$

subtract preceding equation

$$S_n - rS_n = g_1 - g_1 r^n$$

$$S_n - rS_n = g_1 - g_1 r^n$$

$$(1-r)S_n = g_1$$

$$S_n = \frac{g_1(1-r^n)}{1-r} \quad \text{as long as } r \neq 1$$



## **Assignment 2** (Sequences/Series)

a worksheet

