Quickly Check your HW. Then do the following Warm Up

Warm Up

Solve $30(2.1)^n = 6000$

$30(2.1)^n = 6000$

divide by 30

$(2.1)^n = 200$

$\log(2.1^n) = \log(200)$

$n \cdot \log(2.1) = \log(200)$

$n =$

$$30(2.1)^n = 6000$$

divide by 30

$$\boxed{(2.1)^n = 200}$$

$$\log(2.1^n) = \log(200)$$

$$n \cdot \log(2.1) = \log(200)$$

$$n =$$

Log Form

$$\log_{2.1}(200) = n$$

$$n = \log_{2.1}(200)$$

$$\boxed{n = \frac{\log(200)}{\log(2.1)}}$$

$$= 7.14$$

#6 Solution does not match

deposit \$50 on first BD.

50, 75, 100, ...

① ② ③

a) How much \$ on 16th birthday?

$$U_n = U_1 + d(n-1)$$

$$U_{16} = 50 + 25(16-1)$$

$$= \boxed{\$425}$$

b) How much would he have deposited in total on 16th Birthday?

Translation: Find the Sum of the first 16 terms

$$S_n = \frac{n}{2}\left[2u_1 + d(n-1)\right]$$

$$= \frac{16}{2}\left[2(50) + 25(16-1)\right]$$

$$\boxed{= \$3,800}$$

Today

Finding the Sum of Geometric Sequences

(and of course the n^{th} term form those same sequences) example

$2, 10, 50, 250, \ldots\ldots$

U

A sequence is geometric if each term can be obtained from the previous term by multiplying by the same number.

This number is the constant ratio, r

$u_1 \quad u_2 \quad u_3$

$2, 10, 50, 250, \ldots\ldots \quad \ldots \quad \_\_, \_\_$

$u_n \quad u_{n+1}$

$u_1 \quad u_2 \quad u_3$

common ratio $= r = \dfrac{u_2}{u_1}$

$= \dfrac{u_3}{u_2}$

$= \dfrac{u_4}{u_3}$

$r = \dfrac{u_{n+1}}{u_n}$

$r = \dfrac{u_n}{u_{n-1}}$

W

What is the common ratio of

135, 90, 60, 40, . . .

If the terms are getting smaller, then the common ratio must be less than 1

Note

You are expected to show work, using good notation in this unit.

Just fiddling around with a calculator won't work out well for you.

W

Is the following sequence geometric?

0.5 , 1 , 2 , 4 , 8 , 16 , ...

1 2 3

If so, what is common ratio? $r = 2$

How many applications of 2 do you need to get from the first term to the 3rd term?

to the 4th term?

$U_n = 0.5(2)^{n-1}$

to the 85th term?

to the nth term?

• if Geometric •.......

Your best friend

will be

Explicit formula for Geometric Sequences

$=$

n = # of terms

any term of interest

first term

common ratio

Explicit formula for Geometric Sequences

$$U_n = U_1 r^{n-1}$$

n = # of terms

any term of interest

first term

common ratio

Y

Let's try it!

What's the 23rd term of this sequence?

$$\frac{1}{9}, -\frac{1}{3}, 1, -3, 9, \ldots$$

Find the ratio:

$$r = \frac{u_2}{u_1} = \frac{-\frac{1}{3}}{\frac{1}{9}} = -3$$

$$\frac{1}{-\frac{1}{3}} =$$

$$\frac{-3}{1} =$$

$$u_{23} = \left(\frac{1}{9}\right)(-3)^{23-1}$$

$$= 3,486,784,401$$

$$3,490,000,000$$

or

$$3.49 \times 10^{9}$$

Z

The first term of a geometric sequence is 4 and the last term is 26,244. If there are 9 terms in the sequence, what is the common ratio?

$$26244 = 4(r)^{9-1}$$

$$26244 = 4r^{8}$$

$$r^{8} = \frac{26244}{4}$$

$$r = \sqrt[8]{\frac{26244}{4}}$$

$$r = \pm 3$$

$$u_n = u_1 + d(n-1)$$

$$u_n = u_1(r)^{n-1}$$

The first term of a geometric sequence is 4 and the last term is 26,244. If there are 9 terms in the sequence, what is the common ratio?

$(-3)^8$

Alternative wording for the same

find r if $u_1 = 4$ and $u_9 = 26{,}244$

Best friend

$U_n = u_1 (r)^{n-1}$

<u>Note</u>

If something grows at 15%, then the common ratio would be (1+.15) or 1.15

100% + 15%

115%

$$U_n = U_1 \cdot r^{n-1}$$

?

$$y = ab^x$$

0 term

B.B.

Geometric Series

the sum of a geometric sequence

Add the first 10 terms of

$4, 2, 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8} \ldots$

$S_{10} = 4 + 2 + 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} \ldots$

Luckily, there's a formula!

To find the sum of the first n terms of a geometric sequence:

See your friendly Formula Packet

Luckily, there's a formula!

To find the sum of the first n terms of a geometric sequence:

$$S_n = \frac{u_1(1-r^n)}{1-r}$$

for finite sequences

$$S_{10} = 4 + 2 + 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} \ldots$$

$$S_{10} =$$

$$= \frac{4\left(1-\left(\frac{1}{2}\right)^{10}\right)}{1-\frac{1}{2}}$$

$$= 8\left(1-\left(\frac{1}{2}\right)^{10}\right)$$

$$= \frac{1023}{128}$$

$$S_n = \frac{u_1(1-r^n)}{1-r}$$

reverse

reverse

$$S_n = \frac{u_1\cancel{(-1)}(r^n-1)}{\cancel{-}(r-1)}$$

$+(9-8)$

$-(8-9)$

Formula Packet

The n^{th} term of an arithmetic sequence	$u_n = u_1 + (n-1)d$
The sum of n terms of an arithmetic sequence	$S_n = \frac{n}{2}(2u_1 + (n-1)d) = \frac{n}{2}(u_1 + u_n)$
The n^{th} term of a geometric sequence	$u_n = u_1 r^{n-1}$ ✓
The sum of n terms of a geometric sequence	$S_n = \frac{u_1(r^n-1)}{r-1} = \frac{u_1(1-r^n)}{1-r}$, $r \neq 1$

$$S_n = \frac{u_1(r^n - 1)}{r - 1}$$

$$S_n = \frac{u_1(1 - r^n)}{1 - r}$$

$$S_n \frac{u_1(1 - r^n)}{1 - r}$$

$$S_n = \frac{u_1(r^n - 1)}{r - 1}$$

$$4\left(\frac{1 - \left(-\frac{3}{4}\right)^{20}}{1 - \left(-\frac{3}{4}\right)}\right)$$

$$4\left(\frac{\left(\frac{-3}{4}\right)^{20} - 1}{\left(\frac{-3}{4}\right) - 1}\right)$$

See your test

Assignment #2

Worksheet

Why does $S_n = \frac{g_1(1-r^n)}{1-r}$ work?

$$S_n = g_1 + g_1 r + g_1 r^2 + \dots\dots + g_1 r^{n-1}$$

Typical Geometric Series

last (or N^{th}) term

multiply by r

$$S_n = g_1 + g_1 r + g_1 r^2 + \dots\dots + g_1 r^{n-1}$$

Typical Geometric Series

last (or N^{th}) term

multiply by r

$$r S_n = g_1 r + g_1 r^2 + g_1 r^3 + \dots\dots g_1 r^{n-1} + g_1 r^n$$

subtract preceeding equation

$$S_n - rS_n = g_1 - g_1 r^n$$

$$S_n - rS_n = g_1 - g_1 r^n$$

$$(1-r)S_n = g_1$$

$$S_n = \frac{g_1(1-r^n)}{1-r}$$ as long as $r \neq 1$

Assignment 2 (Sequences/Series)

a worksheet