Pick up the Warm Up

(a) Imagine flipping a fair coin three times. Give a probability model for this chance process and be sure to justify why it is a valid model. [Hint: start by listing the sample space]

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TTT TTH THH HHH
THT HTH
HTT HHT

# Heads	0	1	2	3
Probability	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

It's valid because all probabilities add up to 1

(b) Define event A as getting 2 or more heads, event B as getting no heads, and event C as getting at least one head. Find the probability of each of these events.

(c) Are any of these probabilities related?

(b) Define event A as getting 2 or more heads, event B as getting no heads, and event C as getting at least one head. Find the probability of each of these events.

$P(A) = P(2 \text{ or more heads}) = \frac{3}{8} + \frac{1}{8} = \frac{3+1}{8} = 4/8$

$P(B) = P(\text{no heads}) = 1/8$

$P(C) = P(\text{at least 1 head}) = \frac{3+3+1}{8} = 7/8$

(c) Are any of these probabilities related?

- The events No heads and at least 1 head are mutually exclusive (can't happen at the same time) so... $P(B \text{ and } C) = 0$

- They are also complementary events so $P(B) = 1 - P(C)$

Random HW Check

- Use a two-way table or Venn diagram to model a chance process and calculate probabilities involving two events.
- Apply the general addition rule to calculate probabilities.

$P(A \text{ or } B) = P(A) + P(B)$

that's not it

Some people believe that the ability to **taco tongue** and **evil eyebrow** is something that you are born with.

Is this true? Are the two abilities somehow related?

Put a tally in **one** of the four cells

	Yes Evil Eyebrow	No Evil Eyebrow	Total
Yes Taco Tongue	|||| 4	||||| 5	
No Taco Tongue	| 1	|| 2	
Total			

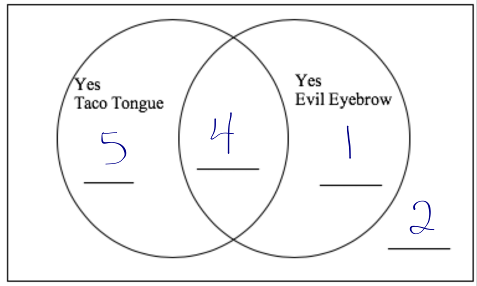
Pick Up the Handout
- do #1 for now

Can You Taco Tongue and Evil Eyebrow?
5.2 Day 2

Some people believe that the ability to taco tongue and evil eyebrow is something that you are born with. Is this true? Are the two abilities somehow related?

1. Collect class data to fill in the following two-way table and Venn Diagram.

	Yes Evil Eyebrow	No Evil Eyebrow	Total
Yes Taco Tongue	4	5	9
No Taco Tongue	1	2	3
Total	5	7	$\boxed{12}$



	Yes Evil Eyebrow	No Evil Eyebrow	Total
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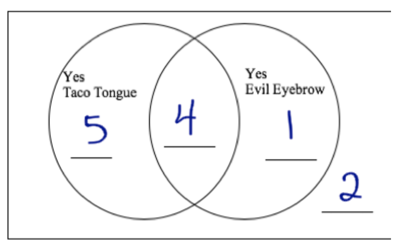


2. Suppose that we randomly choose a student from class. Find the following probabilities.

P(Yes Taco Tongue) = 9/12

P(No Taco Tongue) = 3/12

P(Yes Taco Tongue AND Yes Evil Eyebrow) = 4/12

P(Yes Taco Tongue AND No Evil Eyebrow) = 5/12

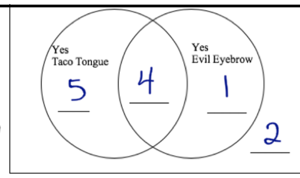
P(Yes Evil Eyebrow) = 5/12

P(No Evil Eyebrow) = 7/12

P(Yes Evil Eyebrow AND No Taco Tongue) = 1/12

P(No Taco Tongue AND No Evil Eyebrow) = 2/12

	Yes Evil Eyebrow	No Evil Eyebrow	Total
Yes Taco Tongue	4	5	9
No Taco Tongue	1	2	3
Total	5	7	$\boxed{12}$



3. Suppose that we randomly choose a student from class. Find the following probabilitie

Mutually Exclusive

P(Yes Evil Eyebrow) = 5/12

P(No Evil Eyebrow) = 7/12

P(Yes Evil Eyebrow OR No Evil Eyebrow) = 12/12

$P(A \text{ or } B) = P(A) + P(B)$

$\frac{5}{12} + \frac{7}{12} = \frac{12}{12}$

4. Suppose that we randomly choose a student from class. Find the following probabilitie

Not Mutually Exclusive

P(Yes Taco Tongue) = 9/12

P(Yes Evil Eyebrow) = 5/12

P(Yes Taco Tongue OR Yes Evil Eyebrow) = 10/12

$P(A \text{ OR } B) = P(A) + P(B) - P(A \text{ and } B)$

$\frac{9}{12} + \frac{5}{12} - \frac{4}{12} = \frac{10}{12}$

Debrief #3

Let's go back and add formal notation to our work.

"If you are **YES Evil Eyebrow**, please stand up"

We'll count out loud then

"If you are **NO Evil Eyebrow**, please stand up"

"If you are **YES Evil Eyebrow** or **NO Evil Eyebrow** please stand up"

We could have simply added the counts from the first two groups.

There are two different uses of the word "**OR**" in everyday life.

When you are asked if you want "soup or salad," the waiter wants you to choose one or the other, but not both.

However, when you order coffee and are asked if you want "cream or sugar," it's OK to ask for one or the other or both.

When you are asked if you want "soup or salad," the waiter wants you to choose one or the other, but not both.

However, when you order coffee and are asked if you want "cream or sugar," it's OK to ask for one or the other or both.

In mathematics and probability, "A or B" means one or the other or both.

Debrief
#4

"If you are **YES Taco Tongue**, please stand up"

"If you are **YES Evil Eyebrow**, please stand up"

"If you are **YES Taco Tongue** **OR** **YES Evil Eyebrow** please stand up"

Big problem. We can't simply add the counts from the first two goups. Why not?

Venn Diagrams and the General Addition Rule

Venn Diagrams and the General Addition Rule

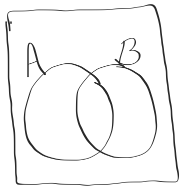
TWO-WAY TABLES VENN Diagram

Venn Diagrams and the General Addition Rule

TWO-WAY TABLES

	B	B^c
A		
A^c		

VENN Diagram

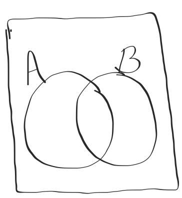


Venn Diagrams and the General Addition Rule

TWO-WAY TABLES

	B	B^c
A	1	2
A^c	3	4

VENN Diagram



Venn Diagrams and the General Addition Rule

TWO-WAY TABLES

VENN Diagram

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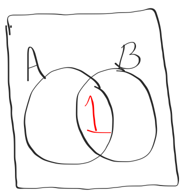

Venn Diagrams and the General Addition Rule

TWO-WAY TABLES

VENN Diagram

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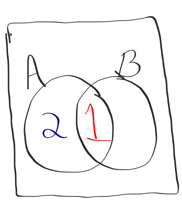

Venn Diagrams and the General Addition Rule

TWO-WAY TABLES

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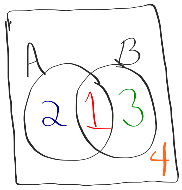

$\cup$ $\cap$

Venn Diagrams and the General Addition Rule

TWO-WAY TABLES

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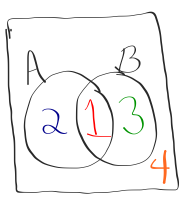
VENN Diagram



General Addition Rule

$$P(A \overset{\cup}{\text{or}} B) = P(A) + P(B) - P(A \overset{\cap}{\text{and}} B)$$

Venn Diagrams and the General Addition Rule

TWO-WAY TABLES

	B	B^c
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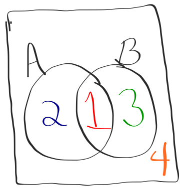
VENN Diagram



General Addition Rule

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

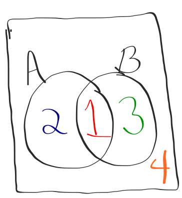
If A and B are Mutually Exclusive $P(A \text{ and } B) = 0$ since they cannot occur together,

Venn Diagrams and the General Addition Rule

TWO-WAY TABLES

	B	B^c
A	1	2
A^c	3	4

VENN Diagram



General Addition Rule

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

If A and B are Mutually Exclusive $P(A \text{ and } B) = 0$ since they cannot occur together, so $P(A \text{ or } B) = P(A) + P(B)$

Check for Understanding but using formal notation. What is the relationship between educational achievement and home ownership? A random sample of 500 U.S. adults was selected. Each member of the sample was identified as a high school graduate (or not) and as a homeowner (or not). The two-way table displays the data. Suppose we choose a member of the sample at random. Define events

G: person is a high school graduate H: person is a homeowner.

	High school graduate	Not a high school graduate
Homeowner	221	119
Not a homeowner	89	71

1. Explain in plain language what $P(G^c)$ means and find the probability.

Probability that the person is not a HS graduate

Check for Understanding but using formal notation. What is the relationship between educational achievement and home ownership? A random sample of 500 U.S. adults was selected. Each member of the sample was identified as a high school graduate (or not) and as a homeowner (or not). The two-way table displays the data. Suppose we choose a member of the sample at random. Define events

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The probability the person is not a high school student is 0.38

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1. Explain in plain language what $P(G^c)$ means and find the probability.

The probability the person is not a high school student is 0.38

$P(G^c) = \frac{190}{500} = 0.38$

Did you use appropriate notation?

	High school graduate	Not a high school graduate	
Homeowner	221	119	= 340
Not a homeowner	89	71	= 160
	310	190	$\boxed{500}$

2. Explain why $P(G \text{ or } H) \neq P(G) + P(H)$. Then find $P(G \text{ or } H)$.

3. Make a Venn diagram to the right to display the sample space of this chance process.

4. Write the event "is not a high school graduate and is a homeowner" in symbolic form and find the probability.

	High school graduate	Not a high school graduate	
Homeowner	221	119	= 340
Not a homeowner	89	71	= 160
	310	190	$\boxed{500}$

2. Explain why $P(G \text{ or } H) \neq P(G) + P(H)$. Then find $P(G \text{ or } H)$.

There are some people who graduated and own a home so they were counted twice

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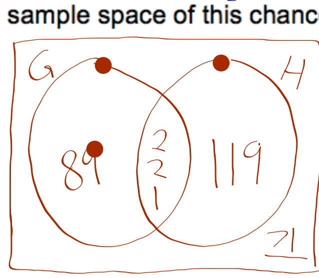
There are some people who graduated and own a home so they were counted twice

$$P(G \text{ or } H) = \frac{310}{500} + \frac{340}{500} - \frac{221}{500}$$

$$= \frac{429}{500} = .858$$

3. Make a Venn diagram to the right to display the sample space of this chance process.



4. Write the event "is not a high school graduate and is a homeowner" in symbolic form and find the probability.

$$P(H \cap G^c) = \frac{119}{500}$$

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	310	190	$\boxed{500}$

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?

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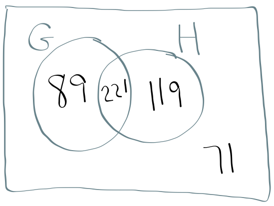


4. Write the event "is not a high school graduate and is a homeowner" in symbolic form and find the probability.

	High school graduate	Not a high school graduate	
Homeowner	221	119	= 340
Not a homeowner	89	71	= 160
	310	190	$\boxed{500}$

2. Explain why $P(G \text{ or } H) \neq P(G) + P(H)$. Then find $P(G \text{ or } H)$.

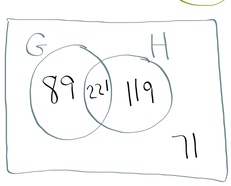
There are some people who graduated and own a home so they were counted twice

$$P(G \text{ or } H) = \frac{310}{500} + \frac{340}{500} - \frac{221}{500}$$

$$= \frac{429}{500} = .858$$

?

3. Make a Venn diagram to the right to display the sample space of this chance process.



4. Write the event "is not a high school graduate and is a homeowner" in symbolic form and find the probability.

$$P(G^c \text{ and } H) = \frac{119}{500} = 0.238$$

BB

Classwork - Part 2

How do we find probabilities from a two-way table?

	High school graduate	Not a high school graduate	Total
Homeowner	221	119	340
Not a homeowner	89	71	160
Total	310	190	500

Define event A as being a high school graduate. Define event B as being a homeowner. Suppose we choose a member of the sample at random. Find the probability that the member:

(a) is a high school graduate.

	High school graduate	Not a high school graduate	Total
Homeowner	221	119	340
Not a homeowner	89	71	160
Total	310	190	500

Define event A as being a high school graduate. Define event B as being a homeowner. Suppose we choose a member of the sample at random. Find the probability that the member:

(a) is a high school graduate.

$P(A) = \underbrace{P(\text{high school grad})}_{\text{Not necessary since event A has been clearly defined}} = \frac{310}{500}$

	High school graduate	Not a high school graduate	Total
Homeowner	221	119	340
Not a homeowner	89	71	160
Total	310	190	500

(b) is a high school graduate and owns a home.

A and B

d) P(A∩B) aren't mutually exclusive and would be counted twice

(c) is a high school graduate or owns a home.

$P(G \cup H) = P(G) + P(H) - P(G \cap H)$

$= \frac{310 + 340 - 221}{500} = \frac{429}{500}$

	High school graduate	Not a high school graduate	Total
Homeowner	221	119	340
Not a homeowner	89	71	160
Total	310	190	500

(b) is a high school graduate and owns a home.

$P(A \text{ and } B) = \frac{221}{500} = .442$

(c) is a high school graduate or owns a home.

$P(A \text{ or } B)$

$= \frac{221 + 89 + 119}{500}$

$= \frac{429}{500}$

(d) Explain why $P(A \text{ or } B) \neq P(A) + P(B)$

$P(A \text{ and } B) = \frac{221}{500}$

$P(A) + P(B) = \frac{310}{500} + \frac{340}{500}$

Outcomes are double counted

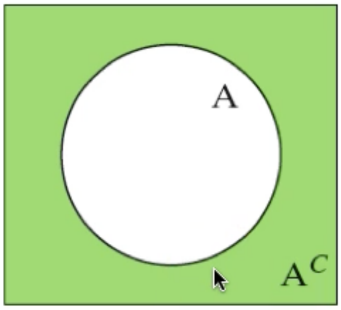
	High school graduate	Not a high school graduate	Total
Homeowner	221	119	340
Not a homeowner	89	71	160
Total	310	190	500

(d) Explain why $P(A \text{ or } B) \neq P(A) + P(B)$

Because the outcomes of $P(A) + P(B)$ are double counted.

Venn Diagrams

and some notation



You can see why

P(A) and P(A^c) add up to 1

Complement Rule:

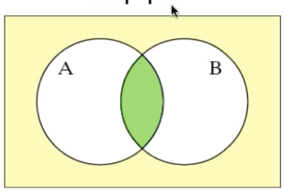
$P(A) + P(A') = 1$

$P(A) = 1 - P(A')$

$P(A') = 1 - P(A)$

AND

$A \cap B$

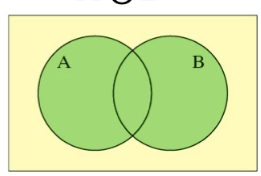


Intersection

(football)

OR

$A \cup B$



Union

left Pacman

Right Pacman

HINT: To keep the symbols straight, remember

∪ for **U**nion and

∩ for i**n**tersection.

Where are the best tacos?

A survey of all students at a large high school revealed that, in the last month, 38% of them had dined at Taco Bell, 16% had dined at Chipotle, and 9% had dined at both. Suppose we select a student at random. What's the probability that the student has dined at Taco Bell or Chipotle in the last month?

a)

b) Now create a **Venn Diagram** to display the sample space in a different way.

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a)

b) Now create a **Venn Diagram** to display the sample space in a different way.

3. Where are the best tacos?

A survey of all students at a large high school revealed that, in the last month, 38% of them had dined at Taco Bell, 16% had dined at Chipotle, and 9% had dined at both. Suppose we select a student at random. What's the probability that the student has dined at Taco Bell or Chipotle in the last month?

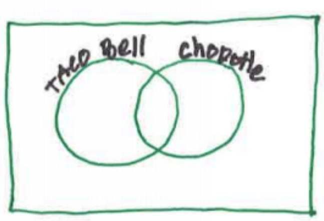
General Addition Rule

= P(Taco Bell OR Chipotle)

= P(Taco Bell) + P(Chipotle) − P(Taco Bell and Chipotle)

= .38 + .16 − .09 = .45

Now create a **Venn Diagram** to display the sample space in a different way.



Left Pacman

Right Pacman

football

Take Home LCQ - due no later than the start of class tomorrow.

5.2 .....41, 47, 49, 51, 53, 55-58

and study pp.318-325