6.3 Day 2 (more on binomial distributions)

✓CALCULATE the mean and standard deviation of a binomial random variable. INTERPRET these values.

✓When appropriate, USE the Normal approximation to the binomial distribution to CALCULATE probabilities.*

Pick Up the handout and do #1

Lesson 6.3: Day 2: Will the SHS girls' soccer team win?

When the time runs out in a soccer game and the score is tied, the game will go to a shootout. Each team gets to choose 5 players to kick penalty kicks. Whichever team makes the most penalty kicks wins. Suppose the IRISH soccer team makes 60% of their penalty kicks, what are the chances they will win the game?

1. Is this a binomial setting? Explain.

B I N S

When the time runs out in a soccer game and the score is tied, the game will go to a shootout. Each team gets to choose 5 players to kick penalty kicks. Whichever team makes the most penalty kicks wins. Suppose the IRISH soccer team makes 60% of their penalty kicks, what are the chances they will win the game?

1. Is this a binomial setting? Explain.

B I N S

Success: Make penalty

failure: miss

each kick is independent

$n=5$

$p=0.6$ for all kicks

So.... binomial situation

$B(5, .6)$

2. Fill in the table below showing the probability of making X penalty kicks. Mr. C may show you a shortcut.

Goals (*X*)	0	1	2	3	4	5
Probability						

P(0 successes)

$P(3)$

${}_5C_3\,(.6)^3(.4)^2$

ENTER VALUES of X in L_1

can use binompdf( ) → L_2

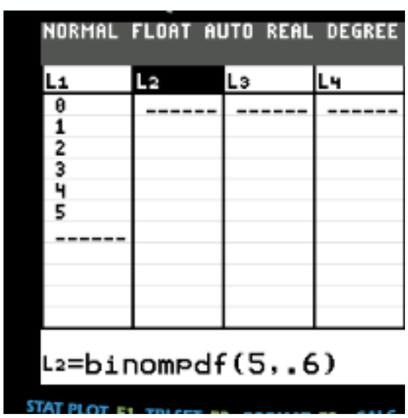
2. Fill in the table below showing the probability of making X penalty kicks. Mr. C may show you a shortcut.

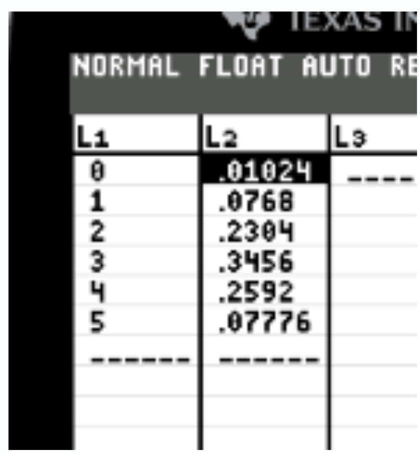
Goals (X)	0	1	2	3	4	5
Probability						

Technology Assist

L_1	L_2
0	
1	
2	
3	
4	
5	

binompdf(5, .6) → L_2





AP 6.3 Day 2 Binom Distrib in in sampling

November 20, 2019

1. Is this a binomial setting? Explain.

Binary: Success → make Penalty / Each shot / Trials / Same prob. $B(5, 0.6)$
Failure → don't make / Independent / $n=5$ / $p=0.6$

2. Fill in the table below showing the probability of making X penalty kicks. Mr. C may show you a shortcut.

Goals (*X*)	0	1	2	3	4	5
Probability	.01024	.0768	.2304	.3456	.2592	.07776

3. Find and interpret the mean of the probability distribution. Show your work.

4. Find and interpret the standard deviation of the distribution.

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for rand.var. distrib

$\sum x \cdot p$

$\mu = 0(.01024) + 1(.0768) + \cdots$

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3. Find and interpret the mean of the probability distribution. Show your work.

for rand.var. distrib

$\sum x \cdot p$

$\mu = 0(.01024) + 1(.0768) + \cdots = 3$

After many shootouts,

1-Var stat L_1, L_2

4. Find and interpret the standard deviation of the distribution.

3. Find and interpret the mean of the probability distribution. Show your work.

for rand.var distrib

$\Sigma x \cdot p$

$\mu = 0(.01024) + 1(.0768) + \ldots = 3$

After many shootouts, we expect the average number of goals made to be 3 out of the 5 attempted.

4. Find and interpret the standard deviation of the distribution.

3. Find and interpret the mean of the probability distribution. Show your work.

shortcut for binomial

$\mu = np$

for rand.var distrib

$\Sigma x \cdot p$

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After many shootouts, we expect the average number of goals made to be 3 out of the 5 attempted.

4. Find and interpret the standard deviation of the distribution.

$\sum (x_i - \mu)^2 \cdot p(x)$

$\sigma = (0-3)^2(.01024) + (1-3)^2(.0768) + \ldots = 1.09$

3. Find and interpret the mean of the probability distribution. Show your work.

shortcut $\mu = np$

for rand.var distrib

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We expect the number of goals made in a shootout to typically vary by 1.09 goals from the mean of 3 out of 5 goals.

3. Find and interpret the mean of the probability distribution. Show your work.

shortcut $\mu = np$

for rand.var distrib

$\sum x \cdot p$

$\mu = 0(.01024) + 1(.0768) + \ldots = 3$

After many shootouts, we expect the average number of goals made to be 3 out of the 5 attempted.

4. Find and interpret the standard deviation of the distribution.

shortcut for binom.

$\sigma = \sqrt{np(1-p)}$

$\sum (x_i - \mu)^2 \cdot p(x)$

$\sigma = (0-3)^2(.01024) + (\quad)^2(\quad) + \ldots = 1.09$

We expect the number of goals made in a shootout to typically vary by 1.09 goals from the mean of 3 out of 5 goals.

II. Probability and Distributions

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ $\quad$ $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$

Probability Distribution	Mean	Standard Deviation
Discrete random variable, X	$\mu_X = E(X) = \sum x_i \cdot P(x_i)$	$\sigma_X = \sqrt{\sum (x_i - \mu_X)^2 \cdot P(x_i)}$
If X has a **binomial** distribution with parameters n and p, then: $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$ where $x = 0, 1, 2, 3, \ldots, n$	$\mu_X = np$ ✓	$\sigma_X = \sqrt{np(1-p)}$ ✓

5. What is the probability that the team scores at least one goal?

6. If the other team is expected to make 3 goals, what is the probability that the SHS girls' team wins?

5. What is the probability that the team scores at least one goal?

$$P(\text{at least } 1) = 1 - P(\text{none})$$
$$= 1 - .01024 = .98976$$

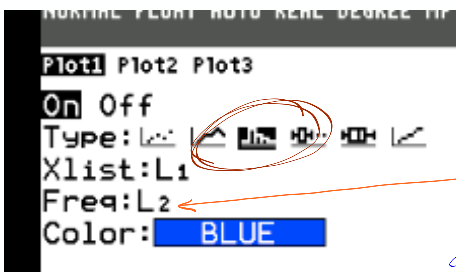
6. If the other team is expected to make 3 goals, what is the probability that the SHS girls' team wins?

$$P(4 \text{ or } 5) = P(4) + P(5)$$
$$= .2592 + .07776$$
$$= .337$$
$$.33696$$

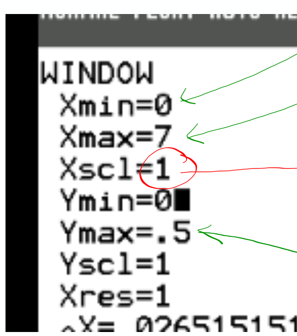
7. Use technology to make a histogram of the probability distribution and then Describe its shape.

L_1	L_2
0	
1	
2	
3	
4	
5	

binompdf



freq: L_2

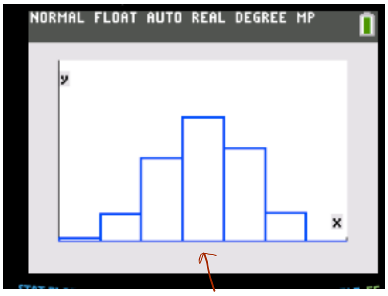


Smallest value of binom. rand. variable

A bit larger than rand. var

Must be 1

Larger than your highest prob



3

Symmetric
with Single Peak
at 3 shots
made

Textbook Site

Extra Applets
-Probability
→ Binomial Distrib

http://bcs.whfreeman.com/webpub/statistics/spa3e/analyze_data/prob.html

Describing Binomial Distributions

Important ideas:

Describing Binomial Distributions

Important ideas:

Shape:

Center:

Variability:

Describing Binomial Distributions

Important ideas:

Shape: Make a histogram → Quick sketch on your paper

Center: $\mu = np$

Variability: $\sigma = \sqrt{np(1-p)}$

Pop Quiz
Guessing

POP QUIZ GUESSING: Mr. Miller's class is very difficult. It's so hard that when he gave a pop quiz recently, the students just guessed on every question! Each student in the class guesses an answer from A through E on each of the 10 multiple-choice questions. Hannah is one of the students in this class. Let Y = the number of questions that Hannah answers correctly.

Note: B: **Success** (correct answer) **Failure** (Not correct)
I: Each shot is independent.
N: Set # of trials, $n = 10$
S: Same probability, $p = 0.2$

THEREFORE: ____________________

1. Use technology to make a histogram of the probability distribution of Y. Describe its shape.

POP QUIZ GUESSING: Mr. Miller's class is very difficult. It's so hard that when he gave a pop quiz recently, the students just guessed on every question! Each student in the class guesses an answer from A through E on each of the 10 multiple-choice questions. Hannah is one of the students in this class. Let Y = the number of questions that Hannah answers correctly.

Note: B: **Success** (correct answer) **Failure** (Not correct)
I: Each shot is independent.
N: Set # of trials, $n = 10$
S: Same probability, $p = 0.2$

THEREFORE: this setting represents a binomial situation.

1. Use technology to make a histogram of the probability distribution of Y. Describe its shape.

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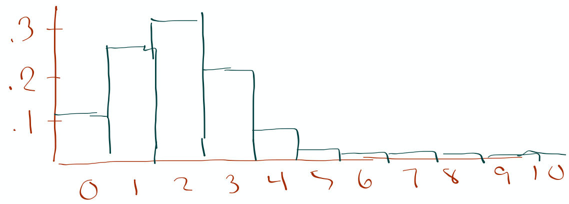


Skewed right with a single peak at 2 questions correct

2. Calculate the mean of Y and then complete the interpretation.

After ____________________ quizzes, we expect the average number of correct _____________ is ________ questions.

3. Calculate the standard deviation of Y and then complete the interpretation.

The number correct on a quiz of 10 questions _______________ varies by ___________ from the mean of 2 questions.

2. Calculate the mean of Y and then complete the interpretation. $\mu = np$

After ____________________ quizzes, we expect the average number of correct ______________ is ________ questions.

3. Calculate the standard deviation of Y and then complete the interpretation.

$\sigma = \sqrt{np(1-p)}$

The number correct on a quiz of 10 questions _________________ varies by _____________ from the mean of 2 questions.

2. Calculate the mean of Y and then complete the interpretation. $\mu = np = 10 \times 0.2 = 2$

After many many *quizzes, we expect the average number of correct* out of 10 *is* 2 *questions.*

3. Calculate the standard deviation of Y and then complete the interpretation.

$\sigma = \sqrt{np(1-p)} = \sqrt{10(.2)(.8)} = 1.26$

The number correct on a quiz of 10 questions typically *varies by* 1.26 *from the mean of 2 questions.*

Binomial Distributions in Statistical Sampling

Almost all real-world sampling, such as taking an SRS from a population of interest, is done without replacement.

Sampling without replacement leads to a violation of the Independent condition of the binomial setting.

But, when the population is much larger than the sample, a count of successes in an SRS of size n has an ***approximately*** binomial distribution.

In practice, the binomial distribution gives a good approximation to situations that don't have replacement (non-independence) as long as we sample less than 10% of the population. This is called the 10% condition.

When taking a random sample of size n from a population of size N, we can use a binomial distribution to model the count of successes in the sample as long as $n < 0.10N$. We refer to this as the **10% condition**.

Describing Binomial Distributions

Important ideas:

Shape: Make a histogram → Quick sketch on your paper

Center: $\mu = np$

Variability: $\sigma = \sqrt{np(1-p)}$

10% CONDITION
if $n < 10\% N$
can use binomial setting

PEW RESEARCH CENTER: A recent report from the Pew Research Center estimates that 71% of teenagers aged 13-17 use Snapchat. Assume this claim is true. Suppose that some researchers are going to contact a random sample of 300 teenagers to find out if they use Snapchat. Let X = the number of teens in a random sample of size 300 who use Snapchat.

(a) Explain why X can be modeled by a binomial distribution even though the sample was selected without replacement.

(b) Use a binomial distribution to estimate the probability that 200 or more teens in the sample use Snapchat.

(a) Explain why X can be modeled by a binomial distribution even though the sample was selected without replacement.

300 is definitely less than 10% of all teenagers aged 13-17

(b) Use a binomial distribution to estimate the probability that 200 or more teens in the sample use

. . . . . ~~197~~ ~~198~~ ~~199~~ 200 201 202 . . .

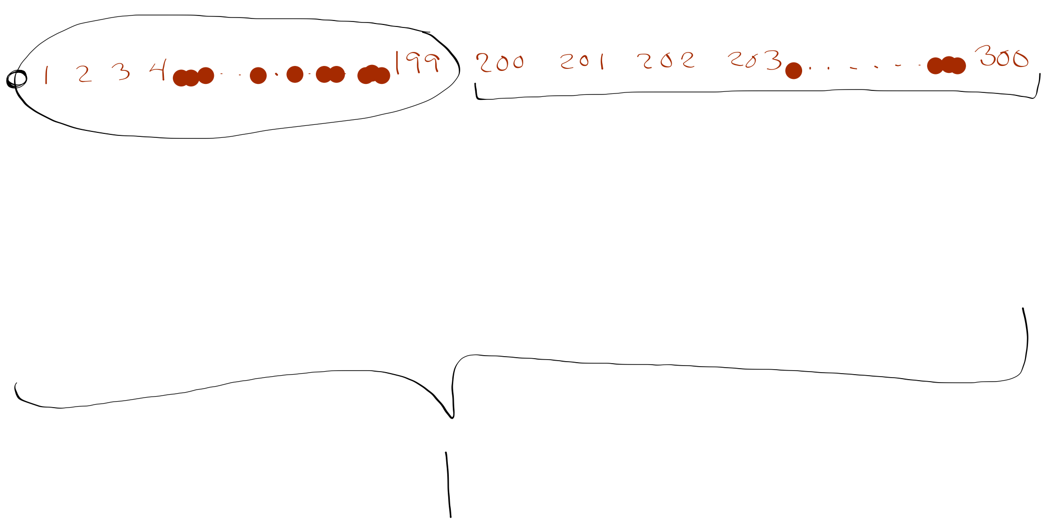
$P(X \geq 200)$

$= 1 - (X \leq 199)$

$= 1 -$ binomcdf [trials: 300, p: 0.71, X-value: 199]

$= 1 - 0.044$

$= 0.956$



6.3....91, 93, 95, 99, 101, 105

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