Review Problems – Sets and Venn Diagrams
(Probability in a later review) Key

1 **Question 1**

Consider the following sets:

$$U = \{1, 2, 3, 4, 5, 6, 7, 8\}$$

$$A = \{2, 3, 4, 5\} \qquad B = \{2, 4, 6, 8\}$$

a.   Determine whether each of the following is true or false. Write the appropriate word in the answer space below.

∅ stands for "Empty" Set
∅ is a subset of every set.

(i) $\{3\} \in A$ Yes, 3 is an element of A

(ii) $\emptyset \subset B$ ⟵ *Odd question* *so true*

b.   Determine each of the following:
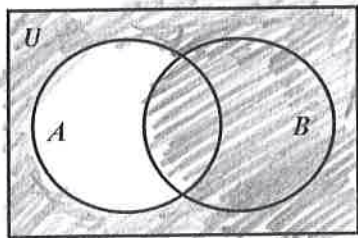
(i) $A' = \{1, 6, 7, 8\}$

(ii) $A' \cap B = \{6, 8\}$ is the intersection

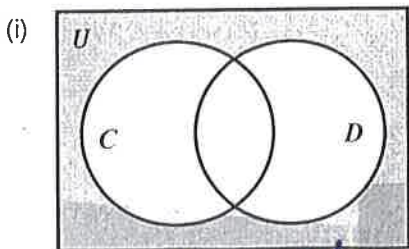(iii) $A \cup B = \{2, 3, 4, 5, 6, 8\}$

(iv) U' → any number that is not one of the integers 1, 2, 3, ...8
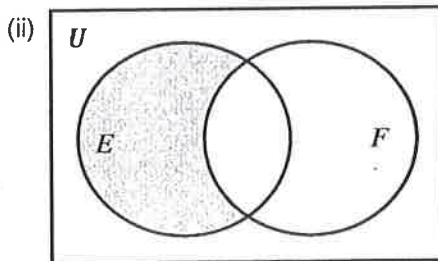
2 **Question 2**

a.   Shade $A' \cup B$ in the Venn diagram shown below:

b.   Identify the shaded region in each of the Venn diagrams shown below:

(i)

$(C \cup D)'$

(ii)

$E \cap F'$ is one possibility

3 **Question 3**

Consider the following sets:

$$U = \{Positive\ Integers\}$$

$$G = \{x \mid x < 4,\ x \in Z^+\} \qquad H = \{x \mid 2 \le x \le 6, x \in Z^+\}$$
1, 2, 3   2, 3, 4, 5, 6

a.   Identify the following sets:

(i) $G' = \{5, 6, 7, ...\}$
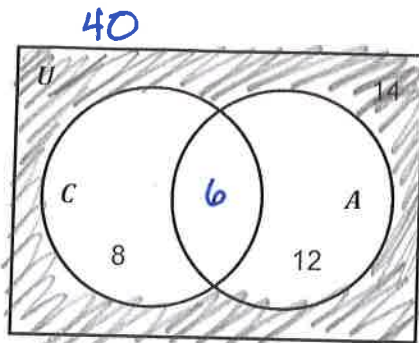
(ii) $G \cap H = \{2, 3\}$

b.   Identify 3 subsets of G. $\{1\}$ $\{1, 2\}$ and $\{2, 3\}$ for example

$\boxed{4}$ ## Question 4

A high school language department is offering Chinese (C) and Arabic (A) to new students. The Venn diagram below shows the number of students signed up for these classes.

40

$40 - (8+12+14) = 6$

There are 40 new students in the school. How many of the new students are:

a. (i)   taking Chinese and Arabic?   6

(ii)   taking Chinese only?   8

b.   In the Venn diagram shown above shade the region corresponding to $(C \cup A)'$.

$\boxed{5}$ ## Question 5

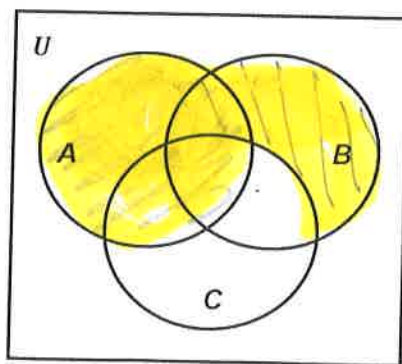a. (i)   Consider the following set: $X = \{a, b, c\}$.

$\{a,b\}$ $\{a,c\}$
$\{b,c\}$

List all the subsets of X having at least 2 elements.

(ii)   Calculate the total number of subsets of $\{a, b, c\}$.   a   ab   ac
b   bc   c   so $\boxed{6}$

b.   Consider the Venn diagram shown below:

Shade the region in the diagram above which represents $A \cup (B \cap C')$.

6

Question 6

Consider the following sets:

$$U = \{all\ IB\ students\}$$
$$M = \{all\ IB\ students\ studying\ Mathematics\}$$
$$B = \{all\ IB\ students\ studying\ Biology\}$$
$$F = \{all\ IB\ students\ studying\ French\}$$

same as "and"

a. Describe the following sets in words:

(i) $M \cup B$ students who study math or biology or both

(ii) $M' \cap F$ students who don't study math but also do study French

(iii) $M' \cup (B \cap F)$ students who don't study math or students who study bio and french

7

Question 7

a. If $n(A) = 9$ and $n(B) = 16$ and $n(A \cup B) = 20$, then calculate $n(A \cap B)$?   so $n(A \cap B) = 5$

$20 = 9 + 16 - x$
$20 = 25 - x$
$x = 5$

b. A student affairs committee U has three sub-committees:
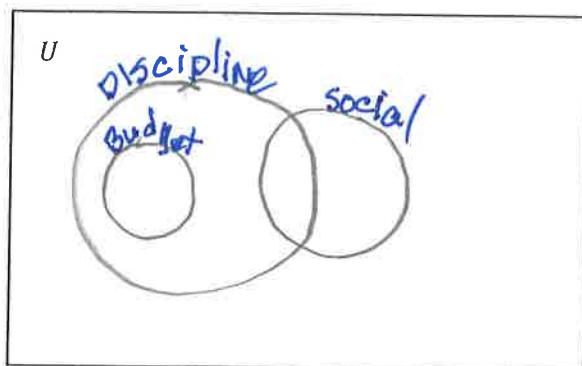
$$D = \{Discipline\ Review\} \qquad B = \{Budgeting\}$$
$$S = \{Social\ Events\}$$

No student belongs to both Budgeting and Social Events.

All members of Budgeting also belong to Discipline Review.

Some students belong to both Discipline Review and Social Events.

Construct and clearly label a Venn diagram representing the given information on the rectangular space provided below.

Question 8

8

a. A survey was given to 140 students to find out their opinions about ways of reducing traffic accidents and 10 said that they had no opinion.
The main findings were:

80 students favoured increasing the penalty (P);

40 favoured increasing the minimum driving age (A);

60 students favoured lowering the speed limit (S);

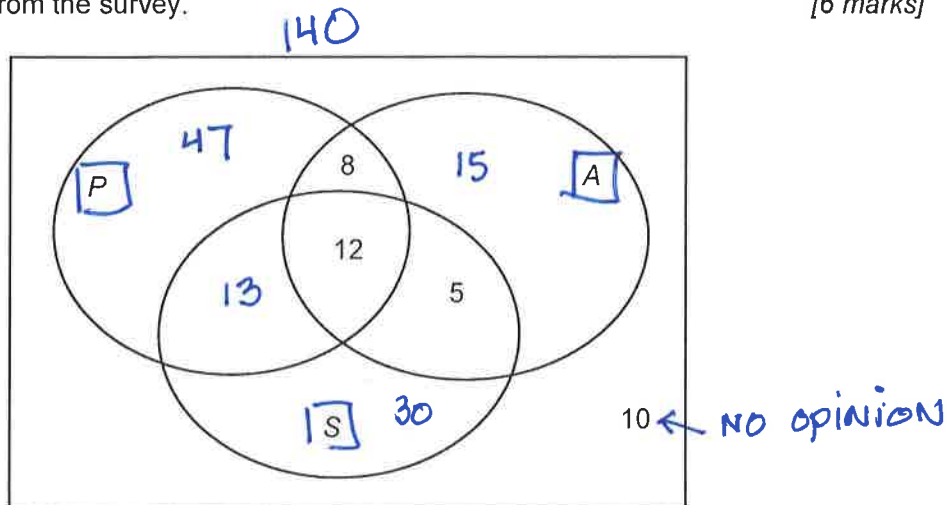25 favoured lowering the speed limit and increasing the penalty;

17 wanted to lower the speed limit and increase the minimum driving age;

20 wanted to increase the minimum driving age and increase the penalty;

→ 12 students favoured all three changes.

6

(i) Copy and clearly label the Venn diagram provided below.
Complete the diagram to represent all information obtained
from the survey. **[6 marks]**

140

47   8   15
P   A
      12
13   5

S   30

10 ← NO OPINION

(ii) Determine the number of students who favoured

(a) lowering the speed limit only. **30** **[1 mark]**

(b) increasing the penalty and lowering the speed limit, but
not increasing the driving age. **13** **[1 mark]**

b. Question continued on next page . . .

b. The results of voting preference poll for 70 voters aged 18 – 38 are shown in the table below.

		Political Choice			
		Socialist (S)	Conservative (C)	New Alliance (N)	Total
Age	18 - 24 (A1)	4	7	12	23
	25 - 31 (A2)	9	4	8	21
	32 - 38 (A3)	15	6	5	26
	Total	28	17	25	70

Given that A1, A2 and A3 are the three age groups and that S, C and N are the political choices then determine the number of voters in each of the following sets: [6 marks]

(i)   A2 ∩ C   4 voter

(ii) A3 ∪ N   46 ← 15+6+5+8+12

(iii) A1 ∪ (S ∩ A2)   32 voters
      23 + 9

(14 total marks)

9

Question 9

10, 11, 12, ... 19, 20

a. Given the following information:

→ ← means integers

$$U = \{x \mid 10 \leq x \leq 20, x \in Z\}$$

$$A = \{12, 14, 16, 18, 20\} \qquad B = \{\text{multiples of 5 between 9 and 21}\}$$

10, 15, 20

$$C = \{x \mid 14 < x \leq 18, x \in Z\}.$$

14 15 16 17 18

Determine each of the following: [6 marks]

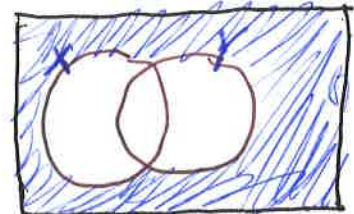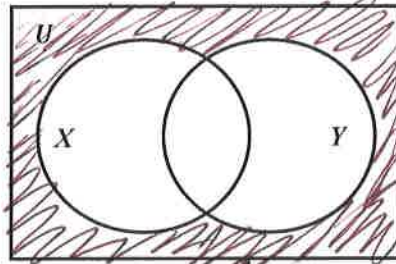(i) $A' = $ 10, 11, 13, 15, 17, 19

(ii) $A \cap B' = $ 12, 14, 6, 18

(iii) $B \cup (A \cap C) = $ 10, 14, 15, 16, 18, 20

10, 15, 20   14, 16, 18

$B' = $ 11, 12, 13, 14, 16, 17, 18, 19

b. Consider the Venn diagram shown below: [6 marks]

(i) Copy this diagram and use it to shade the solution of $(X \cup Y)'$.

(ii) Copy the above diagram a second time and use it to shade the solution of $X' \cap Y'$.

(iii) Hence or otherwise, interpret the relationship between $(X \cup Y)'$ and $X' \cap Y'$.

Since the shaded Venn diagrams are equivalent,

$(X \cup Y)'$ and $X' \cap Y'$ must be equivalent

(12 total marks)

9 continued c. In a survey 80 students were asked about the type of personal computer they used. The following information was gathered:
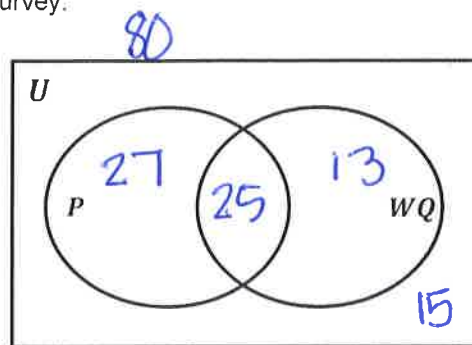
52 students used a 12 inch Peach laptop

38 used a 6 inch WQ Home tablet

15 used neither the laptop or the tablet.

Let P = {12 inch Peach laptop users} and WQ = {6 inch WQ Home tablet users}

(i) Copy and clearly label the Venn diagram shown below.
Complete the diagram to represent all information obtained
from the survey. **[4 marks]**

80

U

27 25 13

P   WQ

15

$80 - 15 = 65$
65 is the UNION
$n(P \cup WQ) = 65$

$\boxed{ } = \boxed{} + \boxed{} - \boxed{}$
$65 = 52 + 38 - X$
$X = 25$

(ii) How many students are using the Peach and not the WQ ?   **27** **[1 mark]**

(iii) Use appropriate set notation to describe the students
"not using the Peach or the WQ". **[1 mark]**

$(P \cup WQ)'$

(12 total marks)

10

a. The probability of a dolphin sighting on North Beach on any given day is 0.9, and the probability of sighting a dolphin on any given day is independent of sighting on another day.

What is the probability of sighting a dolphin on North Beach

(i) next Tuesday and Wednesday?

$P(\text{Dolphin and Dolphin}) = (0.9)(.9) = .81$ or 81%

since independ.

(ii) next Tuesday or next Wednesday? 0.9

b. If it is known that 60% of the dolphin population at North Beach is male, then determine the probability that the first three dolphins sighted this week will be male. Write your answer correct to 2 significant figures.

Working .....

$$P\left(\begin{array}{c}\text{FIRST} \\ \text{DOLPHIN} \\ \text{IS} \\ \text{male}\end{array} \text{ and } \begin{array}{c}\text{2nd Dolphin} \\ \text{is male}\end{array} \text{ and } \begin{array}{c}\text{3rd Dolphin} \\ \text{is male}\end{array}\right)$$

$$= (.9)(.9)(.9)$$

$$= .729 = .73$$

(a) (i) _____ .81 or 81% _____
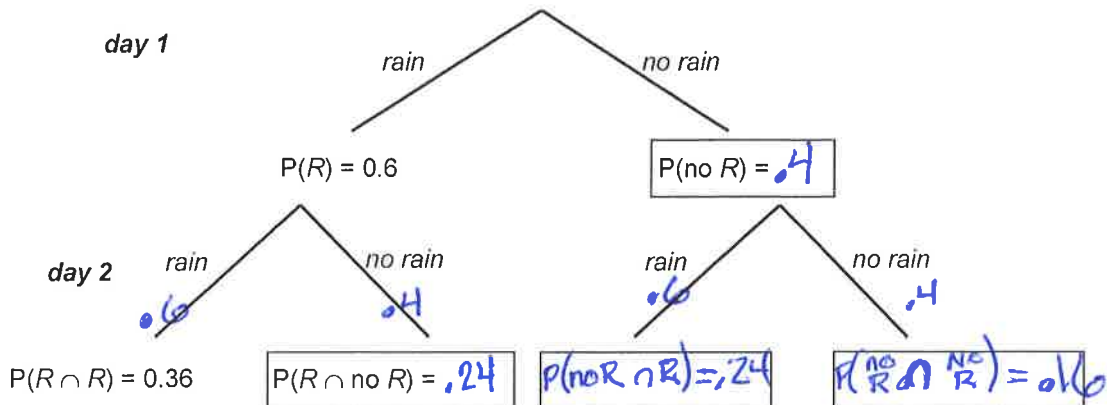
(ii) _____ 0.9 _____

(b) _____ 0.73 or 73% _____

(6 marks)

11

Assume that the probability that it will rain (R) in London on any given day is 0.6.

a. Complete the tree diagram shown below by filling in the rectangluar spaces to show all possible probabilities along with correct labels for two consecutive days.

day 1

rain   no rain

$P(R) = 0.6$ $P(\text{no } R) = .4$

day 2   rain   no rain   rain   no rain

.6   .4   .6   .4

$P(R \cap R) = 0.36$ $P(R \cap \text{no } R) = .24$ $P(\text{no } R \cap R) = .24$ $P(\overset{no}{R} \cap \overset{no}{R}) = .16$

b. Calculate the expected number of days with rain in London for the month of April (30 days).

Working...... 60% of 30

$(0.6)(30)$

18 days

a. Write the missing information in the rectangles shown on the tree diagram above.

b. _____18 days_____

(6 marks)

Question 9

Jar A shown below contains three balls number 1 ball and one each of numbers 3 and 4 balls. *Jar B* contains two number 2 balls and one number 8 ball.

One ball is selected from each of the jars shown below. The numbers on the balls are noted and are added together.

jar A   **jar B**
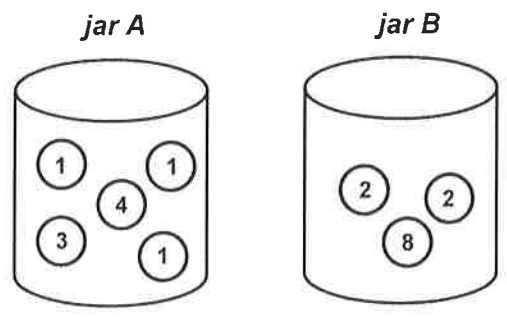
tough question

a. Construct and clearly label a probability tree diagram to illustrate the possible outcomes and their corresponding probabilities. **[6 marks]**

b. By using your tree diagram, or otherwise, determine the probability of the sum being

4 ways to get at least a 6

(i) at least 6. $\left(\frac{3}{5}\right)\left(\frac{1}{3}\right) + \left(\frac{1}{5}\right)\left(\frac{1}{3}\right) + \left(\frac{1}{5}\right)\left(\frac{2}{3}\right) + \left(\frac{1}{5}\right)\left(\frac{1}{3}\right) = \frac{7}{15}$ or 46.7% **[2 marks]**

(ii) less than 6 given that a 3 is drawn on the first ball. **[2 marks]**

$\dfrac{2/3}{1} = 2/3$

JAR A   JAR B
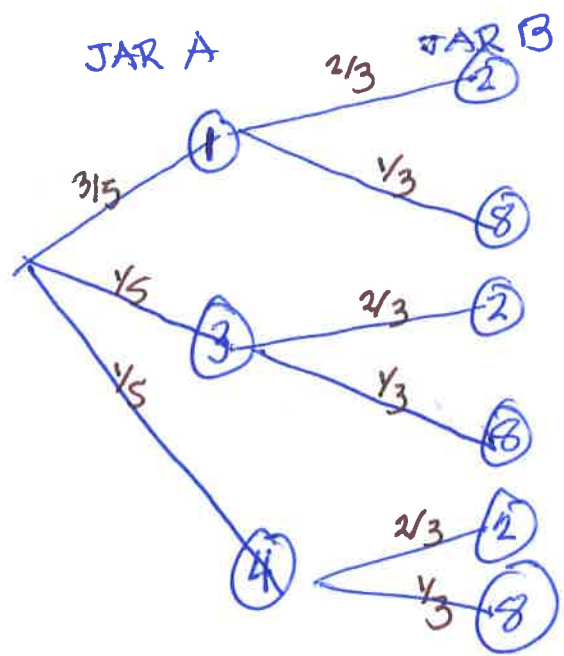
b. Mrs. Crabapple drives to work each day from Shelbyville to
    Springfield. During the journey she has to pass through
    8 intersections with traffic lights.

    Each light's colour at any given moment is independent of the
    others but is such that the light is

    green (G) for 40 seconds ,

    yellow (Y) for 5 seconds, and

    red (R) for 15 seconds.

(i)   As Mrs. Crabapple approaches any intersection, what
       Is the probability that the traffic light is green?   *[2 marks]*

(ii)   How many red lights can Mrs. Crabapple expect to
       be see as she approaches the 8 intersections?   *[2 marks]*

(iii) What is the probability that Mrs. Crabapple gets a red
       light at the first intersection and a yellow light at the second
       intersection and a green light at the third intersection?   *[2 marks]*

(i) $\dfrac{40}{60}$ or $2/3$

[13 total marks]

(ii) $P(red) = \dfrac{15}{60} = \dfrac{1}{4}$ or 25%

she can expect to see 25% of 8 lights
to be red.

$(.25)(8) = $ **2 lights**

(iii) Red   Yellow   green
       $\left(\dfrac{1}{4}\right)\left(\dfrac{5}{60}\right)\left(\dfrac{2}{3}\right) = \dfrac{1}{72}$ or 1.39%