Review Set 20B

(2) (a) $y = 3x^2 - x^4$

$f'(x) = 6x - 4x^3$

(b) $y = \frac{x^3 - x}{x^2}$

$= \frac{x^3}{x^2} - \frac{x}{x^2}$

$= x - \frac{1}{x}$

$= x - 1x^{-1}$

$f'(x) = 1 + x^{-2}$

(c) $y = 2x + x^{-1} - 3x^{-2}$

$f'(x) = 2 - x^{-2} + 6x^{-3}$

or

$2 - \frac{1}{x^2} + \frac{6}{x^3}$

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(3) Find tangent
$y = x^3 - 3x + 5$ at $x = 2$

point of tangency $(2, 7)$

$f'(x) = 3x^2 - 3$

$f'(2) = 3(2)^2 - 3 = 9$

$y - 7 = 9(x - 2)$

$y = 9x - 11$

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(4) $y = 2x + x^{-1}$

To find where tangent is horizontal, set derivative equal to 0

$f'(x) = 2 - x^{-2}$

$= 2 - \frac{1}{x^2}$

$0 = 2 - \frac{1}{x^2} \Rightarrow \frac{1}{x^2} = 2 \quad x^2 = \frac{1}{2}$

$x = \pm\sqrt{\frac{1}{2}}$

two points $(\sqrt{\frac{1}{2}}, 2.83)$ $(-\sqrt{\frac{1}{2}}, -2.83)$ or $(.707, 2.83)$ and $(-.707, -2.83)$

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(5) $f(x) = 7 + x - 3x^2$

a) $f(3)$

$= 7 + (3) - 3(3)^2$

$= -17$

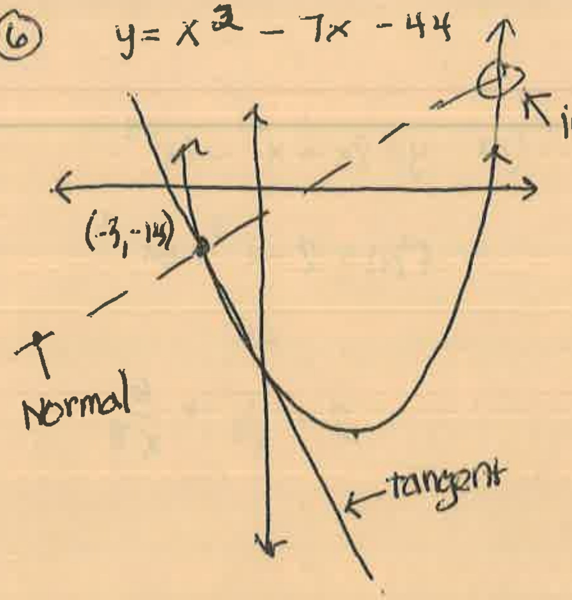
(b) $f'(x) = 1 - 6x$

$f'(3) = 1 - 6(3)$

$= -17$

NOT usual

⑥ $y = x^2 - 7x - 44$



- $f'(x) = 2x - 7 \quad f'(-3) = 2(-3) - 7 = -13$

so -13 is the gradient of the tangent at $x = -3$

- the gradient of the NORMAL is $\frac{1}{13}$

- EQUATION OF NORMAL

$$(y - -14) = \frac{1}{13}(x - -3)$$

or $(y+14) = \frac{1}{13}(x+3)$

$$y + 14 = \frac{1}{13}x + \frac{3}{13}$$

$$y = \frac{1}{13}x - \frac{179}{13}$$

Use GDC to graph in Y_2

intersection is at $(10.1, -13.0)$

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⑦ $f(x) = a - \dfrac{b}{x^2}$. Point of tangency $(-1,-1)$

tangent equation is $y = -6x - 7$ ↓ gradient $= -6$

$= a - bx^{-2}$

$f'(x) = 2bx^{-3} = \dfrac{2b}{x^3}$

since gradient of tangent $= -6$ when $x = -1$

$\dfrac{2b}{x^3} = -6$

$\dfrac{2b}{(-1)^3} = -6$

$\dfrac{2b}{-1} = -6$

$2b = 6$

$b = 3$

Since $(-1,-1)$ is on $f(x) = a - \dfrac{b}{x^2}$

↓

$-1 = a - \dfrac{3}{(-1)^2}$

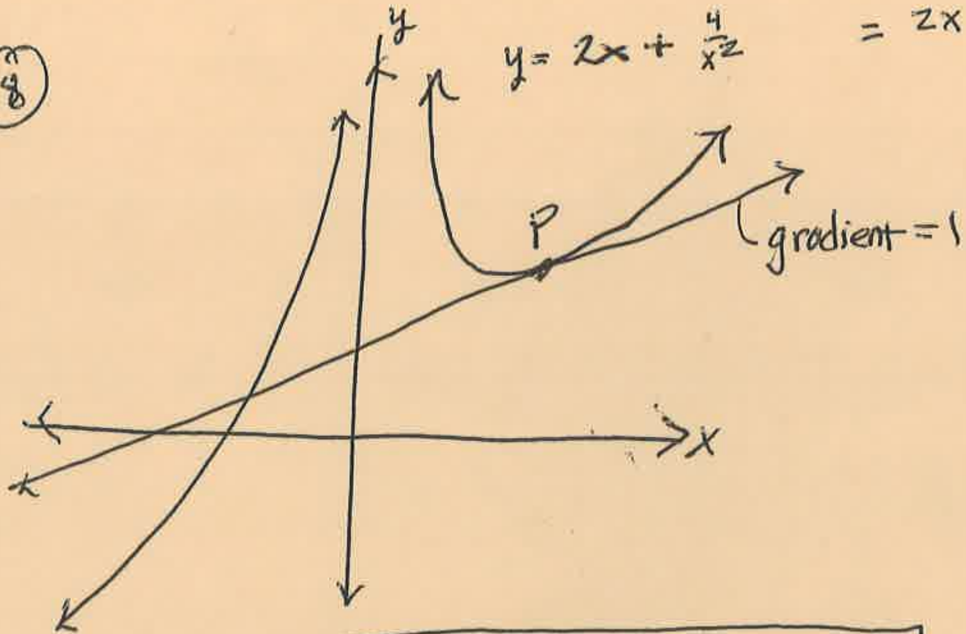
$-1 = a - 3$

$a = 2$

so $f(x)$ would be $f(x) = 2 - \dfrac{3}{x^2}$

(8)

$y = 2x + \frac{4}{x^2} = 2x + 4x^{-2}$



(a) $f'(x) = 2 - 8x^{-3} = 2 - \frac{8}{x^3}$

$1 = 2 - \frac{8}{x^3}$

$-1 = \frac{-8}{x^3}$

$x^3 = 8$

$x = 2$

$P(2, 5)$ f(2)

(b) Equation of tangency

$\boxed{y - 5 = 1(x-2)}$

or

$y = x + 3$

(c) To find x-intercept

$y = 0$

$y = x + 3 \quad 0 = x + 3$

$x = -3$

or $(-3, 0)$

(d) Normal gradient would be -1

$\boxed{y - 5 = -1(x-2)}$

$y - 5 = -x + 2$

$y = -x + 7$