

1

TURN IN  
Take Home LCQ

2

**You have 5  
minutes to check  
your HW with the  
solutions (ask any  
questions  
afterwards)**



Sheldon High School  
HIGH SCHOOL

- ACADEMICS +
- STUDENTS +
- PARENTS +
- STAFF +
- DEPARTMENTS +
- ABOUT +

Look for  
Sheldon Library  
"Links"



Links

- [Barleby.com \(Poetry, quotes, reference, etc.\)](#)

- [Poet's Corner](#)

#### Library Sites

- [Answerland: Chat With a Librarian](#)
- [Internet Public Library](#)
- [Library of Congress](#)
- [Eugene Public Library](#)
- [U. of Oregon Libraries](#)

#### Mathematics

- [The Math Forum@Drexel: Ask Dr. Math](#)
- [Khan Academy](#)
- [IB Math Studies Datasets](#)



#### Biography

- [A&E Biography](#)

#### Careers/Colleges

- [Occupational Outlook Handbook](#)
- [Oregon State University](#)
- [University of Oregon](#)
- [Lane Community College](#)

#### Fine Arts & Museums

- [All Music Blog](#)
- [Art History Resources](#)
- [Artcyclopedia](#)
- [Ask Art](#)

#### Foreign Language

- [Spanish Yahoo!](#)
- [Google Translation Service](#)

#### Health

- [Center for Disease Control](#)
- [MEDLINEplus](#)
- [National Institute of Drug Abuse](#)
- [Mental Health Net](#)
- [ChooseMyPlate.gov](#)
- [National Institutes of Health](#)
- [World Health Organization](#)
- [HealthLinks USA](#)

#### Oregon Government

- [Oregon Blue Book](#)
- [State of Oregon](#)
- [Eugene](#)
- [Lane County](#)
- [Oregon Legislature](#)
- [Oregon Revised Statutes](#)
- [Planet Eugene](#)

#### Reading Lists

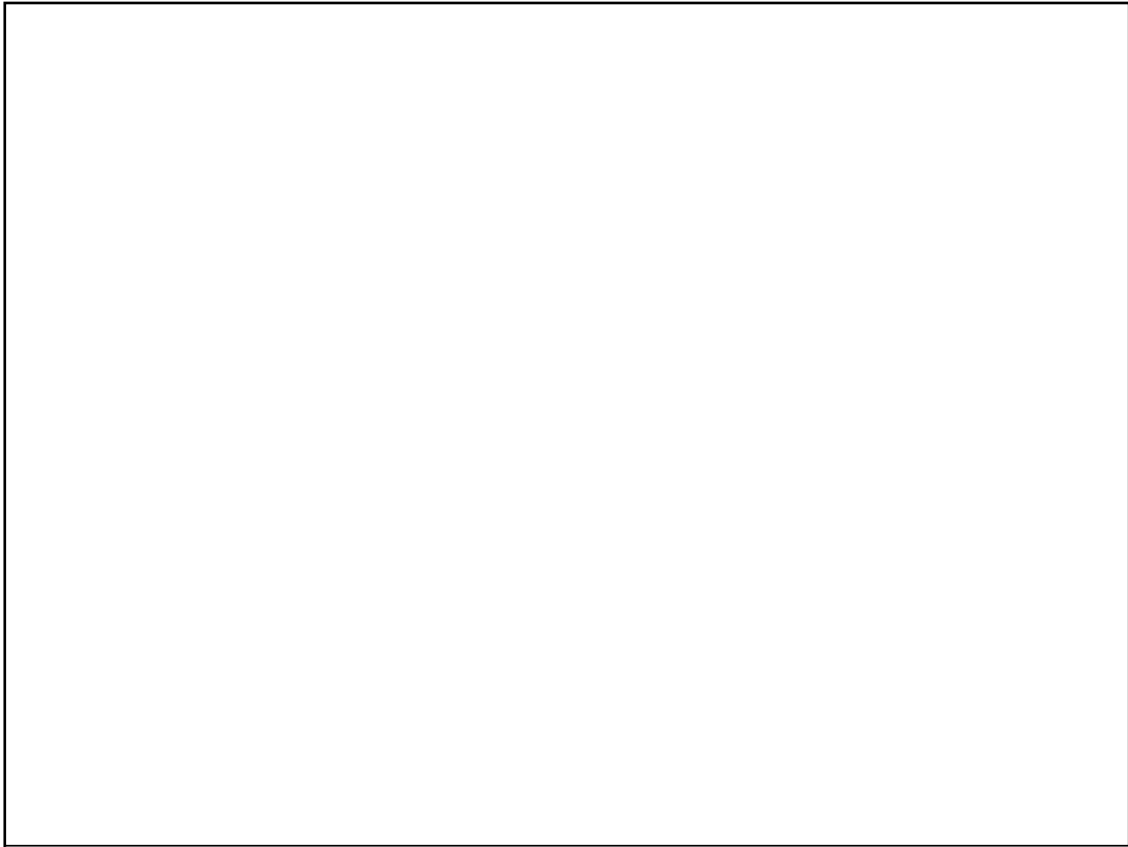
- [Multicultural Reading List](#)
- [Western Classic Novels](#)
- [Pulitzer Prize Winners](#)
- [National Book Award](#)
- [Teen Reads](#)
- [What Should I Read Next?](#)
- [Outstanding Books for the College Bound](#)

#### Research Tools

- [CRAP Test-source evaluation](#)
- [CRAAP Test-worksheet](#)
- [Cooperative Library Instruction Project Tutorials](#)
- [Purdue OWL](#)

#### Science

- [The Why Files](#)
- [Deep Impact](#)
- [WebElements](#)



# Schedule

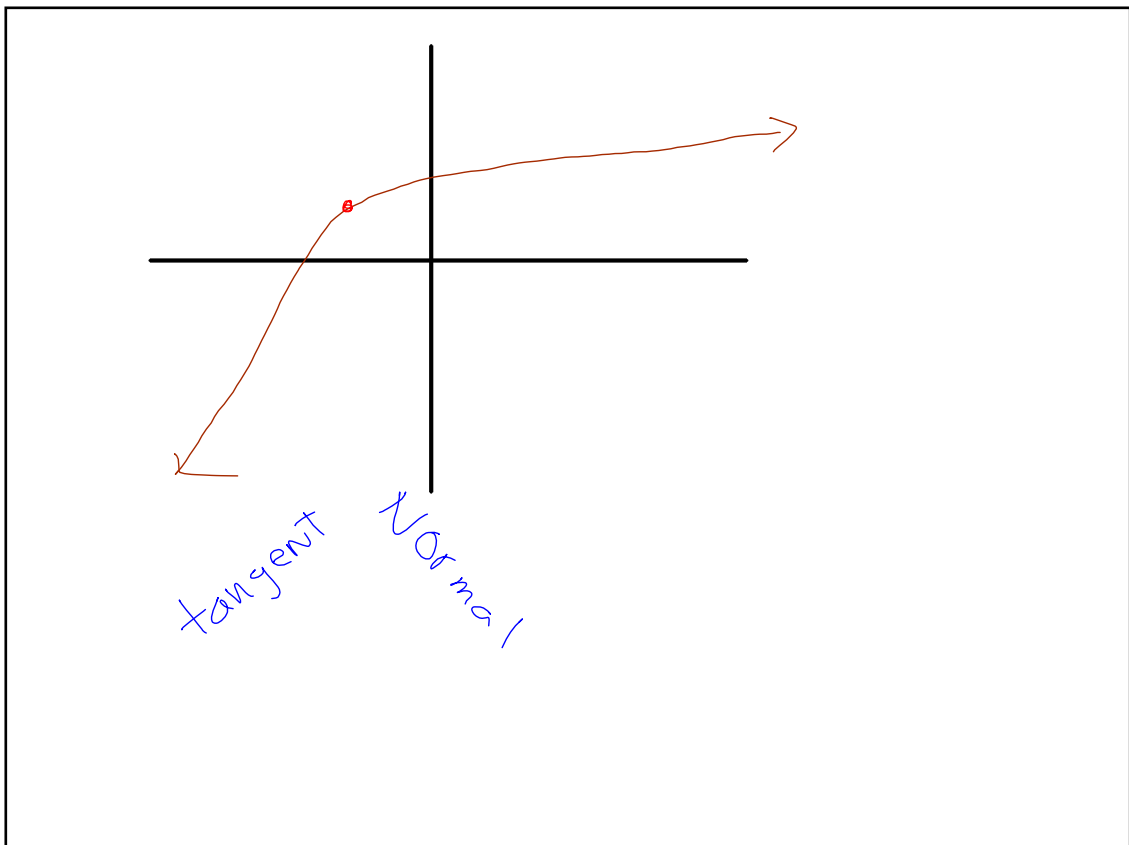
Th NORMALS + OPTIMIZATION ✓

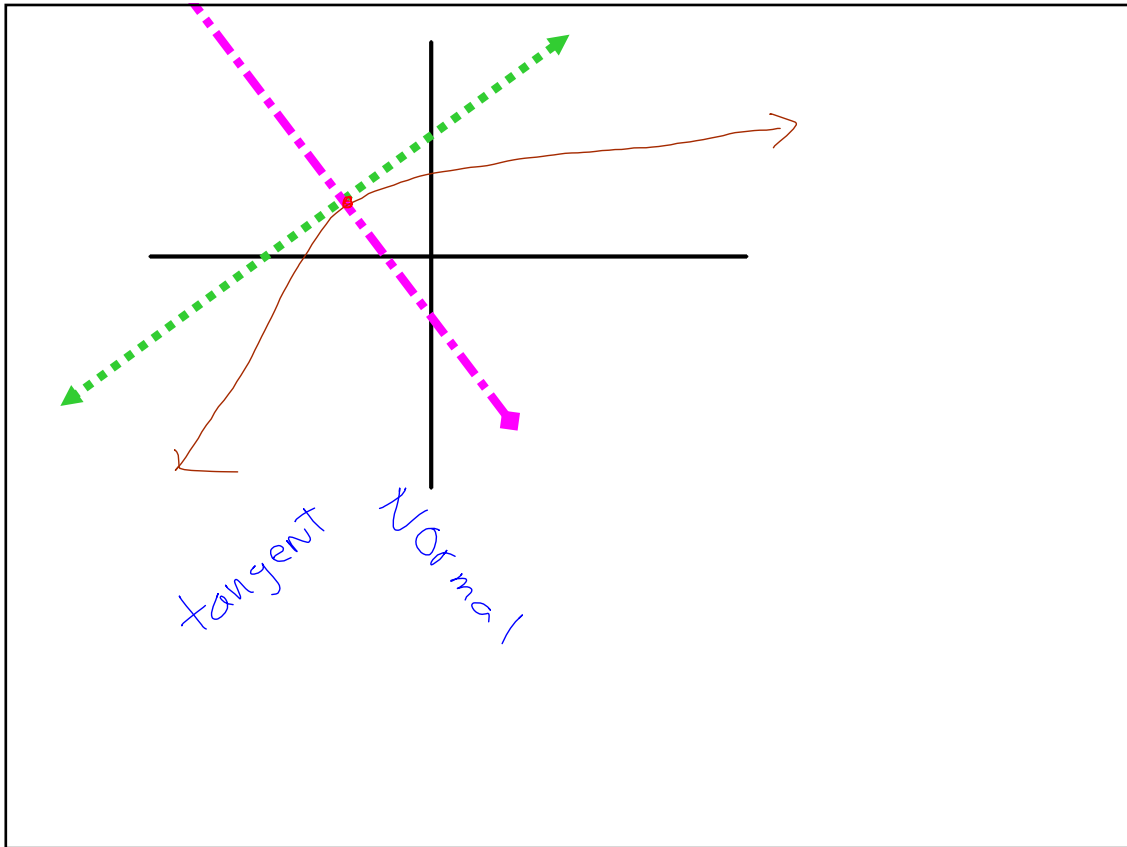
Fri Review ✓

Mon QUIZ ON CALCULUS ✓

## Using the derivative:

1. Find the equation of a **NORMAL**
2. Optimize a situation.





Do the first problem  
on the Notes 4.0  
handout

*sketches are valuable !!!*

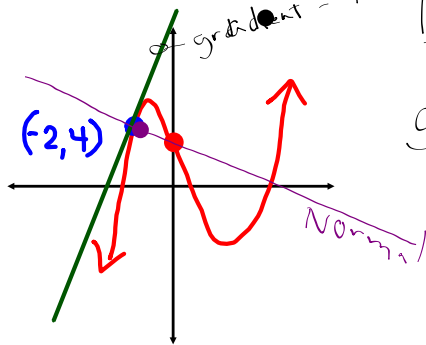
① Find the equation of the **Tangent** and the **Normal** for the equation  $y = x^3 - 5x + 2$  at the location  $x = -2$

P.O.T.  $f(-2)$

$$f(-2) = (-2)^3 - 5(-2) + 2$$

$$= 4$$

$(-2, 4)$



$$f'(x) = 3x^2 - 5$$

$$f'(-2) = 3(-2)^2 - 5 = 7$$

$$y - 4 = 7(x + 2) \text{ Tangent}$$

gradient of Normal  $-\frac{1}{7}$

$$y - 4 = -\frac{1}{7}(x + 2) \text{ Normal}$$

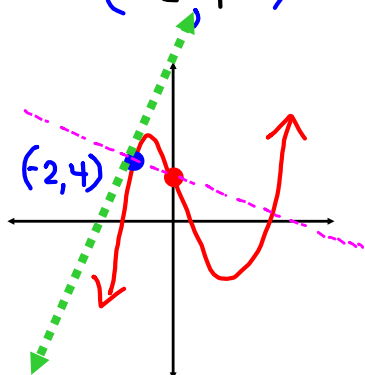
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P.O.T.  $f(-2)$

$$f(-2) = (-2)^3 - 5(-2) + 2$$

$$= 4$$

$(-2, 4)$



GRADIENT  $f'(x) =$

$$f'(-2) =$$

Equation of Tangent

Equation of Normal

② Find the equation of the **Tangent** and the **Normal** for the equation

$$y = \frac{1}{x} + 2 \quad \text{at the location } (-1, 1)$$

$$y = x^{-1} + 2$$

$$f'(x) = -|x|^{-2}$$

$$= -\frac{1}{x^2}$$

$$f'(-1) = -\frac{1}{(-1)^2} =$$

$$= -1$$

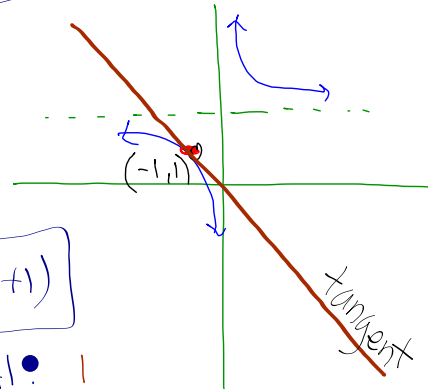
equation of tangent

$$y - 1 = -1(x + 1)$$

slope Normal: 1

$$y - 1 = 1(x + 1)$$

$$x - 1 = (x + 1)$$



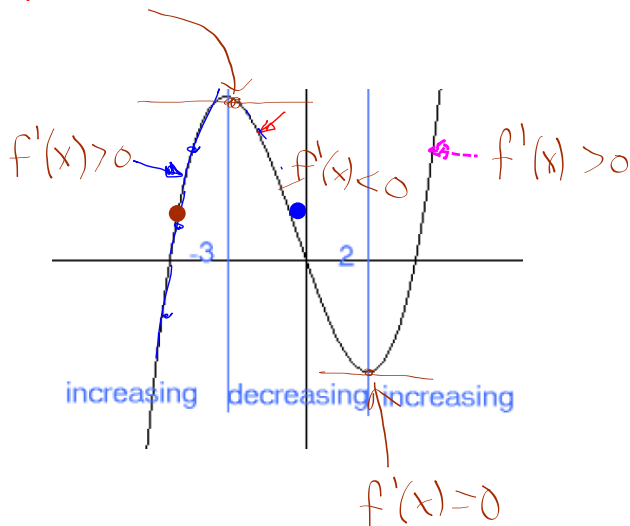
A

$$f'(x) < 0$$

$$f'(x) = 0$$

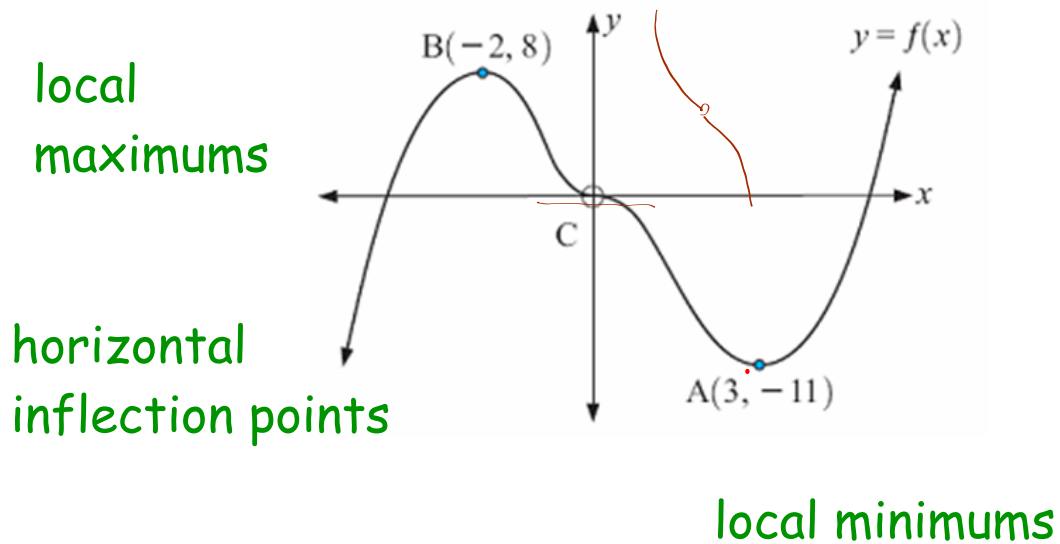
$$f'(x) > 0$$

Stationary  
Points



any point on a curve where the tangent line is horizontal is a **STATIONARY point**

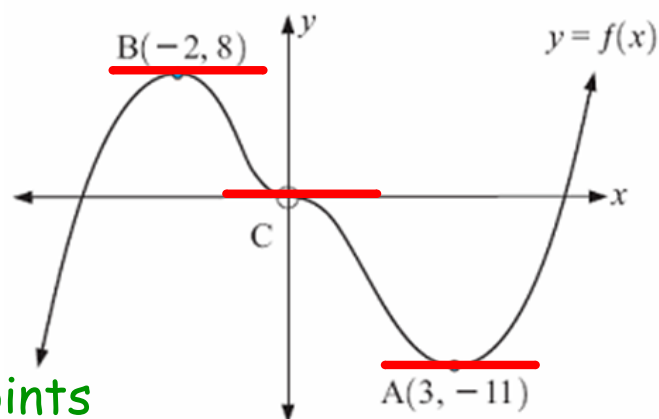
B



local maximums

$$f'(x) = 0$$

horizontal inflection points



local minimums



Finding "stationary" points means to find out the locations (x-values)

where the ....

Horizontal tangents have a gradient of zero

To find all of those places on any given function:

- (1) find the *gradient function* and
- (2) set it equal to zero

$$f'(x) = 0$$

Same as  
"find the derivative!"

- (3) Solve to find x-values (if any) which are the locations where the tangents have a gradient of zero!

Skip  
3

③ Find the <sup>locations</sup> equation(s) of any horizontal tangents of

$$f(x) = \frac{1}{3}x^3 - x + 2$$

$$f'(x) = x^2 - 1$$

$$x^2 - 1 = 0$$

$$x^2 = 1$$

$$\sqrt{\quad} \quad \sqrt{\quad}$$

$$x = \pm 1$$

$$\left(1, \frac{4}{3}\right)$$

$$f(1)$$

$$\frac{1}{3}(1)^3 - 1 + 2$$

$$\frac{1}{3} + 1$$

$$\frac{2}{3}$$

$$\left(-1, \frac{8}{3}\right)$$

$$f(-1)$$

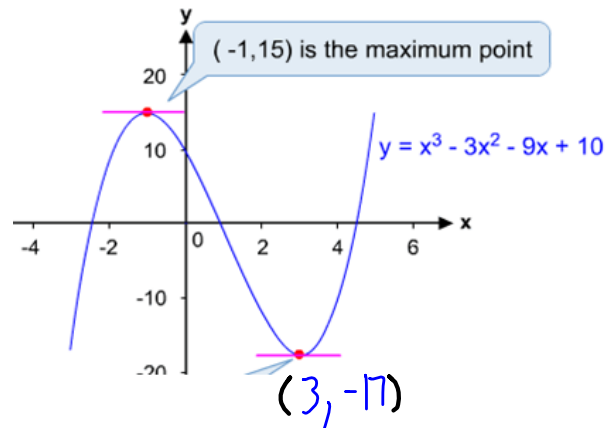
$$\frac{1}{3}(-1)^3 - (-1) + 2$$

$$-\frac{1}{3} + 1 + 2$$

$$\frac{2}{3} + 3$$

④

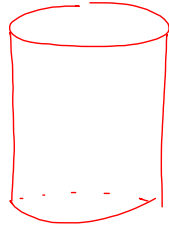
Find the stationary points of the curve  $y = x^3 - 3x^2 - 9x + 10$



## Optimization

- can be challenging to get started
- You just need some experience

Typically tougher questions



Create a cylindrical package that can hold a  $\frac{1}{2}$  liter

but wait...

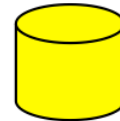
there are two variables?

What's the best radius?

### What if you wanted the minimum material to make a cylinder with a required volume?



In this case you would have two variables (radius and height) and one fixed quantity (volume)



Think about  
Why is having two variables a problem?



In order to differentiate, you need an expression for the quantity you want to minimise (or maximise) in terms of **just one variable**

## Working with a cylinder

First, use the fixed volume to eliminate one of the variables (either the height or radius)

When you have an expression for the quantity of material needed to make the cylinder in terms of just one variable, differentiate it and put the derivative = 0

Solve this equation to find the value of the variable that gives a minimum (or maximum)

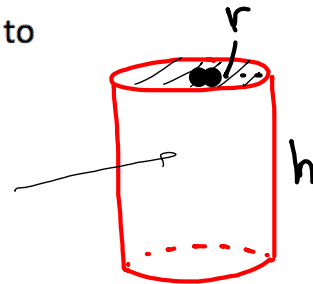
Then find the value of the other variable and the minimum (or maximum) that you require

## Minimum material to make a can

Say you want to find the minimum metal needed to make a can to hold 500 ml (the same as 500 cm<sup>3</sup>)

$$V =$$

SA  
of metal

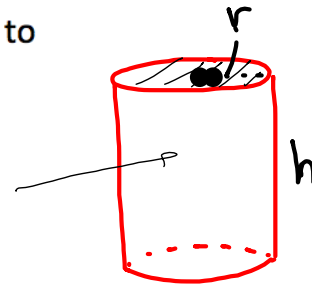


500 ↗

## Minimum material to make a can

Say you want to find the minimum metal needed to make a can to hold 500 ml (the same as 500 cm<sup>3</sup>)

$$V = \pi r^2 h$$



SA of metal  $M =$

500

## Minimum material to make a can

Say you want to find the minimum metal needed to make a can to hold 500 ml (the same as 500 cm<sup>3</sup>)

$$V = \pi r^2 h$$

SA of metal

$$M = 2\pi r^2 + 2\pi r h$$

$$= 2\pi r^2 + 2\pi r \left( \frac{500}{\pi r^2} \right)$$

$$M = 2\pi r^2 + \frac{1000}{r}$$

$$M = 2\pi r^2 + 1000r^{-1}$$

$$\frac{dM}{dr} = 4\pi r - \frac{1000}{r^2}$$

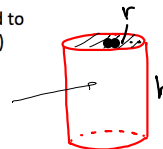
$$4\pi r - \frac{1000}{r^2} = 0$$

$$4\pi r^3 - 1000 = 0$$

$$4\pi r^3 = 1000$$

$$r^3 = \frac{1000}{4\pi}$$

$$r = 4.30 \text{ cm}$$



$$500 = \pi r^2 h$$

$$h = \frac{500}{\pi r^2}$$

$$2\pi(2r)$$

$$1000 \cdot -1 \cdot r^{-2}$$

d

October 17, 2019

$$M = 2\pi r^2 + 2\pi rh$$

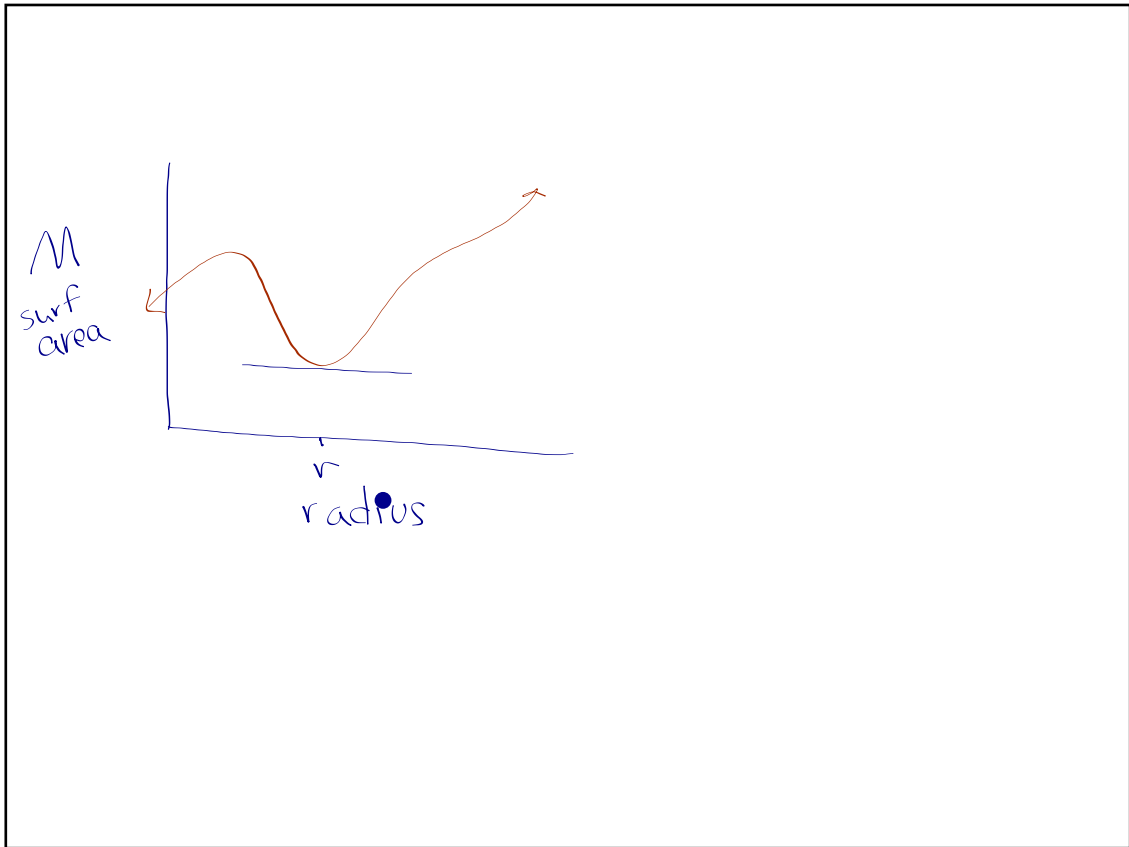
$$500 = \pi r^2 h$$

$$M = 2\pi r^2 + 2\pi rh$$

$$500 = \pi r^2 h$$

$$\dots h = \frac{500}{\pi r^2}$$

$$= 4\pi r - \frac{1000}{\sqrt{2}}$$



$$4\pi r^2(4\pi r) - \frac{1000}{r^2} = 0 \cdot r^2$$

$$4\pi r^3 - 1000 = 0$$

$$4\pi r^3 = 1000$$

$$r^3 = \frac{1000}{4\pi}$$

$$r = \sqrt[3]{\frac{1000}{4\pi}}$$



## Assignment

① Calculus packet:  
and p. 582...Review Set A..... 1-8

② the Box Problem

B.B.