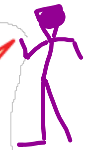
Warm Up

Convert each term of the functions below to the form

ax^n where $n \in \mathbb{Z}$

means **integers**

Remember that $\dfrac{1}{x^n} = x^{-n}$.



$$f(x) = \frac{10}{x} + \frac{5}{x^2} =$$

HW HOTLINE

TURN YOUR Draft #1 Hard Copy to basket

$$f(x) = \frac{10}{x} + \frac{5}{x^2} =$$

$10x^{-1} + 5x^{-2}$

$10x$

$$g(x) = \frac{3x^2 - 6x + x^3}{x^3} =$$

ax^n

$$= \frac{3x^2}{x^3} - \frac{6x}{x^3} + \frac{\cancel{x^3}\,1}{\cancel{x^3}\,1}$$

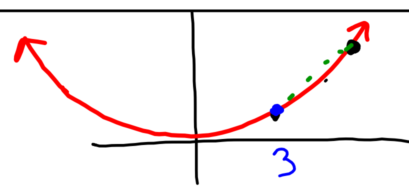
$$= \frac{3}{x} - \frac{6}{x^2} + 1$$

$$= 3x^{-1} - 6x^{-2} + 1$$

$$g(x) = \frac{3x^2 - 6x + x^3}{x^3} =$$

Questions on HW

1a $y = x^2$ $\underline{x=3}$



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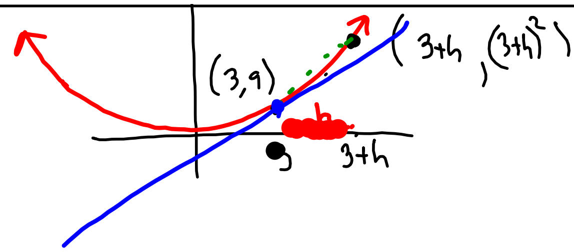

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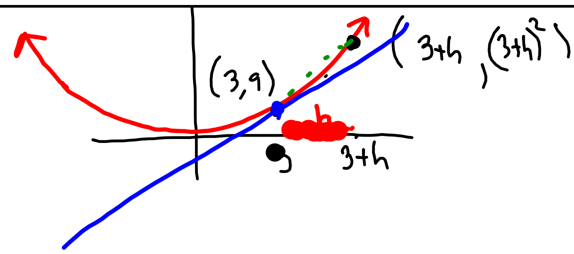


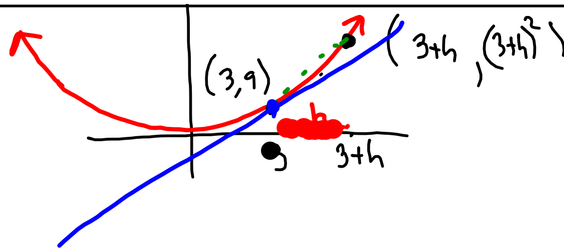
$$m = \frac{(3+h)^2 - 9}{\cancel{3}+h - \cancel{3}}$$

$$= \frac{\cancel{9}+6h+h^2 - \cancel{9}}{h}$$

$$= \frac{6h+h^2}{h} = \frac{\cancel{h}(6+h)}{\cancel{h}} = 6+h$$

1a

$y = x^2 + 2x$ $\quad x=3$



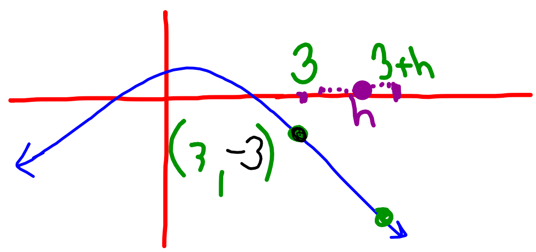
$$m_{gr} = \frac{(3+h)^2 - 9}{\cancel{3}+h - \cancel{3}}$$

$$= \frac{\cancel{9}+6h+h^2 - \cancel{9}}{h}$$

$$= \frac{6h+h^2}{h} = \frac{\cancel{h}(6+h)}{\cancel{h}} = 6+h$$

As $h \to 0$ then gradient is 6

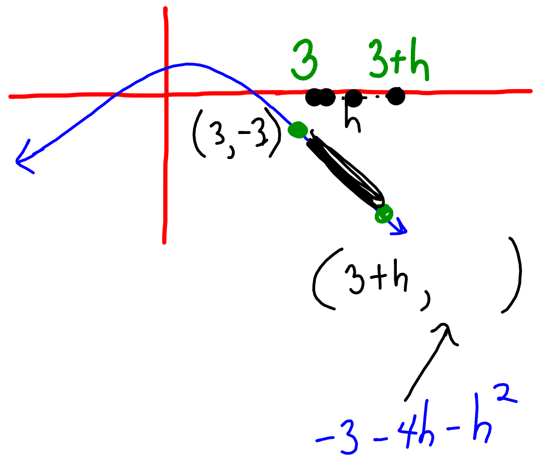
1e $\boxed{y = 2x - x^2}$ at $x=3$



(3+h,)

1e $y = 2x - x^2$ at $x=3$

$3+h \quad 3+h$



$$2(3+h) - (3+h)^2$$

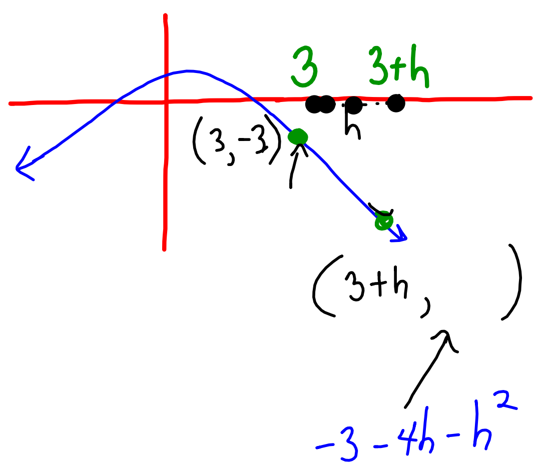
$$6+2h - (3+h)(3+h)$$

$$6+2h - [9+6h+h^2]$$

$$= 6+2h-9-6h-h^2$$

$$= -3-4h-h^2$$

1e $y = 2x - x^2$ at $x=3$



Gradient =

$$\frac{-3-4h-h^2 - -3}{3+h-3}$$

$$= \boxed{\frac{-4h-h^2}{h}} = \frac{\cancel{h}(-4-h)}{\cancel{h}}$$

$$= -4-h$$

as $h \to 0$
gradient is -4

Last class, we found the gradient of curves at a specific point

(a) drawing a tangent and estimating

(b) with GDC

K. Basic Differential Calculus

Calculate the derivative at a specific point.

1. Graph the function, $f(x)$, and obtain an appropriate window.
2. Select **2nd** then **TRACE** , the select $\frac{dy}{dx}$,
3. enter the appropriate $x - value$, then **ENTER**

Draw Tangent Line (& calculate it's equation)

1. Graph the function, $f(x)$, and obtain an appropriate window.
2. Select **2nd** then **DRAW,** then **TANGENT**,
3. enter the appropriate $x - value$, then **ENTER**

TODAY'S AIM

Find derivatives directly

for functions in the form

$$f(x) = ax^n$$

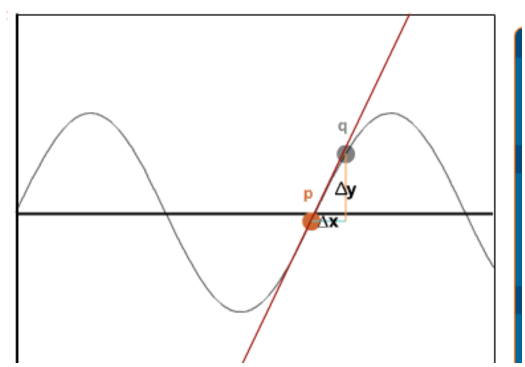
or

$$f(x) = ax^n + bx^{n-1} \ldots$$

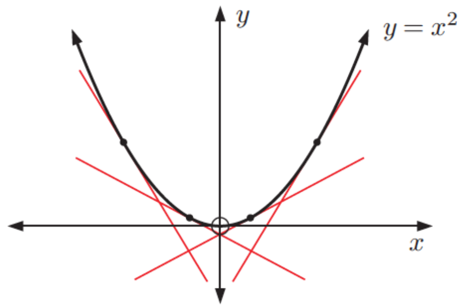
Pick Up

Notes 2.0

What we are about to look at will require you to focus on on the gradients of all of the tangents of a function



A Wouldn't it be cool if there was a **magic function** that could quickly give you the **gradients** at any x-value you want, for any function



That function is called:

The **Gradient** Function

or more commonly called

The **Derivative** function

B

Gradient is a rate of change

The Derivative is a function that that you can use to generate the gradient at any x-value.

other symbols for it:

B

$f'(x)$ or $\frac{dy}{dx}$

$f(x) = x^2$

$f'(x) = 2x$

c

The gradient function (a.k.a derivative function) is created from the original function, f(x).

An example: $f(x) = x^2$

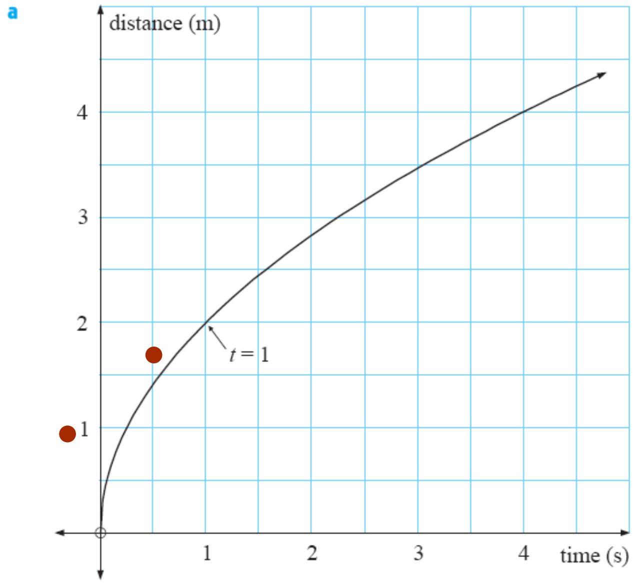
its derivative, $f'(x)$, is $2x$

Differentiation is the process of finding the derivative of a function

- Before we look at some patterns, lets find the gradient one more time with our GDC

D) Find the instaneous rate of change at x = 1 second

$f(x) = 2\sqrt{x}$



✓ Graph (zoom 6)

✓ 2nd Calc $\frac{dy}{dx}$

✓ x=1 (for example) enter

it turns out the derivative function is

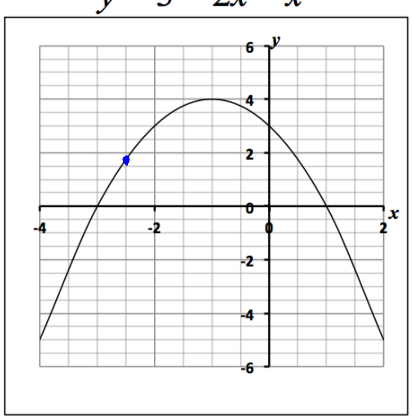
$$f'(x) = \frac{1}{\sqrt{x}}$$

$$f'(1) = \frac{1}{\sqrt{1}} = 1$$

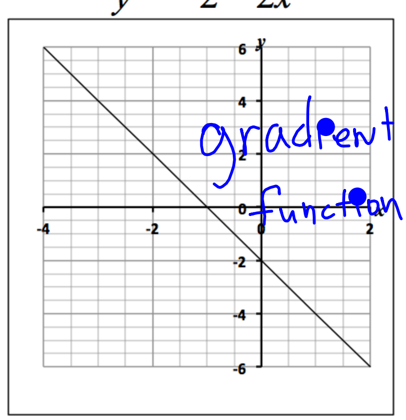
First estimate visually

E) See if you can see a connection between these two graphs?

$y = 3 - 2x - x^2$



$y = -2 - 2x$



Simple Patterns of Differentiation

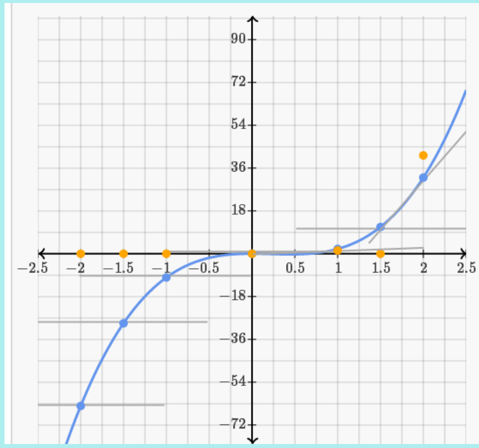
F

for functions in the form $y = ax^n$ where $n \in \mathbb{Z}$

integers

G

f(x)	f'(x)
x^2	$2x$
x^3	$3x^2$
x^4	$4x^3$
x^5	$5x^4$
x^7	$7x^6$



tp://www.khanacademy.org/math/calculus/differential-calculus/e/derivative_intuition

H

f(x)	f'(x)
$7x^2$	$7\cdot(2x) = 14x$
$-5x^3$	$-5(3x^2) = -15x^2$
$4x^{10}$	$40x^9$

If $y = x^n$ then $\dfrac{dy}{dx} = nx^{n-1}$.

$y = ax^n \quad \dfrac{dy}{dx} = anx^{n-1}$

I

f(x)	f'(x)
$x^4 + 2x^2$	$4x^3 + 4x$
$x^5 - 3x^2$	$5x^4 - 6x$
$\frac{2}{x} = 2x^{-1}$	$\frac{dy}{dx} = 2 \cdot -1x^{-2} = \boxed{-2x^{-2}} = \boxed{-\frac{2}{x^2}}$
$\frac{3}{x^2} = 3x^{-2}$	$\frac{dy}{dx} = 3 \cdot -2x^{-3} = -6x^{-3}$ or $\boxed{-\frac{6}{x^3}}$

Derivative of simple functions

J

f(x)	f'(x)
x	$1(x^0) = 1$
$9x$	$9 \bullet x^0 = 9$
$13x^0$	$13 \cdot 0 \cdot x^{-1} = 0$
-50	0

derivative of any constant is 0

K

Function	$f(x)$	$f'(x)$
a constant	a	0
x^n	x^n	nx^{n-1}
a constant multiple of x^n	ax^n	anx^{n-1}
multiple terms	$u(x) + v(x)$	$u'(x) + v'(x)$

Classwork

p. 571 1-4

work as a group

$y = x^6$

$y = 6x^5$

EXERCISE 20C

1 Find the gradient function $\frac{dy}{dx}$ for:

a $y = x^6$ **b** $y = \frac{1}{x^5}$ **c** $y = x^9$ **d** $y = \frac{1}{x^7}$

a) $\frac{dy}{dx} = 6x^5$

b) $y = x^{-5}$

$\frac{dy}{dx} = -5x^{-6}$

or $-\frac{5}{x^6}$

c) $\frac{dy}{dx} = 9x^8$

d) $y = x^{-7}$

$\frac{dy}{dx} = -7x^{-8}$

or $-\frac{7}{x^8}$

2 For $f(x) = x^5$, find:

a $f(2)$ **b** $f'(2)$ **c** $f(-1)$ **d** $f'(-1)$

a) $f(2)$

$= 2^5$

$= 32$

b) $f'(x) = 5x^4$

$\therefore f'(2) = 5(2)^4$

$= 80$

c) $= (-1)^5$

$= -1$

d) $f'(-1)$

$= 5(-1)^4$

$= 5$

3 Consider $f(x) = \frac{1}{x^4}$.

a Find $f'(x)$. **b** Find and interpret $f'(1)$.

$f(x) = x^{-4}$

$f'(x) = -4x^{-5}$

$= \frac{-4}{x^5}$

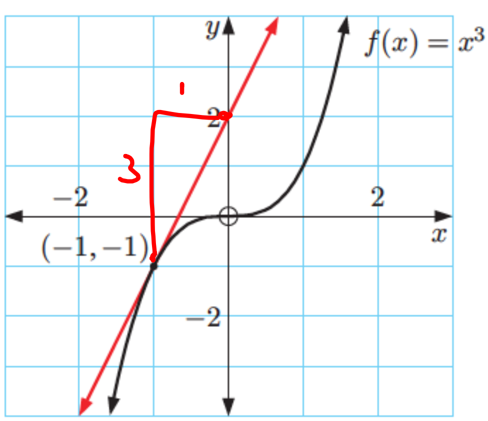
$f'(1) = -\frac{4}{(1)^5} = -4$

-4 is the gradient of the tangent at $x=1$

4 The graph of $f(x) = x^3$ is shown alongside, and its tangent at the point $(-1, -1)$.

a Use the graph to find the gradient of the tangent. $\frac{3}{1} = \underline{\underline{3}}$

b Check your answer by finding $f'(-1)$.



$f'(x) = 3x^2$

$f'(-1) = 3(-1)^2 = \underline{\underline{3}}$

Brain Break

Find $f'(x)$ *for............*

$$f(x) = 5x^3 + 6x^2 - 3x + 2$$

$$f'(x) = 5(3x^2) + 6(2x) - 3(1) + 0$$

$$f'(x) = 15x^2 + 12x - 3$$

Papa Bear

$$f(x) = 7x - \frac{4}{x} + \frac{3}{x^3}$$

$$= 7x - 4x^{-1} + 3x^{-3}$$

First rewrite f(x)

$$f'(x) = 7(1) - 4(-1x^{-2}) + 3(-3x^{-4})$$

$$f'(x) = 7 + 4x^{-2} - 9x^{-4}$$

or $$f'(x) = 7 + \frac{4}{x^2} - \frac{9}{x^4}$$

The Big Kahuna

$$f(x) = \frac{x^2 + 4x - 5}{x}$$

$$f(x) = \frac{x^2}{x} + \frac{4x}{x} - \frac{5}{x}$$

$$f(x) = x + 4 - 5x^{-1}$$

$$f^{'}(x) = 1 + 0 + 5x^{-2}$$

$$f^{'}(x) = 1 + \frac{5}{x^2}$$

Assignment

p.573. Calculus packet

1aceghj , 2, 4ace , 7, 8

p.563

1

Schedule

Wed -brief intro to Calc

-Prepare for Final or Look at Mock 1

Thur CALC

Fri CALC

Mon Calc

Tues CALC + QUIZ

Wed Paper 2 / Final

Thur - Go over paper 2
- IB Test INFO
or Final

Part 1: Estimating Slopes of Tangent Lines from a Graph

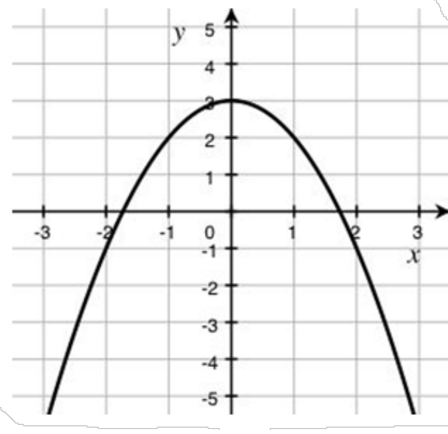
What is f'(x) at x = −2 ?

What is f'(x) at x = −1 ?

What is f'(x) at x = 0 ?

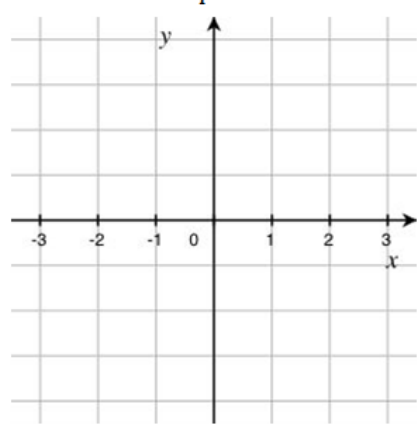
What is f'(x) at x = 1 ?

What is f'(x) at x = 2 ?



We can take all the different values of the derivative at different points and make them into a table or a graph. For example:

x	f'(x)
−2	
−1	
0	
1	
2	



Remember that the table and graph for f(x) and f'(x) are completely different!

Derivatives, Numerically and Physically

Part 1: Average Speed vs Instantaneous Speed

(1) If I drive to New York City (about 180 miles) in 3 hours, what was my average speed?

(2) I go driving for three hours. For the first 2 hours, I drive at a speed of 50 mph. For the last hour, I drive at 80 mph.

-What was my average speed?

-What was my instantaneous speed during each part of the trip?

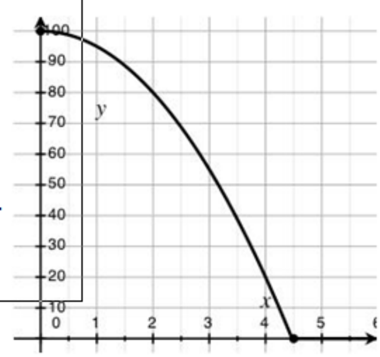
(3) What kind of speed does your car's speedometer read?
What kind of speed does Calculus enable us to find?

(4) On a graph, average speed is the ____________ of a ____________ line.
On a graph, intantaneous speed is the ____________ of a ____________ line.

Part 2: Falling Objects

The following equation tells you *where* a falling object that was dropped from 100 meters will be after x seconds. This equation only works if you use meters. (In feet, we would use -16 instead of -5 for the first coefficient. Ask me if you're curious.)

$$y = -5x^2 + 100$$



(1) On a graph of distance and time, the speed is given by ____________ (or the ____________!).

(2) Use the graph to estimate the **average** speed between when the object falls and when it hits the ground.

(3) Use the graph to estimate the **instantaneous** speed at

0 seconds

1 second

2 seconds

Remember that the graph has units (and intervals of 10 on the y-axis)! Find a slope with units.

Before we proceed we need to review some index (exponent) facts

Function	Can be written as:
$\sqrt{x}$	$x^{\frac{1}{2}}$
$\sqrt[3]{x}$	$x^{\frac{1}{3}}$
$\frac{1}{x}$	x^{-1}
$\frac{1}{\sqrt{x}}$	$x^{-\frac{1}{2}}$

f(x)	f '(x)
$\sqrt{x}$	
$\sqrt[3]{}$	
$\frac{1}{x}$	
$\frac{1}{\sqrt{x}}$	

$f(x)$	$f'(x)$
$3\sqrt[3]{x}$	
$8\sqrt[3]{x}$	
$\frac{2}{x}$	
$\frac{1}{3x}$	