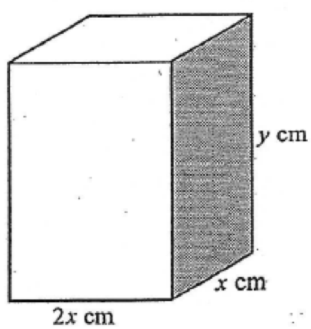
HW Questions

Agenda

1. Discuss HW
   See your LCQ
2. Warm Up
3. Information about Test *
4. Work on study questions.



$SA = 2(\text{Top}) + 2(\text{Side}) + 2(\text{Front})$

$300 = 2(2x \cdot x) + 2(x y) + 2(2x \cdot y)$

$300 = 4x^2 + 2xy + 4xy$

$300 = 4x^2 + 6xy$

(a) Show that $4x^2 + 6xy = 300$.

$4x^2 + 6xy = 300 \Rightarrow 6xy = 300 - 4x^2$ **(2)**

(b) Find an expression for y in terms of x.

$y = \frac{300 - 4x^2}{6x}$ **(2)**

(c) Hence show that the volume V of the box is given by $V = 100x - \frac{4}{3}x^3$.

$$\text{Volume} = (2x)(x)y \qquad y = \frac{300 - 4x^2}{6x}$$

$$= (\cancel{2x})(x)\left(\frac{300 - 4x^2}{3\cancel{6x}}\right)$$

$$x\left(\frac{300}{3}\right) - \frac{x\,4x^2}{3}$$

(d) Find $\frac{dV}{dx}$.

$$= 100 - 4x^2$$

$$\frac{dV}{dx} = 100 - 4x^2$$

(e) (i) Hence find the value of x and of y required to make the volume of the box a maximum.

(ii) Calculate the maximum volume.

(5)
(Total 13 marks)

maximum Volume ocurrs when Tangent is flat (gradient = 0)

Set $\frac{dV}{dx}$ equal to 0

$$100 - 4x^2 = 0$$

$$4x^2 = 100$$

$$x^2 = \pm 5$$

ignore negative dimension

$x = 5$ is the optimum dimension

(e) (i) Hence find the value of x and of y required to make the volume of the box a maximum.

(ii) Calculate the maximum volume.

(5)

(Total 13 marks)

maximum Volume ocurrs when tangent is flat (gradient $= 0$)

Set $\frac{dV}{dx}$ equal to 0

$$100 - 4x^2 = 0$$

$$4x^2 = 100$$

$$x^2 = \pm 5$$

ignore negative dimension

$x = 5$ cm

MAX Volume

$$V = 100x - \frac{4}{3}x^3$$

$$= 100(5) - \frac{4}{3}(5)^3$$

$$= 333 \text{ cm}^3$$

$$\text{Volume} = 2x^2y$$

$$333 = 2(5)^2 y$$

$y = 6.66$ cm

2

The cost per person, in euros, when x people are invited to a party can be determined by the function

$$C(x) = x + \frac{100}{x} = x + 100x^{-1}$$

(a) Find $C'(x)$.

(3)

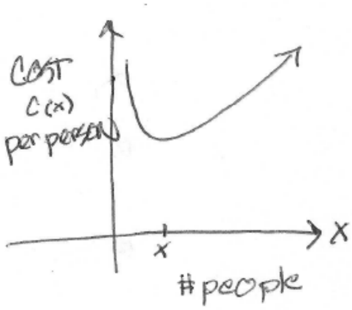
(b) Show that the cost per person is a minimum when 10 people are invited to the party.

(2)

(c) Calculate the minimum cost per person.

(2)

(Total 7 marks)



(a) $C'(x) = 1 - 100x^{-2} = 1 - \frac{100}{x^2}$

(b) To find minimum cost (where tangent is flat) set $C'(x) = 0$ and solve

$1 - \frac{100}{x^2} = 0$ multiply by x^2

$$1 - \frac{100}{x^2} = 0$$

(b) To find minimum cost (where tangent is flat)

set $C'(x) = 0$ and solve

$1 - \frac{100}{x^2} = 0$ multiply by x^2

$x^2 - 100 = 0$

$x^2 = 100$

$x = \pm 10$ ignore negative

so, 10 people would give the minimum cost

(c) Minimum Cost $= C(10) = x + \frac{100}{x}$

$= 10 + \frac{100}{10} = 20$

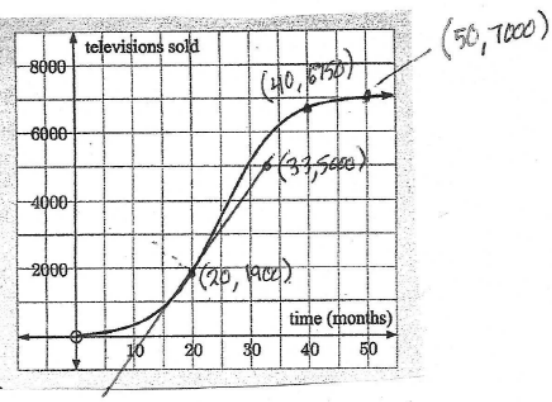
So the minimum cost per person is \$20
(which occurs when 10 people are invited)

p582 Review Set 20A

1.

The total number of televisions sold over many months is shown on the graph alongside.
Estimate the rate of sales:

- **a** from 40 to 50 months
- **b** from 0 to 50 months
- **c** at 20 months.



40 to 50

a) $\frac{\Delta y}{\Delta x} = \frac{7000 - 6750}{50 - 40}$

$\doteq 25$ televisions/month

b) 0 to 50 $\frac{\Delta y}{\Delta x} = \frac{7000 - 0}{50 - 0}$

$= 140$ TV's/month

c) $\frac{\Delta y}{\Delta x} \doteq \frac{5000 - 1900}{33 - 20} \doteq 239$ TV's/mo.

(2) a) $7x^3$

$f'(x) = 21x^2$

b) $x^2 - x^3$

$f'(x) = 2x - 3x^2$

c) $(2x-3)^2 = (2x-3)(2x-3)$

$= 4x^2 - 12x + 9$

$f'(x) = 8x - 12$

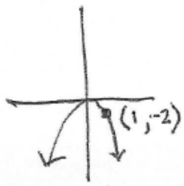
(3) $f(x) = x^4 - 3x - 1$

(a) $f'(x) = 4x^3 - 3$

(b) $f'(2) = 4(2)^3 - 3 = 29$

(c) $f'(0) = 4(0)^3 - 3 = -3$

(4) $y = -2x^2$
at $x = 1$



$f'(x) = -4x$

$f'(1) = -4(1)$

$= -4$

↑ gradient

equation

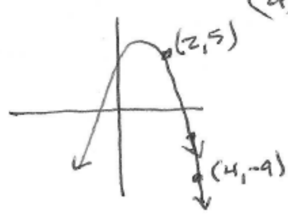
$y - -2 = -4(x-1)$

or

$y = -4x + 2$

Review Set A

(5) $f(x) = -2x^2 + 5x + 3$



(a) Avg rate of change

$\frac{\Delta y}{\Delta x} = \frac{5 - -9}{2-4} = \frac{14}{-2} = -7$

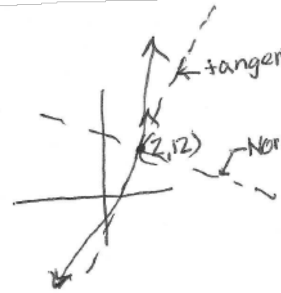
(b) instantaneous rate at $x = 2$

$f'(x) = -4x + 5$

$f'(2) = -4(2) + 5 = -3$

(6) $y = x^3 + 3x - 2$ at $x = 2$

Find NORMAL



$f'(x) = 3x^2 + 3$

$f'(2) = 3(2)^2 + 3 = 15$ ← gradient of tangent

gradient of normal $= -\frac{1}{15}$

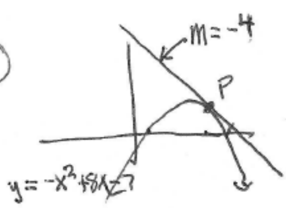
Equation of Normal using (2,12) and $m = -1/15$

$y - 12 = -\frac{1}{15}(x - 2)$

$y - 12 = -\frac{1}{15}x + \frac{2}{15}$

$\boxed{y = -\frac{1}{15}x + \frac{182}{15}}$

(7)



Find where tangent has gradient $= -4$

$f'(x) = -2x + 8$

$-4 = -2x + 8$

$2x = 4$

$x = \cancel{2}$ 6

(6, 5)

f(2)

(8) $y = ax^3 - 3x + 3$

when $x = 2$

$f'(x) = 21$

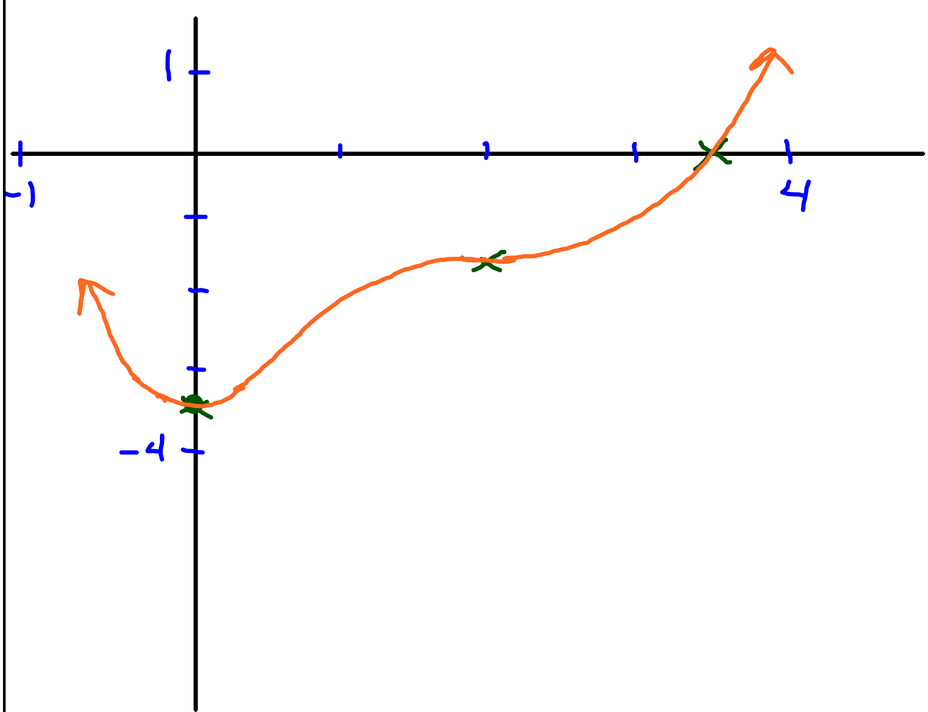
$f'(x) = 3ax^2 - 3$

$21 = 3ax^2 - 3$

$21 = 3a(2)^2 - 3$

$24 = 12a$

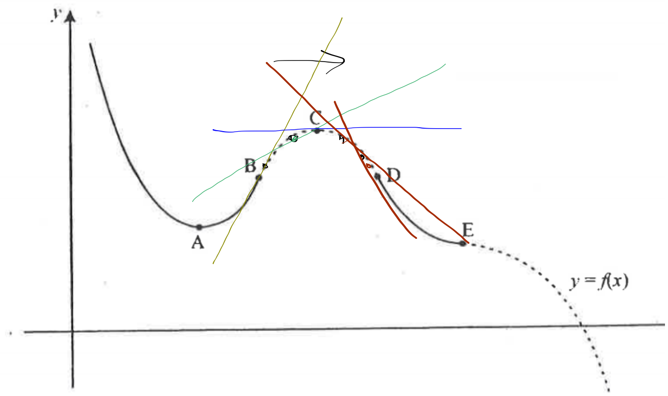
$a = 2$



- WARM UP

back side only!

2. The letters A to E are placed at particular points on the curve $y = f(x)$.



(a) What is the gradient of the curve $y = f(x)$ at the point marked C?

gradient $= 0$ at C.

(b) In passing from point B, through point C, to point D what is happening to $\frac{dy}{dx}$? Is it decreasing or increasing?

✓ Calculator skills: On typical or non-typical functions.... use GDC to:

- Caclulate the gradient at a given location
- Calculate the equation of a tangent line at a given location

3. $f(x) = -x^2 + 2^x - \sqrt{x}$

$f'(3) = -.743$

The Quiz on Introductory Calculus will be **Monday**

List of Quiz Items

and Summary Sheet

is available

NOTATION

$f(x)$

$5x^2 - 6x + 1$

$f'(x)$

In-class → Practice
(You will turn-in today) (but stretch us)

At home → Study Problems from textbook [solutions posted]

- Study Problems

Review Set B

→ 1 - 6

7, 8 ← nice challenge questions for those going for a "7"

I'll be posting solutions

Start
Pink Sheet
(together)

two options

A) work quietly on your own

B) work as a class together

Unit 3 Practice - Introduction to Differential Calculus Name ____________

IB Math Studies October 22, 2019

$f(x) = \frac{48}{x} + 3x^2 - 58$

1. Consider the function $f(x) = \frac{48}{x} + kx^2 - 58$, where $x > 0$ and k is constant.

The graph of the function passes through the point with coordinates (4,2).

$2 = \frac{48}{4} + k(4)^2 - 58$

$2 = 12 + 16k - 58$

$k = 3$

a. Find the value of k. [2]

b. Using your value of k, find $f'(x)$. [3]

$f(x) = 48x^{-1} + 3x^2 - 58 \to f'(x) = -48x^{-2} + 6x$

$= -\frac{48}{x^2} + 6x$

$\boxed{-\frac{48}{x^2} + 6x = 0}$

P is the minimum point of the graph of $f(x)$.

-22

c. Use your answer to part (b) to show that the minimum value of $f(x)$ is 22. [3]

$-48 + 6x^3 = 0$

$6x^3 = 48$

$x^3 = \frac{48}{6}$

$x = \sqrt[3]{\frac{48}{6}}$

$x = 2$

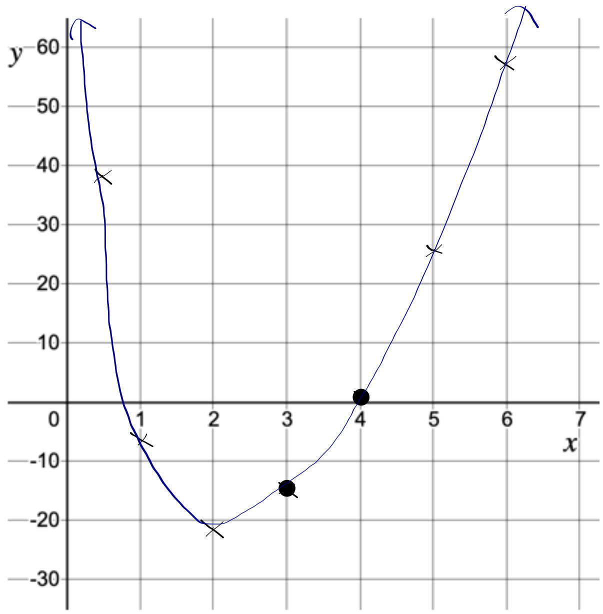
$f(2) = -22$

d. Write down the two values of x which satisfy $f(x) = 0$. [2]

x-intercepts are $x = 3.90$ and $x = .861$

e. Sketch the graph of $y = f(x)$ for $0 \le x \le 6$ and $-30 \le y \le 60$. [4]

e. Sketch the graph of $y = f(x)$ for $0 \leq x \leq 6$ and $-30 \leq y \leq 60$.



2. Consider the function $g(x) = x^3 + kx^2 - 15x + 5$

a. Find $g'(x)$. [3]

The tangent to the graph of $y = g(x)$ at $x = 2$ is parallel to the line $y = 21x + 7$.

b. i. Show that $k = 6$.

ii. Find the equation of the tangent to the graph of $y = g(x)$ at $x = 2$. Give your answer in the form$y = mx + c$. [5]

c. Use your answer to part (a) and the value of k, to find the x-coordinates of the stationary points of the graph of $y = g(x)$. [3]

d. i. Find $g'(-1)$.

d. i. Find $g'(-1)$.

ii. Hence justify that g is decreasing at x=-1.

e. Find the y-coordinate of the local minimum.

Turn page

3. Consider the function $f(x) = \frac{96}{x^2} + kx$, where k is a constant and $x \neq 0$.

a. Write down $f'(x)$. [2]

The graph of $y = f(x)$ has a local minimum at point $x = 4$.

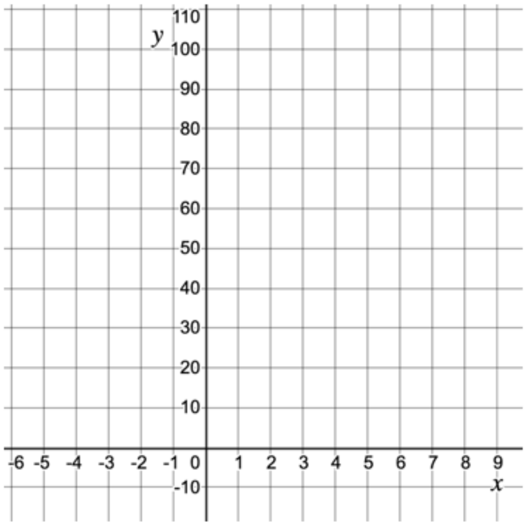
b. Show that $k = 3$. [2]

c. Find $f(2)$. [2]

d. Find $f'(2)$. [2]

e. Find the equation of the normal to the graph of $y = f(x)$ at the point where $x = 2$.
Give your answer in the form $ax + by + d = 0$ where $a, b, d \in \mathbb{Z}$.

f. Sketch the graph of $y = f(x)$, for $-5 \leq x \leq 10$ and $-10 \leq y \leq 100$.



g. Write down the coordinates of the point where the graph of $y = f(x)$ intersects the x-axis. [2]

h. State the values of x for which f(x) is decreasing. [2]

REVIEW SET 20B

1 **a** **i** 5 **ii** $4\frac{1}{2}$ **iii** 4.1

b $f'(x) = 2x+2$ **c** gradient $= 4$, as $x \to 1$, $f'(x) \to 4$

2 **a** $\dfrac{dy}{dx} = 6x - 4x^3$ **b** $\dfrac{dy}{dx} = 1 + x^{-2}$

c $\dfrac{dy}{dx} = 2 - x^{-2} + 6x^{-3}$

3 $y = 9x - 11$ **4** $\left(-\frac{1}{\sqrt{2}}, -2\sqrt{2}\right)$ and $\left(\frac{1}{\sqrt{2}}, 2\sqrt{2}\right)$

5 **a** -17 **b** -17 **6** $(10.1, -13.0)$ **7** $a = 2$, $b = 3$

8 **a** P(2, 5) **b** $y = x + 3$ **c** $(-3, 0)$ **d** $y = -x + 7$

3 $y = 9x - 11$ **4** $\left(-\frac{1}{\sqrt{2}}, -2\sqrt{2}\right)$ and $\left(\frac{1}{\sqrt{2}}, 2\sqrt{2}\right)$

5 **a** -17 **b** -17 **6** $(10.1, -13.0)$ **7** $a = 2, \;\; b = 3$

8 **a** $\text{P}(2, 5)$ **b** $y = x + 3$ **c** $(-3, 0)$ **d** $y = -x + 7$

REVIEW SET 20C

1 **a** $f'(x) = 4x^3 + 6x^2 + 6x$ **b** $f'(x) = -6x^{-4} - 4x^{-5}$

c $f'(x) = -x^{-2} + 8x^{-3}$

2 **a** -5 **b** -12 **c** $\frac{7}{9}$ **d** -1 **3** $y = -24x + 36$

4 $S'(t) = 0.9t^2 - 36t + 550 \;\; \text{g sec}^{-1}$

This gives the instantaneous rate of change in weight, in grams per second, for a given value of t.

4 $S'(t) = 0.9t^2 - 36t + 550 \;\; \text{g sec}^{-1}$

This gives the instantaneous rate of change in weight, in grams per second, for a given value of t.

5 $y = -\frac{1}{2}x + \frac{13}{2}$ **6** $a = 3, \;\; b = 7$

7 $(-1.32, \; -0.737)$ and $(1.32, \; -1.26)$

8 **a** $f'(x) = 3x^2 - 8x + 4$

b $f'(1) = -1.$ This is the gradient of the tangent to the curve at the point $x = 1$.

c **i** 0 **ii** $y = 1$