

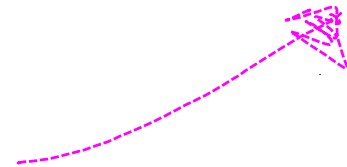
If someone rolls 2 die,
what is the probability
they don't roll a
pair of 3's ?

← Warm
Up

Toss 4 COINS

What is the probability
of getting exactly
2 tails.

Questions
on HW ???



AND FINALLY, A **SUBTRACTION RULE**: FOR ANY EVENT E,

$$P(E) = 1 - P(\text{NOT } E)$$

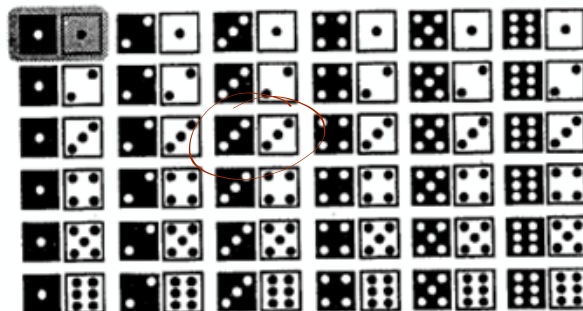
THIS IS USEFUL WHEN $P(\text{NOT } E)$ IS EASIER TO COMPUTE THAN $P(E)$. FOR
INSTANCE, LET E BE THE EVENT, A DOUBLE-1 IS **NOT** THROWN. THE EVENT
NOT-E, A DOUBLE-1 IS THROWN, HAS PROBABILITY $P(\text{NOT } E) = \frac{1}{36}$.

SO

$$P(E) = 1 - P(\text{NOT } E)$$

$$= 1 - \frac{1}{36}$$

$$= \frac{35}{36}$$



Complementary Events

$$P(\text{rains}) = 1 - P(\text{does not rain})$$

$$P(\text{it does not snow}) = 1 - P(\text{it snows})$$

$$P(E) = 1 - P(E')$$

TOSS 4 COINS
at a time

Listing the
Sample
Space

What is the probability
of getting exactly 2 tails ?

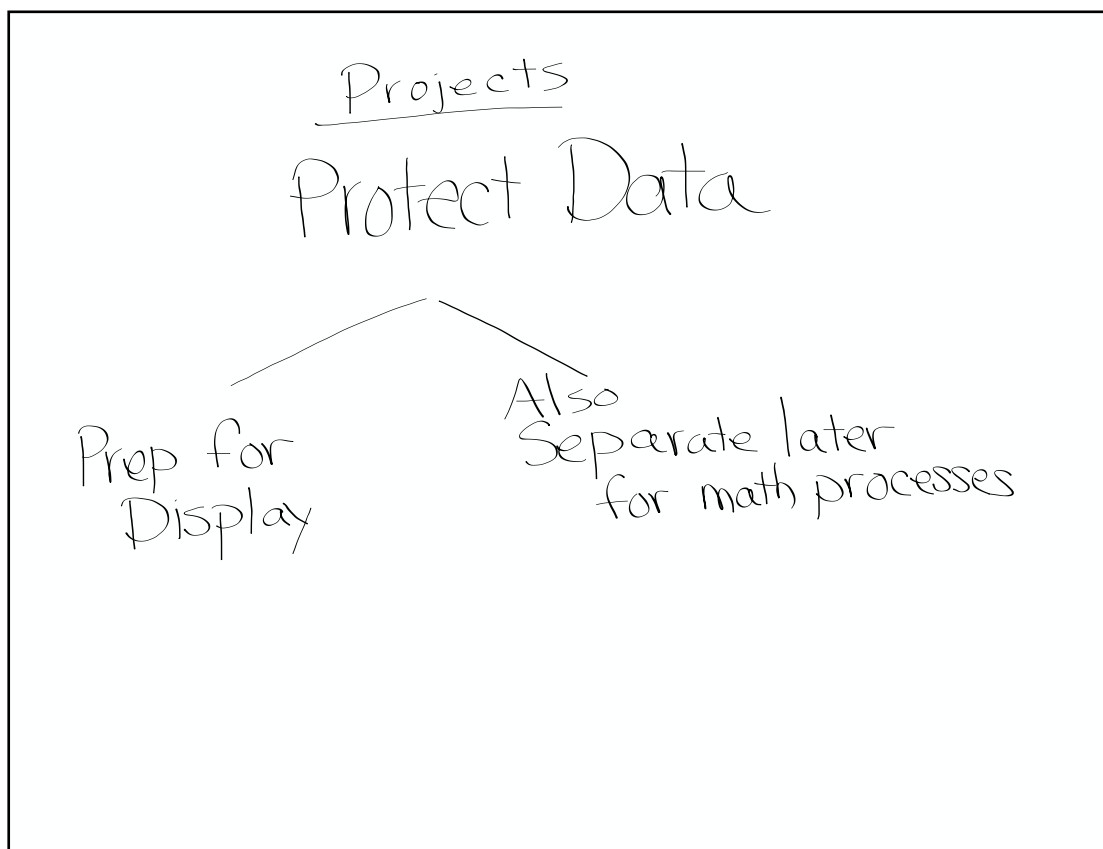
Strategy: List all of the outcomes

HHH HHT TTH TTT
 HTH THT
 THH HTT

being systematic
can be helpful

TTTT HTTT HHTT HHTT HHHH
 THTT HHTH HHTH
 TTHT HTTH HTTH
 TTTH TTHH TTHH
 THTH
 THTH
 THTH

$$P(\text{exactly two tails}) = \frac{6}{16}$$



1. Enter raw data into a spreadsheet.
2. ^{Separate} Adjust the headers to minimize width
 - Wrap text if necessary.
 - Adjust width so you can see text and numbers
3. Copy all columns and paste into a new section of spreadsheet.

New Data Table for Draft 2 and Final Draft

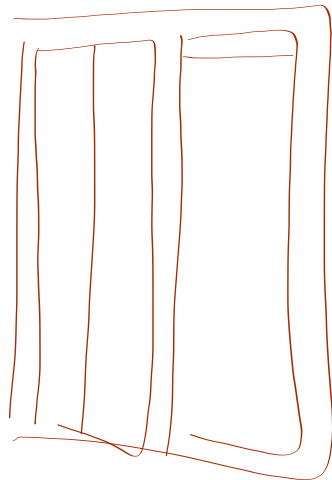
File Edit View Insert Format Data Tools Add-ons Help [Last edit was](#)

100% \$ % .0 .00 123 Arial 14 B

	A	B	C	D	E
1	Rank (Density of Restaurants)	State	McDonald's Locations per 100,000 People	% of Population With Diabetes	
2	1	Ohio	7.1	11.10%	
3	2	Michigan	6.4	11.20%	
4	3	Kansas	6.3	9.40%	
5	4	Maryland	6.3	10.80%	
6	5	Louisiana	6.3	12.10%	
7	6	Arkansas	6.2	13.50%	
8	7	Missouri	6.1	11.50%	
9	8	Tennessee	6	12.70%	
10	9	Indiana	5.9	11.50%	
11	10	Wisconsin	5.9	9.80%	
12	11	Kentucky	5.9	13.10%	
13	12	Alabama	5.8	14.60%	
14	13	Illinois	5.7	10.40%	
15	14	Oklahoma	5.6	12.00%	
16	15	Iowa	5.6	9.30%	
17	16	Wyoming	5.5	8.30%	

4. Decide on a reasonable font size
(not too small).

6. Consider splitting the page if you have $\geq \frac{4}{5}$ of data.

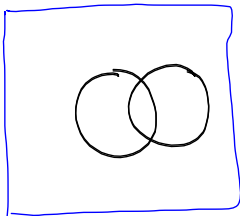


• Questions on the
Homework ?

Check HW
from Fri

p. 80* 9

factory 56 workers , 47 day shift , 29 night shift



EXERCISE 14C

p. 461 ●● 1

1 The table shows data from a survey conducted at five schools on the rate of smoking amongst 15 year old students.

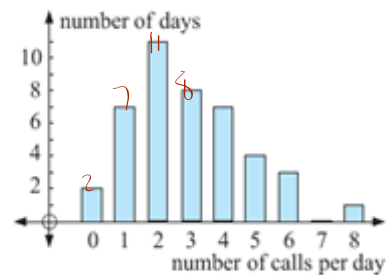
- a What is the probability that a randomly chosen female 15 year old student at school C is a smoker?
- b What is the probability that a randomly chosen 15 year old student at school E is a smoker?
- c If a 15 year old is chosen at random from the five schools, what is the probability that he or she is a smoker?

School	No. of 15 year olds		No. of smokers	
	Male	Female	Male	Female
A	45	51	10	11
B	36	42	9	6
C	52	49	13	13
D	28	33	9	10
E	40	39	7	4
Total	201	214	48	44

p. 456 ●● 3

3 Betul keeps records of the number of phone calls she receives over a period of consecutive days.

- a For how many days did the survey last? 43
- b Estimate Betul's chance of receiving:
 - i no phone calls on one day
 - ii 5 or more phone calls on a day
 - iii less than 3 phone calls on a day.



465

#37

How many possible three child families?

GGG GGB GBB BBB
 GBG BGB
 BGG BBG

EXERCISE 14C

1 The table shows data from a survey conducted at five schools on the rate of smoking amongst 15 year old students.

a What is the probability that a randomly chosen female 15 year old student at school C is a smoker?

b What is the probability that a randomly chosen 15 year old student at school E is a smoker?

c If a 15 year old is chosen at random from the five schools, what is the probability that he or she is a smoker?

School	No. of 15 year olds		No. of smokers	
	Male	Female	Male	Female
A	45	51	10	11
B	36	42	9	6
C	52	49	13	13
D	28	33	9	10
E	40	39	7	4
Total	201	214	48	44

$$\frac{13}{49} = 0.265$$

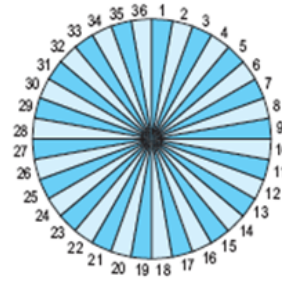
$$\frac{7+4}{40+39} = \frac{11}{79} = 0.139$$

$$\frac{48+44}{201+214} = 0.222$$

p. 465

4 A dart board has 36 sectors, labelled 1 to 36. Determine the probability that a dart thrown at the board hits:

- a a multiple of 4
- b a number between 6 and 9 inclusive
- c a number greater than 20
- d 9
- e a multiple of 13
- f an odd number that is a multiple of 3.

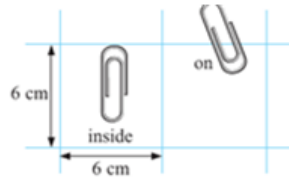


6 List the six different orders in which Antti, Kai and Neda may sit in a row. If the three of them sit randomly in a row, determine the probability that:

- | | |
|-------------------------------|------------------------------------|
| a Antti sits in the middle | b Antti sits at the left end |
| c Antti sits at the right end | d Kai and Neda are seated together |

1 When a batch of 145 paper clips were dropped onto 6 cm by 6 cm squared paper it was observed that 113 fell completely inside squares and 32 finished up on the grid lines. Find, to 2 decimal places, the estimated probability of a clip falling:

a inside a square b on a line.



$$\frac{113}{145} = .7793 \dots$$

$$= 0.779$$

$$\frac{32}{145} = 0.2206 \dots$$

$$= 0.221$$

2

Length	Frequency
0 - 19	17
20 - 39	38
40 - 59	19
60+	4

Jose surveyed the length of TV commercials (in seconds). Find to 3 decimal places the estimated probability that a randomly chosen TV commercial will last:

- a 20 to 39 seconds b more than a minute
c between 20 and 59 seconds (inclusive)

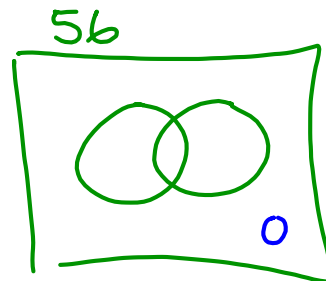
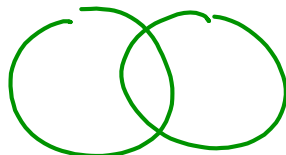
$$a) \frac{38}{78} \div 0.487$$

$$b) \frac{4}{78} = 0.051$$

$$c) \frac{38+19}{78} = 0.731$$

p. 80
9

56 people factory 47 day 29 night



$$\text{Venn diagram} = 0 + 0 - \text{Venn diagram}$$

$$56 = 47 + 29 - \text{Venn diagram}$$

P. 465
#6

AKN ANK KAN KNA NAK NKA

a) $\frac{2}{6}$

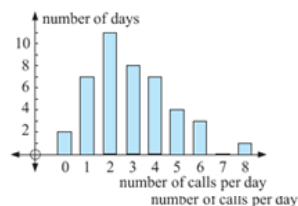
3

Betul keeps records of the number of phone calls she receives over a period of consecutive days.

a For how many days did the survey last?

b Estimate Betul's chance of receiving:

- i no phone calls on one day
- ii 5 or more phone calls on a day
- iii less than 3 phone calls on a day.



$$2+7+11+8+7+4+3+0+1 = 43 \text{ days}$$

$$i) P(0) = \frac{2}{43} = 0.047$$

$$ii) P(\geq 5 \text{ calls}) = \frac{4+3+0+1}{43} = \frac{10}{43} = 0.186$$

$$iii) P(< 3) = \frac{2+7+11}{43} = \frac{20}{43} = 0.465$$

4 Pat does a lot of travelling in her car and she keeps records on how often she fills her car with petrol. The table alongside shows the frequencies of the number of days between refills. Estimate the likelihood that:

- a there is a four day gap between refills
- b there is at least a four day gap between refills.

Days between refills	Frequency
1	37
2	81
3	48
4	17
5	6
6	1

$$\text{Total frequency} = 37+81+48+17+6+1 = 190$$

$$a) \frac{17}{190} = 0.089 \quad b) \frac{17+6+1}{190} = 0.126$$

- 2 The given table shows complaints received by the Telecommunications Ombudsman concerning internet services over a four year period.

Reason	1998/99	1999/00	2000/01	2001/02
Access	585	1127	2545	-
Billing	1822	2102	3136	3582
Contracts	242	440	719	836
Credit control	3	44	118	136
Customer Service	12	282	1181	1940
Disconnection	n/a	n/a	n/a	248
Faults	86	79	0	2384
Privacy	93	86	57	60
Provision	172	122	209	311
Total	3015	4282	7965	9497

- a What is the probability that a complaint received in 2000/01 is about customer service?

$$\frac{1181}{7965} = 0.148$$

- b What is the probability that a complaint received at any time during the 4 year period related to billing?

$$\frac{1822 + 2102 + 3136 + 3582}{3015 + 4282 + 7965 + 9497} = 0.430$$

- c What is the probability that a complaint received in 2001/02 did *not* relate to either billing or faults?

$$\frac{836 + 136 + 1940 + 248 + 60 + 311}{9497} = 0.372$$

Next Test

Quiz this Friday (Nov. 1)

Sets/VennDiag/Probability

Goal:

Finding Compound
Probabilities

handout

or	A or B
and	A and B

can get
complicated

INDEPENDENCE and the special multiplication rule.

TWO EVENTS E AND F ARE *INDEPENDENT* OF EACH OTHER IF THE OCCURRENCE OF ONE HAS *NO INFLUENCE* ON THE PROBABILITY OF THE OTHER. FOR INSTANCE, THE ROLL OF ONE DIE HAS NO EFFECT ON THE ROLL OF ANOTHER (UNLESS THEY'RE GLUED TOGETHER, MAGNETIC, ETC.!).



INDEPENDENT EVENTS

Independent events are events where the occurrence of one of the events **does not** affect the occurrence of the other event.

If A and B are **independent events** then $P(A \text{ and } B) = P(A) \times P(B)$.

Also:

If A, B and C are all **independent events**, then $P(A \text{ and } B \text{ and } C) = P(A) \times P(B) \times P(C)$.

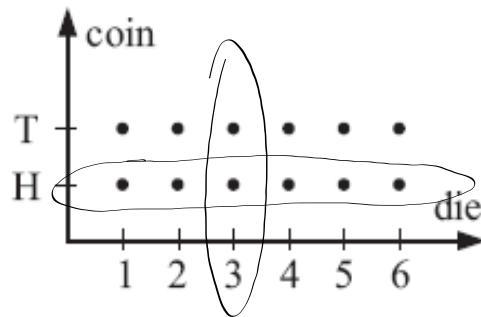
(E)

Example:

A coin and a die are tossed simultaneously. Determine the probability of getting a head and a 3 without the sample space

$$P(\text{h and } 3)$$

$$\frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$$



★

(F)

Rain Example

At a mountain village in New Guinea it rains on average 6 days a week. Determine the probability that it rains on:

a any one day $\frac{6}{7}$

b two successive days

$$P(\text{rain on 1st and rain on 2nd day})$$

c three successive days.

$$\frac{6}{7} \times \frac{6}{7} \times \frac{6}{7} \quad \left(\frac{6}{7}\right)^3$$

G Target Example

Two marksmen fire at a target simultaneously. Jiri hits the target 70% of the time and Benita hits it 80% of the time. Determine the probability that:

- a** they both hit the target

$$(.70)(.80) = .56 = 56\%$$

$$\frac{7}{10} \cdot \frac{8}{10} = \frac{56}{100}$$

- b** they both miss the target

$$\frac{3}{10} \cdot \frac{2}{10} = \frac{6}{100}$$

- c** Jiri hits it but Benita misses

$$\frac{7}{10} \cdot \frac{2}{10} = \frac{14}{100}$$

- d** Benita hits it but Jiri misses.

$$\frac{3}{10} \cdot \frac{8}{10} = \frac{24}{100}$$

I

CONDITIONAL PROBABILITY Situation:



Suppose you are dealt two cards. What is the probability that both are Spades?

$$P\left(\begin{array}{l} \text{1st card} \\ \text{is a} \\ \text{Spade} \end{array} \text{ and } \begin{array}{l} \text{2nd card is} \\ \text{a Spade} \end{array}\right) = \frac{13}{52} \times \frac{12}{51} = \left(\frac{1}{17}\right)$$

BB
+ Lottery
C



CONDITIONAL PROBABILITY:
Event #2 is affected by Event #1

*The chance of getting 1 Spade is $13/52$ since:
There are 13 total spades and 52 total cards*

IMPORTANT: Probability has changed!!
After getting Spade #1, there are only 12
Spades left and there are only 51 cards left.
Probability of the second spade $12/51$

CONDITIONAL PROBABILITY :
Multiplication

**Probability (Spade #1) times Probability
(Spade 2)**

$$\frac{13}{52} \cdot \frac{12}{51} = .0588$$



DEPENDENT EVENTS

Two or more events are **dependent** if they are **not independent**.

Dependent events are events where the occurrence of one of the events *does affect* the occurrence of the other event.

If A and B are dependent events
then

$$P(A, \text{then } B) = P(A \text{ given that } A \text{ has occurred})$$

B.B

Assignment #5
due tomorrow,

p.466 ... 9

p.468 ... 1, 3cefg

p.471 ... 2-3

p.473 ... 1