Unit 3 Practice - Introduction to Differential Calculus Name Key

IB Math Studies October 22, 2019

$= \frac{48}{x} + 3x^2 - 58$

1. Consider the function $f(x) = \frac{48}{x} + kx^2 - 58$, where $x > 0$ and k is constant.

The graph of the function passes through the point with coordinates (4,2).

a. Find the value of k. [2]

$2 = \frac{48}{4} + k(4)^2 - 58$

$2 = 12 + 16k - 58$

$16k = 48$

$k = 3$

b. Using your value of k, find $f'(x)$. [3]

so.... $f(x) = 48x^{-1} + 3x^2 - 58 \rightarrow f'(x) = -48x^{-2} + 6x$ or $= \boxed{-\frac{48}{x^2} + 6x}$

P is the minimum point of the graph of $f(x)$.

c. Use your answer to part (b) to show that the minimum value of $f(x)$ is 22. [3]

Set $f'(x) = 0$

where tangent is flat

$-\frac{48}{x^2} + 6x = 0$ ← multiply by x^2

$-48 + 6x^3 = 0$

$6x^3 = 48$

$x = \sqrt[3]{48/6} = 2$

$f(2) = \frac{48}{2} + 3(2)^2 - 58$

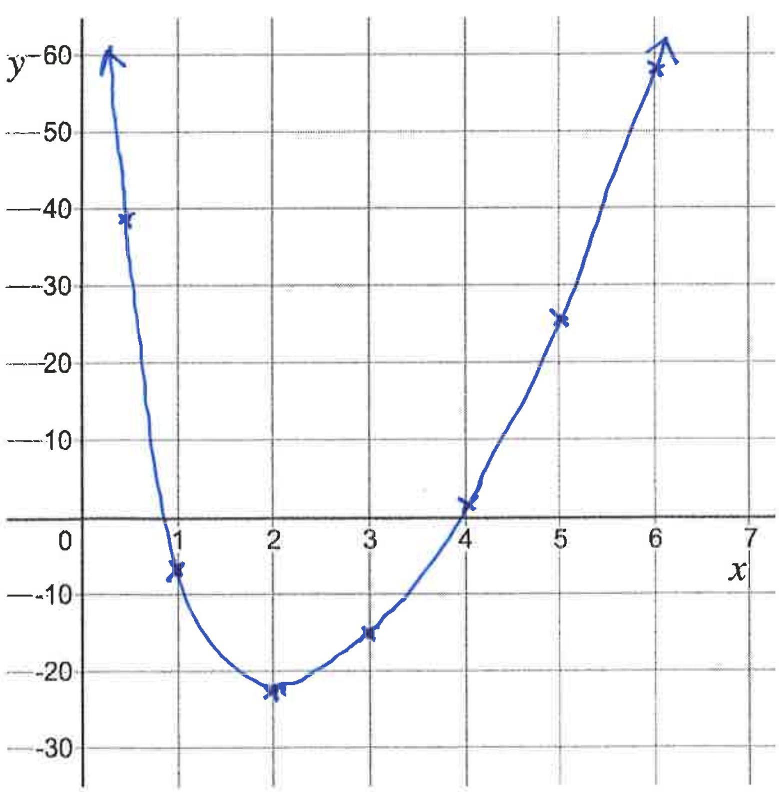
$= -22$

d. Write down the two values of x which satisfy $f(x) = 0$. [2]

these are x-intercepts

$x \approx 0.961$ $x \approx 3.90$

e. Sketch the graph of $y = f(x)$ for $0 \le x \le 6$ and $-30 \le y \le 60$. [4]



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2. Consider the function $g(x) = x^3 \cancel{+} kx^2 - 15x + 5$

$3x^2 + 2kx - 15 = 21$
$3(2)^2 + 2k(2) = 36$
$12 + 4k = 36$
$4k = 24$
$k = 6$

a. Find $g'(x)$. $= 3x^2 + 2kx - 15$ [3]

The tangent to the graph of $y = g(x)$ at $x = 2$ is parallel to the line $y = 21x + 7$.

b. i. Show that $k = 6$.

ii. Find the equation of the tangent to the graph of $y = g(x)$ at $x = 2$. Give your answer in the form $y = mx + c$. [5]

so $g(x) = x^3 + 6x^2 - 15x + 5 \rightarrow$ P.O.T $(2, 7)$ $\boxed{y - 7 = 21(x-2)}$
$g'(x) = 3x^2 + 12x - 15$
$g'(2) = 21$
$y - 7 = 21x - 42$
$y = 21x - 35$

c. Use your answer to part (a) and the value of k, to find the x-coordinates of the stationary points of the graph of $y = g(x)$. [3]

$3x^2 + 12x - 15 = 0$
Set $g'(x) = 0$
$x = -5 \quad x = 1$
$(-5, \quad)$ $f(-5)$ $(1, \quad)$ $f(1)$

d. i. Find $g'(-1)$.

$g'(-1) = -24$

ii. Hence justify that g is decreasing at x=-1. [3]

Since the gradient is negative (-24), the function must be decreasing.

e. Find the y-coordinate of the local minimum. [2]

at $x = 1$

so $f(1) = -3$

the y-coordinate is -3 at the local min.

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3. Consider the function $f(x) = \frac{96}{x^2} + kx$, where k is a constant and $x \neq 0$. $96x^{-2} + Kx = \frac{96}{x^2} + kx$

   a. Write down $f'(x)$. $= -\frac{192}{x^3} + K$ [2]

The graph of $y = f(x)$ has a local minimum at point $x = 4$.

tangent flat

$-\frac{192}{x^3} + K = 0$

$-192 + kx^3 = 0$

$kx^3 = 192$

$K = \frac{192}{(4)^3} = 3$

   b. Show that $k = 3$. [2]

   c. Find $f(2)$. if $f(x) = \frac{96}{x^2} + 3x$, then $f(2) = \frac{96}{2^2} + 3(2) = \underline{\underline{30}}$ [2]

   d. Find $f'(2)$. if $f'(x) = -\frac{192}{x^3} + 3$, then $f'(2) = -\frac{192}{2^3} + 3 = \underline{\underline{-21}}$ [2]

POT $(2, 30)$

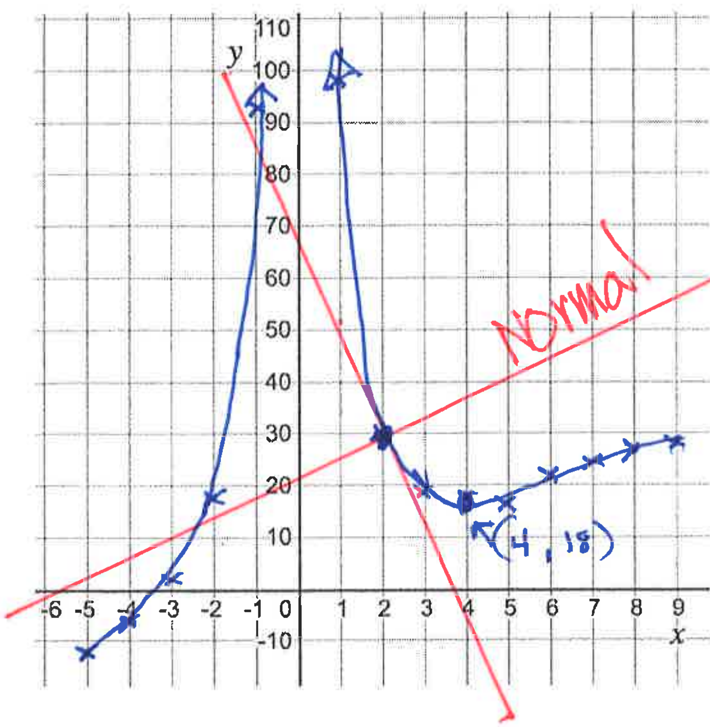
   e. Find the equation of the normal to the graph of $y = f(x)$ at the point where $x = 2$. Give your answer in the form $ax + by + d = 0$ where $a, b, d \in \mathbb{Z}$. [3]

remove fractions

gradient of tangent $= -21$ gradient of Normal $= \frac{1}{21}$ Equation $y - 30 = \frac{1}{21}(x-2)$

$21y - 630 = x - 2$ → CONVERT $\boxed{-X + 21y - 628 = 0}$

   f. Sketch the graph of $y = f(x)$, for $-5 \leq x \leq 10$ and $-10 \leq y \leq 100$.



$y - 30 = \frac{1}{21}(x-2)$

$y - 30 = \frac{1}{21}x - \frac{2}{21}$

$y = \frac{1}{21}x + 29.904$

   g. Write down the coordinates of the point where the graph of $y = f(x)$ intersects the x-axis. [2]

$(-3.17, 0)$

   h. State the values of x for which f(x) is decreasing. [2]

from $0 < x < 4$