SOLUTIONS TO Final Exam Review 2

(1)

$24x^4+31x^3+7x^2+4x+10 \div 3x+2$

$$
\begin{array}{r}
8x^3 + 5x^2 - x + 2 \\
3x+2 \overline{\big)\, 24x^4 + 31x^3 + 7x^2 + 4x + 10} \\
-(24x^4 + 16x^3) \qquad\qquad\qquad \\
15x^3 + 7x^2 \qquad\qquad \\
-(15x^3 + 10x^2) \qquad\qquad \\
-3x^2 + 4x \qquad \\
-(-3x^2 - 2x) \qquad \\
6x + 10 \\
-(6x + 4) \\
6
\end{array}
$$

$\frac{24x^4}{3x} = 8x^3$

$\frac{15x^3}{3x} = 5x^2$

$\frac{-3x^2}{3x} = -x$

$\frac{6x}{3x} = 2$

ANSWER

$8x^3 + 5x^2 - x + 2 + \frac{6}{3x+2}$

This can be confirmed with the Box Method (but NOT with Synthetic)

(2) $P(-3) = -2(-3)^4 + 14(-3)^2 + 6 = -30$

Confirmed

(3) $\frac{-2x^4 + 14x + 6}{x+3}$

$$
\begin{array}{r|rrrrr}
-3 & -2 & 0 & 14 & 0 & 6 \\
 & 0 & 6 & -18 & 12 & -36 \\
\hline
 & -2 & 6 & -4 & 12 & -30
\end{array}
$$

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(4) a) root equation

$(x+1)^2(x+7) = 0$

zero prod. prop

$(x+1)^2 = 0$ is same as $(x+1) = 0$

so roots are

-1 is a double real root

-7 is a single real root

b) $4x^3 - 4x = 0$

$4x(x^2-1) = 0$ differ. of squares

$4x(x+1)(x-1) = 0$

$x=0 \quad x=-1 \quad x=1$

All are single real roots

⑤ $8x^3 - 125 = 0$ ← Difference of cubes! can be factored!

$(2x)^3 - (5)^3 = 0$

$(2x-5)\left[(2x)^2 + (2x)(5) + 5^2\right] = 0$

$(2x-5)(4x^2 + 10x + 25) = 0$

$2x - 5 = 0$

$2x = 5$

$x = 2.5$

$4x^2 + 10x + 25 = 0$

$a = 4$
$b = 10$
$c = 25$

$$x = \frac{-(10) \pm \sqrt{(10)^2 - 4(4)(25)}}{2(4)}$$

$$= \frac{-10 \pm \sqrt{-300}}{8}$$

$$= \frac{-10 \pm i\sqrt{300}}{8} = \frac{-10 \pm 10i\sqrt{3}}{8}$$

$$\frac{-10}{8} \pm \frac{10i\sqrt{3}}{8}$$

The 3 Roots are 2.5, $\frac{-5}{4} + \frac{5i\sqrt{3}}{4}$, $\frac{-5}{4} - \frac{5i\sqrt{3}}{4}$

⑥ $-7-8i$ is a root so $-7+8i$ is also.

shortcut

Sum -14

product $(-7-8i)(-7+8i)$

$49 - 64i^2$

$49 + 64$

113

$y = x^2 - 14x + 113$

or the Longer way

$y = [x - (-7+8i)][x - (-7-8i)]$

$= [x + 7 - 8i][x + 7 + 8i]$

	x	7	$-8i$
x	x^2	$7x$	$-8ix$
7	$7x$	49	$-56i$
$8i$	$8ix$	$56i$	64

$-64i^2$

$y = x^2 + 14x + 113$

(7) $4x^6 + rx^5 + sx^4 + tx^3 + ux^2 + vx + 12 = 0$

a) It will have 6 complex roots, for sure.

note technically a complex root can have:

- both a real and imaginary component. ie. $5+2i$
- just an imaginary component ie. $3i$
- or just a real component ie.. 6

... Also keep in mind roots can have multiplicities (double root, triple, etc)

b) Since 12 is the constant, the possible integral roots are

$12 \cdot 1$ $\quad 6 \cdot 2$ $\quad 4 \cdot 3$

$-12 \cdot -1$ $\quad -6 \cdot -2$ $\quad -4 \cdot -3$

$\rightarrow$ So $\pm 1, \pm 2, \pm 3, \pm 4$ and ± 6

(8) a) $x^3 + 4x^2 + 15x + 22 = 0$

- Possible integral roots $\pm 1, \pm 2, \pm 11, \pm 22$
- From GDC $\rightarrow$ $x = -2$

divide $\dfrac{x^3+4x^2+15x+22}{(x+2)}$

- $(x+2)(\ ?\ ) = 0$

$$\begin{array}{r|rrrr} -2 & 1 & 4 & 15 & 22 \\ & & -2 & -4 & -22 \\ \hline & 1 & 2 & 11 & 0 \end{array}$$

$(x+2)(x^2+2x+11) = 0$

zero product property

$x+2=0$ $\qquad$ $x^2+2x+11=0$

$a=1 \quad b=2 \quad c=11$

$$x = \frac{-(2) \pm \sqrt{(2)^2 - 4(1)(11)}}{2(2)} = \frac{-2 \pm \sqrt{-40}}{2} = \frac{-2 \pm i\sqrt{40}}{2}$$

$$= \frac{-2 \pm 2i\sqrt{10}}{2}$$

$$= -1 \pm i\sqrt{10}$$

The three roots are:
$-2,\ -1+i\sqrt{10},\ -1-i\sqrt{10}$

8b) $x^4+4x^3-17x^2-20x+60=0$

- From GDC → roots are -6 and 2
- $(x+6)(x^3-2x^2-5x+10)=0$

$$\begin{array}{r|rrrrr} -6 & 1 & 4 & -17 & -20 & 60 \\ & & -6 & 12 & 30 & -60 \\ \hline & 1 & -2 & -5 & 10 & 0 \end{array}$$

be sure to show work

- $(x+6)(x-2)(\qquad)=0$
- $(x+6)(x-2)(x^2-5)=0$

$$\begin{array}{r|rrrr} 2 & 1 & -2 & -5 & 10 \\ & & 2 & 0 & -10 \\ \hline & 1 & 0 & -5 & 0 \end{array}$$

$x+6=0$ $\quad$ $x-2=0$ $\quad$ $x^2-5=0 \longrightarrow x^2=5$, $x=\pm\sqrt{5}$

the four roots are 2, -6, $-\sqrt{5}$, and $\sqrt{5}$

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(9) $y=(x-4)^2[\text{quadratic factor from the non-real roots}]$ $\quad$ $2-3i$ must also be a root

Shortcut

Sum of roots $\quad 4$

product $\quad (2-3i)(2+3i)$

$4-9i^2$

$4+9=13$

$y=(x-4)(x-4)(x^2-4x+13)$

$y=[x^2-8x+16][x^2-4x+13]$

	x^2	$-8x$	16
x^2	x^4	$-8x^3$	$16x^2$
$-4x$	$-4x^3$	$32x^2$	$-64x$
13	$13x^2$	$-104x$	208

$$\boxed{y=x^4-12x^3+61x^2-168x+208}$$