

Final Exam Review #1 Detailed Solutions

1 a) $2|x-3| + 7 = 11$

$2|x-3| = 4$
divide

$|x-3| = 2$

$x-3 = 2$ $x-3 = -2$
 $+3$ $+3$

$x = 5$ and $x = 1$

b) $x - 1 = \sqrt{2x-3}$

$x-1 = \sqrt{2x-3}$
square both

$(x-1)^2 = 2x-3$

$(x-1)(x-1) = 2x-3$

$x^2 - 2x + 1 = 2x - 3$ *it's quadratic!*

$x^2 - 4x + 4 = 0$

$(x-2)(x-2) = 0$

$x = 2$

c) $4(x-2)^2 = 16$
divide

$(x-2)^2 = 4$
 $\sqrt{\quad} \quad \sqrt{\quad}$

$x-2 = \pm 2$

$x-2 = \pm 2$

$x-2 = 2$ $x-2 = -2$
 $+2$ $+2$

$x = 4$ $x = 0$

d) $\frac{3}{4}x^2 = \frac{5}{4}x + \frac{1}{2}$
multiply all terms by $\frac{4}{1}$

$4(\frac{3}{4})x^2 = 4(\frac{5}{4})x + 4(\frac{1}{2})$

$3x^2 = 5x + 2$

$3x^2 - 5x - 2 = 0$

$(3x+1)(x-2) = 0$
Z.P.P.

$3x+1 = 0$

$3x = -1$

$x = -\frac{1}{3}$ $x = 2$

e) $5x^3 = 75$
divide

$x^3 = 15$

$x = \sqrt[3]{15}$
exact

f) $85 = 5(3)^{k-1}$
divide

$17 = 3^{k-1}$

either take log of both sides or convert directly to log form

$k-1 = \log_3(17)$

$k = \frac{\log(17)}{\log(3)} + 1$
 ≈ 3.579

g) $-2 = \log_2(x+1)$
convert to exponential form

$2^{-2} = x+1$

$x = 2^{-2} - 1$

$x = \frac{1}{2^2} - 1$

$x = \frac{1}{4} - 1 = -\frac{3}{4}$

h) $\log_3(x) - \log_3(7) = 4$
condense

$\log\left(\frac{x}{7}\right) = 4$

convert

$\frac{x}{7} = 10^4$

$x = 70,000$

2 (a) $x^2 - 4x + 20 = 0$
 $x^2 - 4x = -20$
 $x^2 - 4x + 4 = -20 + 4$
 $(x-2)^2 = -16$
 $x-2 = \pm \sqrt{-16}$
 $x-2 = \pm 4i$
 $x = 2 \pm 4i$

(b) $2x^2 - 3x - 2 = 0$
 divide by 2

$x^2 - \frac{3}{2}x - 1 = 0$

$x^2 - \frac{3}{2}x = 1$

$x^2 - \frac{3}{2}x + \frac{9}{16} = 1 + \frac{9}{16}$

$(x - \frac{3}{4})^2 = \frac{16}{16} + \frac{9}{16}$

$(x - \frac{3}{4})^2 = \frac{25}{16}$

$\sqrt{\quad} \quad \sqrt{\quad}$

$x - \frac{3}{4} = \pm \frac{5}{4}$

$x = \frac{3}{4} \pm \frac{5}{4}$

$\frac{1}{2}$ of b
 $\frac{1}{2} \cdot \frac{3}{2} = -\frac{3}{4}$
 $(-\frac{3}{4})^2 = \frac{9}{16}$

$\frac{3}{4} + \frac{5}{4} = \frac{8}{4} = 2$
 $\frac{3}{4} - \frac{5}{4} = -\frac{2}{4} = -\frac{1}{2}$

4 $x = 2\sqrt{y+1} + 3$
 subtract
 $2\sqrt{y+1} = x-3$
 divide
 $\sqrt{y+1} = \frac{x-3}{2}$
 square both sides
 $y+1 = (\frac{x-3}{2})^2$

in $t^{-1}(x) = (\frac{x-3}{2})^2$

$(y-1)(x-3) = 2$
 divide by $(x-3)$

$y-1 = \frac{2}{x-3}$

$h^{-1}(x) = \frac{2}{x-3} + 1$

$h(x) = \frac{2}{x-1} + 3$

$x = \frac{2}{y-1} + 3$

subtract

$\frac{x-3}{1} = \frac{2}{y-1}$

cross multiply

$g(x) = 3^{x+2}$

$x = 3^{y+2}$

convert

$\rightarrow y+2 = \log_3(x)$

subtract 2

$g^{-1}(x) = \log_3(x) - 2$

$$\begin{aligned}
 \textcircled{5} \quad f(g(x)) &= 2[\quad]^2 - 4 = 2[5x+3]^2 - 4 \\
 &= 2(5x+3)(5x+3) - 4 \\
 &= (10x+6)(5x+3) - 4 \\
 &= 50x^2 + 30x + 30x + 18 - 4 \\
 &= \underline{\underline{50x^2 + 60x + 14}}
 \end{aligned}$$

$$g(f(-5))$$

$$\begin{aligned}
 f(-5) &= 2(-5)^2 - 4 \\
 &= 2(25) - 4 \\
 &= 50 - 4 \\
 &= 46
 \end{aligned}$$

$$\begin{aligned}
 g(46) &= 5(46) + 3 \\
 &= 230 + 3 \\
 &= \underline{\underline{233}}
 \end{aligned}$$

$$\textcircled{6} \quad FV = PV \left(1 + \frac{r}{n}\right)^{nt} = 20,500 \left(1 + \frac{.073}{12}\right)^{12 \cdot 15} = \underline{\underline{\$61,075.31}}$$

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$$\begin{aligned} \textcircled{A} \quad & 9x + 3y + z = 10 \\ \textcircled{B} \quad & 25x + 5y + z = 36 \\ \textcircled{C} \quad & 4x - 2y + z = 15 \end{aligned}$$



$$\begin{aligned} \textcircled{A} \quad & 9x + 3y + z = 10 \\ \textcircled{B} \quad & 25x + 5y + z = 36 \end{aligned} \quad \begin{array}{l} \xrightarrow{-1} \\ \xrightarrow{-1} \end{array} \quad \begin{array}{l} -9x - 3y - z = -10 \\ 25x + 5y + z = 36 \\ \hline 16x + 2y = 26 \end{array}$$

$$\begin{aligned} \textcircled{A} \quad & -9x - 3y - z = -10 \\ \textcircled{C} \quad & + 4x - 2y + z = 15 \\ \hline & -5x - 5y = 5 \end{aligned}$$

$$\begin{aligned} 16x + 2y &= 26 \quad \xrightarrow{5} \\ -5x - 5y &= 5 \quad \xrightarrow{2} \\ \hline 80x + 10y &= 130 \\ -10x - 10y &= 10 \\ \hline 70x &= 140 \end{aligned}$$

$x = 2$

Solution $(2, -3, 1)$

$$\begin{aligned} 16(2) + 2y &= 26 \\ 32 + 2y &= 26 \\ 2y &= -6 \\ y &= -3 \end{aligned}$$

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2, $4+7i$ and $4-7i$ must also be a root

Shortcut
From 2 complex roots create a quadratic function
sum: 8 product $(4+7i)(4-7i)$

$$\begin{aligned} & 16 - 49i^2 \\ & 16 + 49 \\ & 65 \end{aligned}$$

$$y = (x-2)(x^2 - 8x + 65) \leftarrow$$

$$= x^3 - 8x^2 + 65x - 2x^2 + 16x - 130$$

$$y = x^2 - 8x + 65$$

$$y = x^3 - 10x^2 + 81x - 130$$