HW Help

Turn-in take home LCQ

if not turned-in Now I will accept late with penalty if turned in prior to period 1 tomorrow.

Pick Up The WARM UP

and also the Final Exam Information sheet.

$(2x^3 + x^2 + x + 2) \div (x + 1)$

root
-1

-1	2	1	1	2
+		-2	1	-2
	2	-1	2	0

-1	2	1	1	2
		-2	1	-2
	2	-1	2	0

$2x^2 - 1x + 2$

Summary Statement

$$\frac{(2x^3 + x^2 + x + 2)}{(x+1)} = 2x^2 - x + 2$$

OR

$$(2x^3 + x^2 + x + 2) = (x+1)(2x^2 - x + 2)$$

2. Using the Integral Root Theorem (the 3rd Polynomial Theorem from 2 classes ago), list all of the possible integral roots of the polynomial $f(x) = 3x^5 - 2x^2 + 6.$ (possible roots that are integers in other words).

$$f(x) = 3x^5 - 2x^2 + 6.$$

5

$-1 \cdot -6$ $\quad$ $2 \cdot 3$

$1 \cdot 6$ $\quad$ $-2 \cdot -3$

8 possible integral roots

3. Can you predict the remainder of a polynomial division problem, say $\frac{x^4-x^3+20x-48}{x-5}$, without actually dividing? It turns out to be pretty easy. *Polynomial Theorem #4 will shown you how.*

Remainder Theorem: For any number c, when a polynomial $p(x)$ is divided by $(x-c)$, the remainder is $p(c)$. For example, if the polynomial $p(x) = x^4 - x^3 + 20x - 48$ is divided by $(x-5)$, the remainder is $p(5) = 552$.

Note that it follows from the Remainder Theorem that if $p(c) = 0$, then $(x-c)$, is a factor. For example, if $p(x) = x^3 - 3x^2 + 4$, one solution to $x^3 - 3x^2 + 4 = 0$ is $x = -1$. Since $p(-1) = 0$, then $(x+1)$ is a factor.

root $= -2$

With that in mind, predict the remainder of $\frac{x^5+x^4+5}{(x+2)}$ using this Theorem.

$P(-2) = (-2)^5 + (-2)^4 + 5 = -11$

would be the remainder

4. Create a polyomial function, in standard form, with roots 6 *and* $2+2i$

$2-2i$

hint: what do you know about complex roots?
hint: start with with factored form first, then convert to standard form.

$y = (x - \text{ROOT})[x - \text{ROOT}][x - \text{ROOT}]$

4. Create a polyomial function, in standard form, with roots 6 *and* $2+2i$

$2-2i$

hint: what do you know about complex roots?
hint: start with with factored form first, then convert to standard form.

$y=(x-6)[x-(2+2i)][x-(2-2i)]$

$=(x-6)(x-2-2i)(x-2+2i)$

$y=(x-6)[x-(2+2i)][x-(2-2i)]$

$=(x-6)(x-2-2i)(x-2+2i)$

$y=(x-6)(x^2-4x+8)$

	x	-2	$2i$
x	x^2	$-2x$	$\cancel{2ix}$
-2	$-2x$	4	$\cancel{-4i}$
$-2i$	$\cancel{-2ix}$	$\cancel{4i}$	$-4i^2$ 4

$$x^2-4x+8$$

or...

$$y=(x-6)[x-(2+2i)][x-(2-2i)]$$

$$=(x-6)(x-2-2i)(x-2+2i)$$

$$y=(x-6)(x^2-4x+8)$$

$$\boxed{y=x^3-10x^2+32x-48}$$

	x^2	$-4x$	8
x	x^3	$-4x^2$	$8x$
-6	$-6x^2$	$24x$	-48

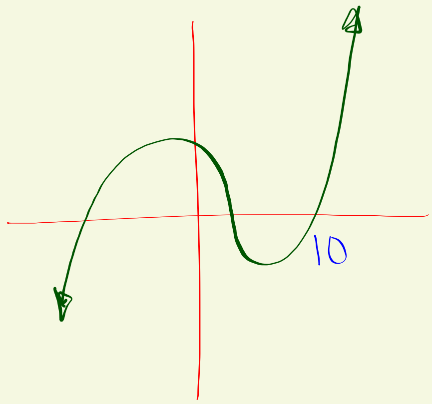
$$y=(x-6)[x-(2+2i)][x-(2-2i)]$$

$$=(x-6)(x-2-2i)(x-2+2i)$$

$$y=(x-6)(x^2-4x+8)$$

sum

product



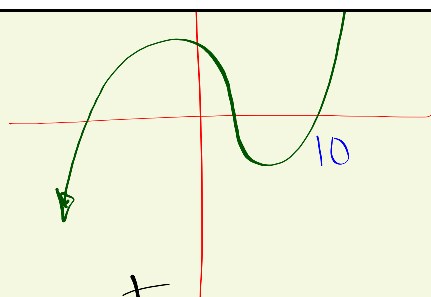
10 is an x-intercept

(10,0) is the x-intercept

10 is a zero of the function

10 is a root of...

But......



10 is an x-intercept

(10,0) is the x-intercept

10 is a zero of the faction

10 is a root of...

but roots can't be written like

HW Questions

8....120-125, 127a

$\boxed{8-123}$ $x^3 - 9x^2 + 19x + 5$

a) $x-2$
b) $x-5$ because 5 is a factor of the last term and 2 and 3 are not
c) $x+3$
d) $x+2$

This leads us to the **INTEGRAL ZERO THEOREM**

$\boxed{B-122}$ $(x-2)(x+3)(x-5)$

a) $x^3-4x^2-11x \cancel{-5}$

b) $\cancel{2x^3}-4x^2-11x+30$

c) $x^3-4x^2-11x+30$

d) $\cancel{2x^3}-4x^2-11x-5$

because the y-intercept $(x=0)$ is
$(-2)(3)(-5) = 30$

and because the leading term would be
$(x)(x)(x) = x^3$, not $2x^3$

$\boxed{120}$ $x^3+5x^2-16x-14 = 0$

c) d) $(x+7)(\text{quadratic factor}) = 0$

$\frac{x^3+5x^2-16x-14}{x+7} =$

$(x+7)(x^2-2x-2) = 0$

$x+7=0$
$x=-7$

$x^2-2x-2=0$

$$\frac{x^3+5x^2-16x-14}{x+7} = x^2-2x-2$$

	x^2	$-2x$	-2
x	x^3	$-2x^2$	$-2x$
7	$7x^2$	$-14x$	-14

121

$$2x^3+3x^2-8x+3=0$$

$x=1$

$$(x-1)(2x^2+5x-3)=0$$

$x-1=0$

$2x^2+5x-3=0$

You'll check
HW Solutions
later

e) $x^3 + 5x^2 - 6x - 14 = 0$

$(x+7)(x^2-2x-2) = 0$

$x^2 - 2x - 2 = 0$

$a = 1$
$b = -2$
$c = -2$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-2)}}{2(1)} = \frac{2 \pm \sqrt{12}}{2}$$

$$= \frac{2 \pm 2\sqrt{3}}{2} = \frac{2(1 \pm \sqrt{3})}{2}$$

Solutions are $x = -7$, $1 \pm \sqrt{3}$

8-121 $2x^3+3x^2-8x+3=0$

$(x-1)(\quad ?? \quad)=0$

$$\frac{2x^3+3x^2-8x+3}{(x-1)} = 2x^2+5x-3$$

$(x-1)(2x^2+5x-3)=0$

$x-1=0$

$x=1$

$2x^2+5x-3=0$

$a=2$
$b=5$
$c=-3$

	$2x^2$	$5x$	-3
x	$2x^3$	$5x^2$	$-3x$
-1	$-2x^2$	$-5x$	3

$$x=\frac{-(5)\pm\sqrt{(5)^2-4(2)(-3)}}{2(2)} = \frac{-5\pm\sqrt{49}}{4} = \frac{-5\pm 7}{4}$$

$x=\frac{-5+7}{4}=\frac{2}{4}=\frac{1}{2}$ $\quad x=\frac{-5-7}{4}=\frac{-12}{4}=-3$

Solutions $x = 1, .5, -3$

8-124

$$\frac{x^3-9x^2+19x+5}{x-5} = x^2-4x-1$$

	x^2	$-4x$	-1
x	x^3	$-4x^2$	$-x$
-5	$-5x^2$	$20x$	5

$x^2-4x-1 \quad a=1, b=-4, c=-1$

$$x=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(-1)}}{2(1)} = \frac{4\pm\sqrt{20}}{2} = \frac{4\pm 2\sqrt{5}}{2}$$

$$=\frac{\cancel{2}(2\pm\sqrt{5})}{\cancel{2}}$$

zeros of $f(x)$ are $5,\ 2+\sqrt{5},\ 2-\sqrt{5}$

8-125

a) $5x^2 - 7x - 6$

$(5x+3)(x-2)$

	x	-2
$5x$	$5x^2$	$-10x$
3	$3x$	-6

$-30x^2$, $3x$, $-10x$, $-7x$

-1	30
-2	15
-3	10
3	-10

b) $(5x+3)(x-2)=0$

$5x+3=0$ $\quad$ $x-2=0$

$5x=-3$

$x=\frac{-3}{5}$ $\quad$ $x=2$

c) the solutions come from the factors set equal to 0.

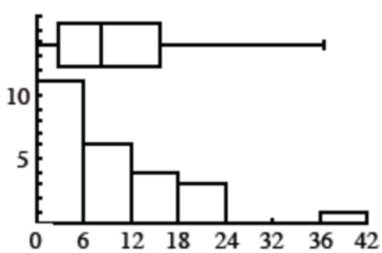
d) 3 and 2 are factors of the constant 6 while 5 is a factor of the lead coefficient

8-126

0	0	0	0	1.7	2.6	2.9	4.2	4.4
5.1	5.6	6.4	8.0	8.9	9.7	10.1	11.2	13.5
15.1	16.3	17.7	21.4	22.0	22.2	36.5		

Checksum 245.5

(a) See the combination histogram boxplot below. The five number summary (for the box plot) is 0, 2.75, 8, 15.7, 36.5.



(b.) Describe the center, shape, spread and outliers.

. The distribution has a right skew and an outlier at 36.5 pounds so the center is best described by the median of 8.0 pounds and the spread by the IQR of 12.95 pounds.

Start a new sheet in your notes.

(lots of space needed)

Aim:

find roots of larger degree polynomials

(3^{rd}, 4^{th}, etc....)

Factor Theorem	Fundamental Theorem of Algebra
Integral Zero Theorem	Remainder Theorem

TASK

Find all of the roots of the polynomial

$$P(x) = x^4 - x^3 - 5x^2 + 3x + 6$$

$5 \quad -6 \quad \sqrt{3} \quad -\sqrt{3}$

$$P(x) = x^4 - x^3 - 5x^2 + 3x + 6$$

✓ List possible Zeros (Roots) that are integers

(Integral Zero Theorem)

6 1 -1

2

$1 \quad 6$
$-1 \quad -6$
$2 \quad 3$
$-2 \quad -3$

Now graph the function see what the possible roots (and factors) are

$x = 2$

$x = -1$

Write the Root Equation:

$$x^4 - x^3 - 5x^2 + 3x + 6 = 0$$

$$(x-2)(x^3 + x^2 - 3x - 3) = 0$$

would change to

$$(x-2)(x+1)(\quad ?? \quad) = 0$$

$$\frac{x^3 + x^2 - 3x - 3}{x+1}$$

$$x^4 - x^3 - 5x^2 + 3x + 6 \qquad (x-2)$$

$$x = 2$$

root

$$\begin{array}{r|rrrrr} 2 & 1 & -1 & -5 & 3 & 6 \\ + & & 2 & 2 & -6 & -6 \\ \hline & 1 & 1 & -3 & -3 & 0 \end{array}$$

$$x^3 + x^2 - 3x - 3$$

$$\frac{x^4 - x^3 - 5x^2 + 3x + 6}{x-2} = x^3 + x^2 - 3x - 3$$

would change to

$$(x-2)(\cancel{x+1})(x^3 + x^2 - 3x - 3) = 0$$

Divide a 2^{nd} time...

$$\frac{x^3 + x^2 - 3x - 3}{(x+1)} =$$

Write the Root Equation:

$$x^4 - x^3 - 5x^2 + 3x + 6 = 0$$

$$(x-2)\underbrace{(x^3 + x^2 - 3x - 3)} = 0$$

$$(x-2)(x+1)(x^2-3) = 0$$

ZPP $x^2 - 3 = 0$

$x^2 = 3$

x

Last two roots?

$$(x-2)(x+1)(x^2-3) = 0$$

two linear factors

one quadratic factor

ZPP $x^3 - 3 = 0$

$x^3 = 3$

$$x = \pm\sqrt{3}$$

$0 = (x-2)(x+1)(x^2-3)$

$x = 2$ $\quad x = -1$ $\quad x^2 - 3 = 0$

$x^2 = 3$

$\sqrt{\ } \quad \sqrt{\ }$

$x = \pm\sqrt{3}$

find all roots

$2, -1, \pm\sqrt{3}$

find all exact x-intercepts (4)

$(2, 0)$ $(-1, 0)$ $(\sqrt{3}, 0)$ $(-\sqrt{3}, 0)$

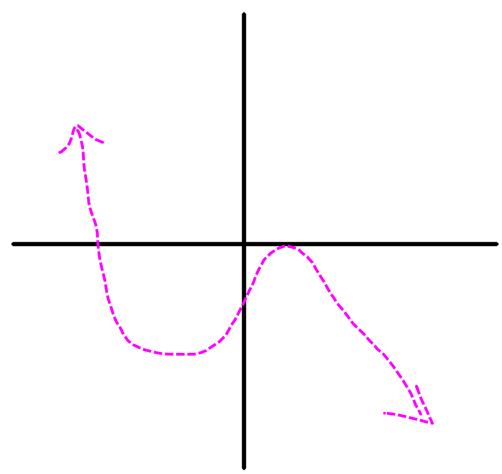
x-intercepts: $-1, 2, \pm\sqrt{3}$

Is there another way we could have found

$$P(x) = x^4 - x^3 - 5x^2 + 3x + 6$$

$$0 = (x-2)(x+1)(x^2-3)$$

Keep your eye out for double roots......



<u>assignment</u>

8...... 132c (ignore poster)

139a, 142, 145-146, 154

Period 2

Turn in your notebook, name/period on front or inside cover with Period 2