PAPER 1 MAY 2014

M14/5/MATSD/SP1/ENG/TZ1/XX

22147403

International Baccalaureate®
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**MATHEMATICAL STUDIES**
**STANDARD LEVEL**
**PAPER 1**

Candidate session number

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | |

Tuesday 13 May 2014 (afternoon)

1 hour 30 minutes

Examination code

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 2 | 2 | 1 | 4 | – | 7 | 4 | 0 | 3 |

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- A clean copy of the ***Mathematical Studies SL** formula booklet* is required for this paper.
- Answer all questions.
- Write your answers in the boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- The maximum mark for this examination paper is *[90 marks]*.

Solutions

*Maximum marks will be given for correct answers. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. Write your answers in the answer boxes provided. Solutions found from a graphic display calculator should be supported by suitable working, for example, if graphs are used to find a solution, you should sketch these as part of your answer.*

**1.** Let $p = \dfrac{2\cos x - \tan x}{\sqrt{y} - z}$.

(a) Calculate the value of $p$ when $x = 45^\circ$, $y = 8192$, and $z = 64$. Write down your full calculator display. *[2]*

(b) Write down your answer to part (a)

(i) correct to two decimal places;

(ii) correct to four significant figures;

(iii) in the form $a \times 10^k$, where $1 \le a < 10$, $k \in \mathbb{Z}$. *[4]*

*Working:*

Also accepted: $\frac{1}{64}$ or $1.5625 \times 10^{-2}$

*Answers:*

(a) 0.015625

(b) (i) 0.02

(ii) .01563

(iii) $1.5625 \times 10^{-2}$

**2.** A class of 13 Mathematics students received the following grades in their final IB examination.

3 5 3 4 7 3 2 7 5 6 5 3 4

For these grades, find

(a) the mode; *[1]*

(b) the median; *[2]*

(c) the upper quartile; *[1]*

(d) the interquartile range. *[2]*

*Working:*

*Answers:*

(a) 3

(b) 4

(c) 5.5

(d) 2.5

**3.** Consider the three propositions $p$, $q$ and $r$.

*$p$: The food is well cooked*
*$q$: The drinks are chilled*
*$r$: Dinner is spoilt*

(a) Write the following compound proposition in words.

$$(p \wedge q) \Rightarrow \neg r$$ *[3]*

(b) Complete the following truth table.

| $p$ | $q$ | $r$ | $p \wedge q$ | $\neg r$ | $(p \wedge q) \Rightarrow \neg r$ |
|---|---|---|---|---|---|
| T | T | T | T | f | f |
| T | T | F | T | T | T |
| T | F | T | f | f | T |
| T | F | F | f | T | T |
| F | T | T | f | f | T |
| F | T | F | f | T | T |
| F | F | T | f | f | T |
| F | F | F | f | T | T |

*[3]*

*Working:*

*Answers:*

(a) If the food is well cooked and the drinks are chilled then dinner is not spoilt.

**4.** A study was carried out to determine whether the country chosen by students for their university studies was influenced by a person's gender. A random sample was taken. The results are shown in the following table.

| | **Country Chosen** | | |
|---|---|---|---|
| | **USA** | **Australia** | **UK** |
| **Male** | 55 | 26 | 40 |
| **Female** | 25 | 31 | 41 |

A $\chi^2$ test was performed at the 1% significance level.
The critical value for this test is 9.210.

(a) State the null hypothesis. *[1]*

(b) Write down the number of degrees of freedom. *[1]*

(c) Write down

(i) the $\chi^2$ statistic;

(ii) the associated $p$-value. *[2]*

(d) State, giving a reason, whether the null hypothesis should be accepted. *[2]*

*Working:*

OR
Since 9.17 < 9.210 we accept the Null Hypothesis.

*Answers:*

(a) Country chosen and gender are independent.

(b) 2

(c) (i) 9.17

(ii) 0.0102

(d) Since 0.0102 > 0.01 we accept null hypothesis.

**5.** ***In this question give all answers correct to two decimal places.***

Dumisani has received a scholarship of 5000 US dollars (USD) to study in Singapore.
He has to travel from South Africa and must change USD for his air fare of 6600 South African rand (ZAR).

The exchange rate is 1 USD = 8.2421 ZAR .

(a) Calculate the number of USD that Dumisani must change to pay for his air fare. *[2]*

On arrival in Singapore, Dumisani changes 3000 USD to Singapore dollars (SGD) to pay for his school fees. There is a 2.8 % commission charged on the exchange.

(b) Calculate the value, **in USD**, of the commission that Dumisani has to pay. *[2]*

The exchange rate is 1 USD = 1.29903 SGD .

(c) Calculate the number of SGD Dumisani receives. *[2]*

*Working:*

(a) $6600 \text{ ZAR} \times \dfrac{1 \text{ USD}}{8.2421 \text{ ZAR}} = 800.77 \text{ USD}$

commission
(b) $0.028 \times 3000 = 84.00$

(c) $3000 - 84 = 2916 \text{ USD}$

$2916 \text{ USD} \times \dfrac{1.29903 \text{ SGD}}{1 \text{ USD}} = 3787.97 \text{ SGD}$

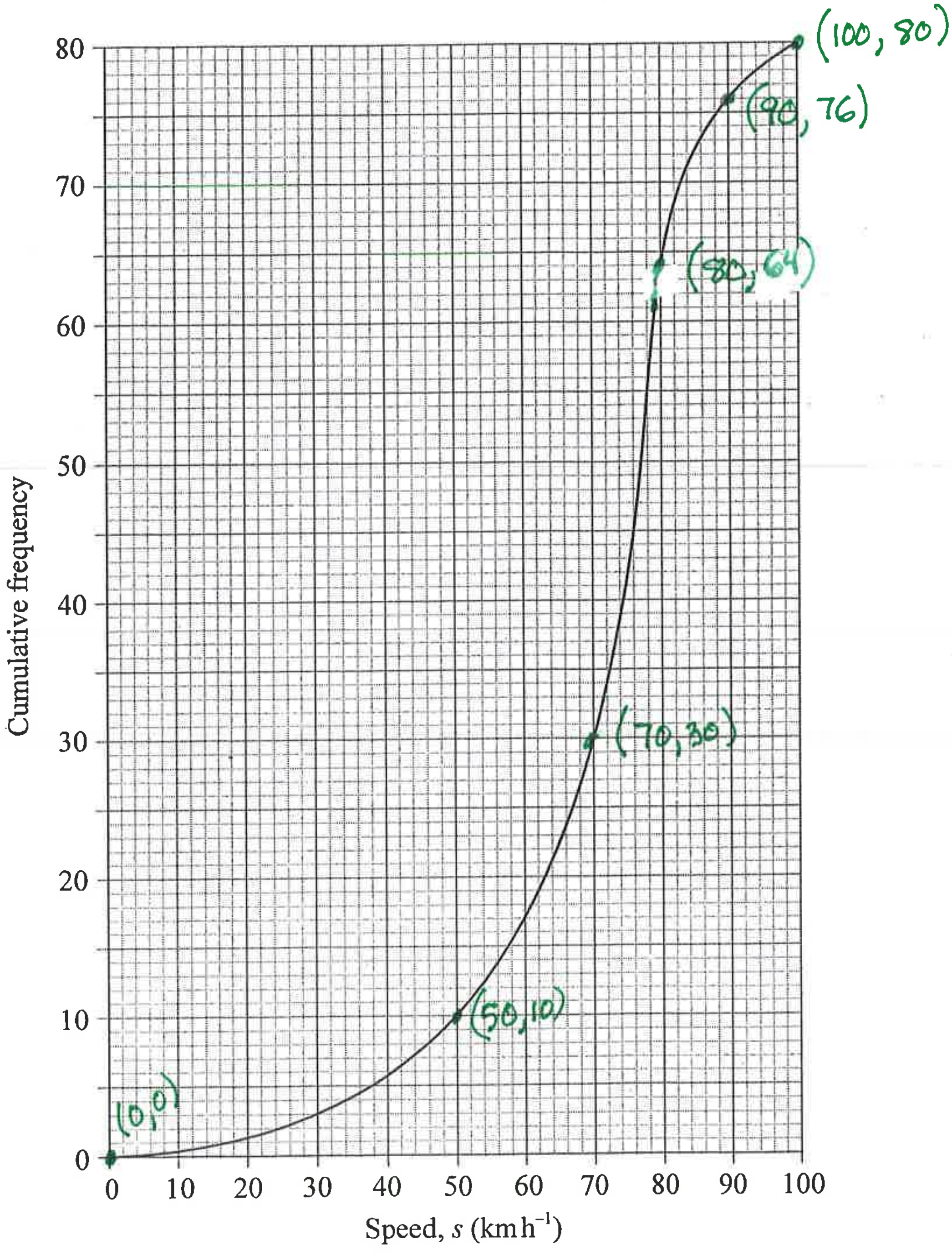
*Answers:*

(a) 800.77 USD

(b) 84 USD

(c) 3787.97 SGD

**6.** The cumulative frequency graph represents the speed, $s$, in $\text{km h}^{-1}$, of 80 cars passing a speed camera.




(a) Write down the number of cars passing the camera with speed of less than or equal to $50 \text{km h}^{-1}$. *[1]*

*(This question continues on the following page)*

*(Question 6 continued)*

(b) Complete the following grouped frequency table for $s$, the speed of the cars passing the camera.

Start by labeling points on graph

| $s\left(\text{km h}^{-1}\right)$ | $0 < s \le 50$ | $50 < s \le 70$ | $70 < s \le 80$ | $80 < s \le 90$ | $90 < s \le 100$ |
|---|---|---|---|---|---|
| Frequency | 10 | 20 | 34 | 12 | 4 |
| Cumul. freq. | 10 | 30 | 64 | 76 | 80 |

*[1]*

(c) Write down the mid-interval value of the $50 < s \le 70$ interval. *[1]*

(d) Use your graphic display calculator to find an estimate of

(i) the mean speed of the cars passing the camera;

76
34
42

(ii) the standard deviation of the speed of the cars passing the camera. *[3]*

*Working:*

c) $50 < s < 70$

$\frac{50+70}{2} = 60$

(ii) see next sheet

d) In List 1 → mid-interval speeds; List 2 → freq.; List 3 → $f \cdot x$

(i)

| List 1 | List 2 | List 3 |
|---|---|---|
| 25 | 10 | |
| 60 | 20 | |
| 75 | 34 | |
| 85 | 12 | |
| 95 | 4 | |
| | 80 | 5400 |

$\Sigma f \cdot x$

$\frac{km}{h}$ same

$\bar{x} = \frac{\Sigma f \cdot x}{n}$

$= \frac{5400}{80} = 67.5$

*Answers:*

(a) 10 cars

(c) 60

(d) (i) 67.5 km h$^{-1}$

(ii) 18.6 km h$^{-1}$

ii

Using mean of 67.5 from part (i) ........

$$s = \sqrt{\frac{\Sigma f(x-\bar{x})^2}{n}} = \sqrt{\frac{27800}{80}} \risingdotseq 18.64135.....$$

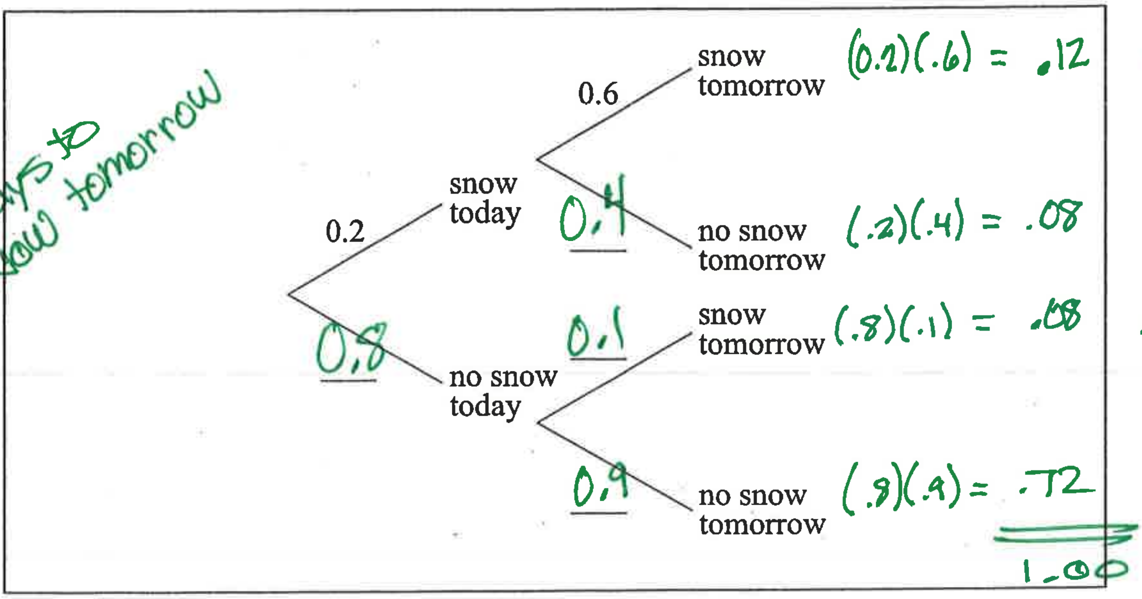
18.6

| $L_1$ | $L_2$ | $L_3$ |
|---|---|---|
| mid inter | freq | $f(x-\bar{x})^2$ |
| 25 | 10 | , |
| 60 | 20 | , |
| 75 | 34 | , |
| 85 | 12 | , |
| 95 | 4 | , |
| | 80 | 27800 ← $\Sigma f(x-\bar{x})^2$ |

**7.** The probability that it snows today is 0.2. If it does snow today, the probability that it will snow tomorrow is 0.6. If it does not snow today, the probability that it will not snow tomorrow is 0.9.

(a) Using the information given, complete the following tree diagram.

2 ways to snow tomorrow




[3]

(b) Calculate the probability that it will snow tomorrow. [3]

*Working:*

(b) P(snow tomorrow)

= P(snow today and snow tomorrow OR no snow today and snow tomorrow)

= (.2)(.6) + (.8)(.1)

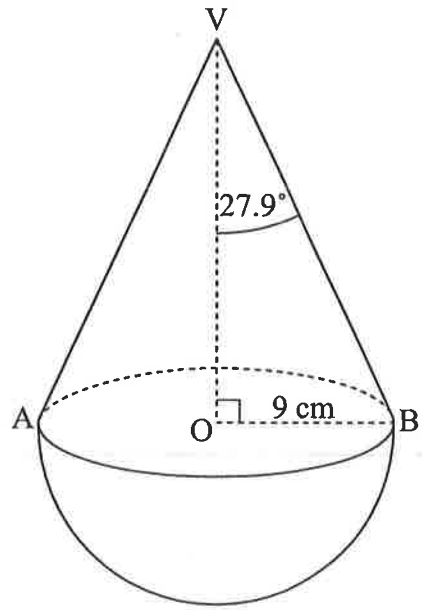
= .2

*Answers:*

(b) .......... .2 ..........

**8.** A child's wooden toy consists of a hemisphere, of radius 9 cm, attached to a cone with the same base radius. O is the centre of the base of the cone and V is vertically above O. Angle OVB is 27.9°.

***diagram not to scale***

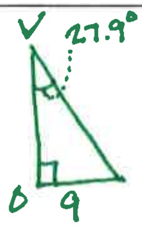



(a) Calculate OV, the height of the cone. *[2]*

(b) Calculate the volume of wood used to make the toy. *[4]*

*Working:*

(a)




Soh Cah Toa

$\tan 27.9° = \frac{9}{VO}$

$VO = \frac{9}{\tan 27.9°} = 17.0\text{cm}$

OR can use Law of Sines

(b) $V_{cone} + V_{hemisphere}$

$= \frac{\pi r^2 h}{3} + \frac{1}{2} \cdot \frac{4\pi r^3}{3}$

$= \frac{\pi (9)^2 (17)}{3} + \frac{1}{2} \cdot \frac{4\pi (9)^3}{3}$

$= 2968.63 \ldots\ldots$ 2970 3 sf.

*Answers:*

(a) 17.0 cm

(b) 2970 cm³

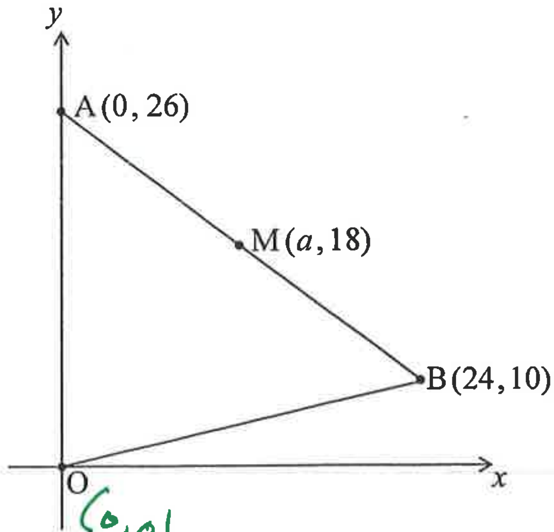
**9.** The diagram shows the points $M(a, 18)$ and $B(24, 10)$. The straight line BM intersects the $y$-axis at $A(0, 26)$. M is the midpoint of the line segment AB.

*diagram not to scale*

(a) Midpoint $\left(\frac{0+24}{2}, \frac{26+10}{2}\right)$

$= (12, 18)$

a




(a) Write down the value of $a$. *[1]*

(b) Find the gradient of the line AB. *[2]*

(c) Decide whether triangle OAM is a right-angled triangle. Justify your answer. *[3]*

*Working:*

b) gradient $\frac{\Delta y}{\Delta x} = \frac{26-10}{0-24} = \frac{16}{-24} = -\frac{2}{3}$

accepted

c) ONE method:

show slopes are opposite reciprocals. Slope of OB would have to be $\frac{3}{2}$

$\frac{10-0}{24-0} = \frac{10}{24} = \frac{5}{12}$ NO

(could also calculate lengths of all 3 sides and check with pythagorean theorem)

*Answers:*

(a) 12

(b) $-2/3$

(c) ..............................

..............................

**10.** Let $f(x) = x^4$.

(a) Write down $f'(x)$. $4x^3$ *[1]*

Point P $(2, 16)$ lies on the graph of $f$.

(b) Find the gradient of the tangent to the graph of $y = f(x)$ at P. *[2]*

(c) Find the equation of the normal to the graph at P. Give your answer in the form $ax + by + d = 0$, where $a$, $b$ and $d$ are integers. *[3]*

*Working:*

(b) $f'(2) = 4(2)^3 = 32$ or you can use GDC to find quickly $\frac{dy}{dx}$

(c) The tangent at $(2,16)$ has gradient 32 so the Normal has a gradient of $-\frac{1}{32}$

its equation would be:

$$\boxed{y - 16 = -\frac{1}{32}(x-2)}$$

also accepted would be

$$y = -\frac{1}{32}x + \frac{257}{16}$$

*Answers:*

(a) $4x^3$

(b) 32

(c) $y - 16 = -\frac{1}{32}(x-2)$

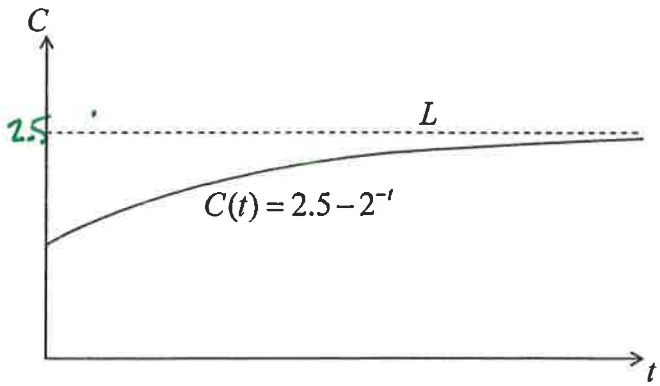
**11.** The amount of electrical charge, $C$, stored in a mobile phone battery is modelled by $C(t) = 2.5 - 2^{-t}$, where $t$, in hours, is the time for which the battery is being charged.

exponential (with reflection)

also equivalent to $C(t) = 2.5 - \left(\frac{1}{2}\right)^t$

$= -\left(\frac{1}{2}\right)^t + 2.5$

*diagram not to scale*




(a) Write down the amount of electrical charge in the battery at $t = 0$. *[1]*

The line $L$ is the horizontal asymptote to the graph.

(b) Write down the equation of $L$. *[2]*

To download a game to the mobile phone, an electrical charge of 2.4 units is needed.

(c) Find the time taken to reach this charge. Give your answer correct to the nearest minute. *[3]*

*Working:*

(a) $C(0) = 2.5 - 2^{-0}$

$= 2.5 - 1$

$= 1.5$

(c) $2.4 = 2.5 - 2^{-t}$

$-2.5 \quad -2.5$

$-.1 = -2^{-t}$

$.1 = 2^{-t}$

$\log .1 = \log 2^{-t}$

$\log .1 = -t \cdot \log 2$

$t = -\frac{\log .1}{\log 2}$

$t \approx 3.32192\ldots$

can also solve graphically

$y_1 = 2.4$

$y_2 = 2.5 - 2^{-t}$

*Answers:*

(a) 1.5

(b) $C = 2.5$

(c) 3 hours 19 min.

or 199 min.

**12.** Ludmila takes a loan of 320 000 Brazilian Real (BRL) from a bank for two years at a nominal annual interest rate of 10 %, **compounded half yearly**. $K=2$

(a) Write down the number of times interest is added to the loan in the two years. *[1]*

(b) Calculate the **exact** amount of money that Ludmila must repay at the end of the two years. *[3]*

Ludmila estimates that she will have to repay 360 000 BRL at the end of the two years.

(c) Calculate the percentage error in her estimate. *[2]*

*Working:*

a) compounded half yearly means interest is paid twice a year or 4 times over 2 years.

b) $FV = 320\,000\left(1+\frac{10}{2\times100}\right)^4$ 2 years at 2 times per year

$= 388962$ BRL

c) ACTUAL 388962
ESTIM 360000

$\%\text{ error} = \left|\frac{360000-388962}{388962}\right|\times100 = 7.445\ldots$

formula IN packet

*Answers:*

(a) 4

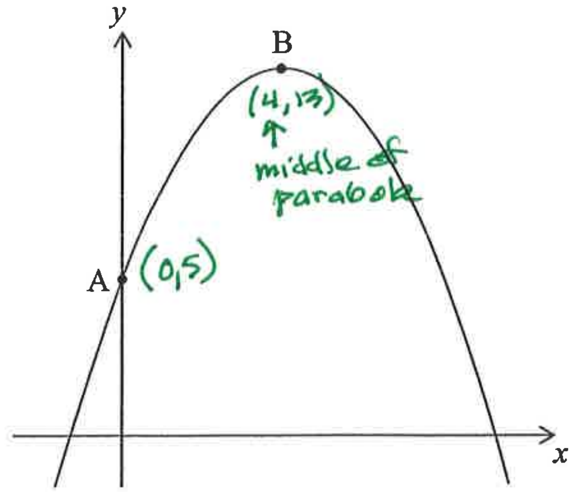
(b) 388962 BRL

(c) 7.45 %

**13.** The graph of the quadratic function $f(x) = ax^2 + bx + c$ intersects the $y$-axis at point $A(0, 5)$ and has its vertex at point $B(4, 13)$.

y-intercept $f(0) = c = 5$

$a(0)^2 + b(0) + c$




(a) Write down the value of $c$. *[1]*

(b) By using the coordinates of the vertex, B, or otherwise, write down **two** equations in $a$ and $b$. *[3]*

(c) Find the value of $a$ and of $b$. *[2]*

*Working:*

(b) from formula packet $x = -\frac{b}{2a}$ to find middle of parabola.

we know $x = 4$

$$\boxed{4 = -\frac{b}{2a}}$$

$f(x) = ax^2 + bx + c \quad (4, 13)$

$13 = a(4)^2 + b(4) + c$

$$\boxed{16a + 4b + 5 = 13}$$

c) $b = -8a$

$16a + 4(-8a) + 5 = 13$

$-16a = 8$

divide by 4

$a = \frac{-1}{2}$

$b = -8\left(\frac{-1}{2}\right) = 4$

*Answers:*

(a) 5

(b) see above

(c) $a = \frac{-1}{2}$ $b = 4$

**14.** Two propositions are defined as follows:

$p$: *Quadrilateral ABCD has two diagonals that are equal in length.*
$q$: *Quadrilateral ABCD is a rectangle.*

(a) Express the following in symbolic form.

could be rewritten

*"A rectangle always has two diagonals that are equal in length."* [2]

(b) Write down in symbolic form the converse of the statement in (a). [1]

(c) Determine, **without** using a truth table, whether the statements in (a) and (b) are logically equivalent. [2]

(d) Write down the name of the statement that is logically equivalent to the converse. [1]

*Working:*

If a Quad is a rectangle then it has two diagonals that are equal in length

$q \to p$

(c) Not equivalent logically since you could have an isosceles trapezoid (which has equal diagonals) but it is Not a rectangle.

An implication ($p \to q$) and its contrapositive ($\neg q \to \neg p$) are logically equivalent but so are the converse ($q \to p$) and the inverse ($\neg p \to \neg q$)

*Answers:*

(a) $q \Rightarrow p$

(b) $p \Rightarrow q$

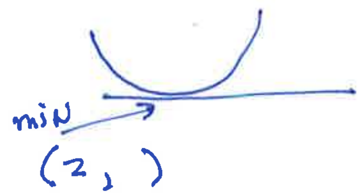
(c)

(d) Inverse

**15.** Consider the curve $y = x^3 + kx$.

(a) Write down $\frac{dy}{dx}$. same as $f'(x)$ *[1]*

The curve has a local maximum at the point where $x = 2$.




(b) Find the value of $k$. *[3]*

(c) Find the value of $y$ at this local maximum. *[2]*

*Working:*

a) $\frac{dy}{dx} = 3x^2 + K$

b) At a minimum, the gradient of the tangent would be 0 (since it is flat)

Set $\frac{dy}{dx}$ equal to 0, $3x^2 + K = 0$ $3(2)^2 + K = 0$

$K = -12$

c) So.... $f(x) = y = x^3 - 12x$

so the point of tangency (y-value) can be found by finding

$f(2) = (2)^3 - 12(2)$

$= -16$

*Answers:*

(a) $3x^2 + K$

(b) $-12$

(c) $-16$

IB SCORING GUIDE FOR PAPER 1, MAY 2014



**1.** (a) $\dfrac{2\cos 45^\circ - \tan 45^\circ}{\sqrt{8192} - 64}$ ***(M1)***

$= 0.015625$ ***(A1)*** ***(C2)***

| **Notes:** Accept $\dfrac{1}{64}$ and also $1.5625 \times 10^{-2}$. |
|---|

(b) (i) 0.02 ***(A1)*(ft)**

(ii) 0.01563 ***(A1)*(ft)**

| **Notes:** For parts (i) and (ii), accept equivalent standard form representations. |
|---|

(iii) $1.5625 \times 10^{-2}$ ***(A2)*(ft)** ***(C4)***

> **Notes:** Award ***(A1)(A0)*** for correct mantissa, between 1 and 10, with incorrect index.
> Follow through from their answer to part (a).
> Where the candidate has correctly rounded their mantissa from part (a) and has the correct exponent, award ***(A0)(A1)***
> Award ***(A0)(A0)*** for answers of the type: $15.625 \times 10^{-3}$.

***[6 marks]***

**2.** (a) 3 ***(A1)*** ***(C1)***

(b) 4 ***(M1)(A1)*** ***(C2)***

| **Note:** Award ***(M1)*** for ordered list of numbers seen. |
|---|

(c) 5.5 ***(A1)*** ***(C1)***

(d) $5.5 - 3$ ***(M1)***

| **Note:** Award ***(M1)*** for 3 and their 5.5 seen. |
|---|

$= 2.5$ ***(A1)*(ft)** ***(C2)***

| **Note:** Follow through from their answer to part (c). |
|---|

***[6 marks]***

**3.** (a) **If** the food is well cooked **and** the drinks are chilled **then** dinner is **not spoilt**. ***(A1)(A1)(A1)*** ***(C3)***

> **Note:** Award ***(A1)*** for "If…then" (then must be seen), ***(A1)*** for the two correct propositions connected with "and", ***(A1)*** for "not spoilt". Only award the final ***(A1)*** if correct statements are given in the correct order.

(b)

| $p$ | $q$ | $r$ | $p \wedge q$ | $\neg r$ | $(p \wedge q) \Rightarrow \neg r$ |
|---|---|---|---|---|---|
| T | T | T | **T** | **F** | **F** |
| T | T | F | **T** | **T** | **T** |
| T | F | T | **F** | **F** | **T** |
| T | F | F | **F** | **T** | **T** |
| F | T | T | **F** | **F** | **T** |
| F | T | F | **F** | **T** | **T** |
| F | F | T | **F** | **F** | **T** |
| F | F | F | **F** | **T** | **T** |

***(A1)(A1)(A1)*(ft)** ***(C3)***

> **Notes:** Award ***(A1)*** for each correct column. The final column must follow through from the previous two columns.

***[6 marks]***

**4.** (a) Country chosen and gender are independent. *(A1)* *(C1)*

| |
|---|
| **Notes:** Accept there is no association between country chosen and gender. Do not accept "not related" or "not correlated" or "influenced". |

(b) 2 *(A1)* *(C1)*

(c) (i) $9.17 (9.16988\ldots)$ *(A1)*

| |
|---|
| **Notes:** Accept 9.169. |

(ii) $0.0102 (0.0102043\ldots)$ *(A1)* *(C2)*

| |
|---|
| **Notes:** Award *(A1)* for 0.010, but *(A0)* for 0.01. |

(d) Since $0.0102 > 0.01$, we accept the null hypothesis. *(R1)(A1)*(ft)

**OR**

Since $9.17 < 9.210$, we accept the null hypothesis. *(R1)(A1)*(ft) *(C2)*

| |
|---|
| **Notes:** To award *(R1)* there should be value(s) given in part (c). If a value is given in (c), we do not need it explicitly stated again in (d).<br>It is sufficient to state a correct comparison.<br>e.g. $p$-value > significance level **OR** $\chi^2_{\text{calc}}$ < critical value<br>Do not award *(R0)(A1)*. Follow through from part (c). |

***[6 marks]***

**5.** (a) $6600 \times \dfrac{1}{8.2421}$ *(M1)*

$= 800.77$ *(A1)* *(C2)*

(b) $3000 \times 0.028$ *(M1)*

$= 84.00$ (accept 84) *(A1)* *(C2)*

(c) $(3000 - 84) \times 1.29903$ *(M1)*

**OR**

$3000 \times 1.29903 \times 0.972$ *(M1)*

$= 3787.97$ *(A1)*(ft) *(C2)*

**Notes:** Follow through from their answer to part (b).

**Note:** Do not penalize in part (c) if conversion process has been reversed consistently ie, multiplication by 8.2421 in part (a) and division by 1.29903 in part (c).

***[6 marks]***

**6.** (a) 10 ***(A1)*** ***(C1)***

(b)

| $s\,(\text{km}\,\text{h}^{-1})$ | $0 < s \le 50$ | $50 < s \le 70$ | $70 < s \le 80$ | $80 < s \le 90$ | $90 < s \le 100$ |
|---|---|---|---|---|---|
| Frequency | **10** | **20** | 34 | **12** | 4 |

***(A1)*(ft)** ***(C1)***

**Note:** Follow through from their answer to part (a).

(c) 60 ***(A1)*** ***(C1)***

(d) (i) $67.5\,(\text{km}\,\text{h}^{-1})$ ***(A2)*(ft)**

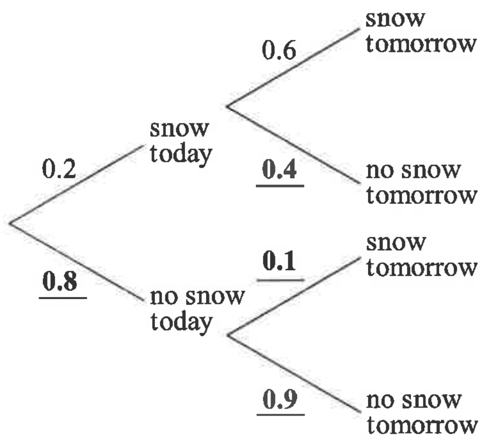
**Notes:** Award ***(M1)*** for an attempt to use the formula for the mean with at least two midpoint values consistent with their answer to part (c). Follow through from their table in part (b).

(ii) 18.6 (18.6413…) ***(A1)*(ft)** ***(C3)***

**Note:** Follow through from their table in part (b).

***[6 marks]***

**7.** (a)




***(A1)(A1)(A1)*** ***(C3)***

| |
|---|
| **Note:** Award ***(A1)*** for each correct pair of probabilities. |

(b) $0.2\times0.6+0.8\times0.1$ ***(A1)*(ft)*(M1)***

| |
|---|
| **Note:** Award ***(A1)*(ft)** for two correct products of probabilities taken from their diagram, ***(M1)*** for the addition of their products. |

$=0.2 \left(\frac{1}{5}, 20\%\right)$ ***(A1)*(ft)** ***(C3)***

| |
|---|
| **Note:** Accept any equivalent correct fraction. Follow through from their tree diagram. |

***[6 marks]***

**8.** (a) $\tan 27.9^\circ = \dfrac{9}{\text{OV}}$ ***(M1)***

> **Note:** Award ***(M1)*** for correct substitution in trig formula.

$\text{OV} = 17.0(\text{cm})\ (16.9980\ldots)$ ***(A1)*** ***(C2)***

(b) $\dfrac{\pi(9)^2(16.9980\ldots)}{3} + \dfrac{1}{2} \times \dfrac{4\pi(9)^3}{3}$ ***(M1)(M1)(M1)***

> **Note:** Award ***(M1)*** for correctly substituted volume of the cone, ***(M1)*** for correctly substituted volume of a sphere divided by two (hemisphere), ***(M1)*** for adding the correctly substituted volume of the cone to *either* a correctly substituted sphere *or* hemisphere.

$= 2970\,\text{cm}^3\ (2968.63\ldots)$ ***(A1)*(ft)** ***(C4)***

> **Note:** The answer is $2970\,\text{cm}^3$, the units are required.

***[6 marks]***

**9.** (a) 12 *(A1)* *(C1)*

> **Note:** Award *(A1)* for $(12, 18)$.

(b) $\dfrac{26-10}{0-24}$ *(M1)*

> **Note:** Accept $\dfrac{26-18}{0-12}$ or $\dfrac{18-10}{12-24}$ (or equivalent).

$= -\dfrac{2}{3} \left(-\dfrac{16}{24}, -0.666666\ldots\right)$ *(A1)* *(C2)*

> **Note:** If either of the alternative fractions is used, follow through from their answer to part (a).
> The answer is now ***(A1)*(ft)**.

(c) gradient of $\text{OM} = \dfrac{3}{2}$ *(A1)*(ft)

> **Note:** Follow through from their answer to part (b).

$-\dfrac{2}{3} \times \dfrac{3}{2}$ *(M1)*

> **Note:** Award *(M1)* for multiplying their gradients.

Since the product is $-1$, OAM is a right-angled triangle *(R1)*(ft)

> **Notes:** Award the final *(R1)* only if their conclusion is consistent with their answer for the product of the gradients.
> The statement that OAM is a right-angled triangle without justification is awarded no marks.

**OR**

$(26-18)^2 + 12^2$ and $12^2 + 18^2$ *(A1)*(ft)

$\left((26-18)^2 + 12^2\right) + (12^2 + 18^2) = 26^2$ *(M1)*

> **Note:** This method can also be applied to triangle OMB.
> Follow through from (a).

Hence a right angled triangle ***(R1)*(ft)**

> **Note:** Award the final ***(R1)*** only if their conclusion is consistent with their ***(M1)*** mark.

**OR**

$\text{OA} = \text{OB} = 26\,(\text{cm})$ an isosceles triangle ***(A1)***

> **Note:** Award ***(A1)*** for $\text{OA} = 26\,(\text{cm})$ and $\text{OB} = 26\,(\text{cm})$.

Line drawn from vertex to midpoint of base is perpendicular to the base ***(M1)***

Conclusion ***(R1)*** ***(C3)***

> **Note:** Award, at most ***(A1)(M0)(R0)*** for stating that OAB is an isosceles triangle without any calculations.

***[6 marks]***

**10.** (a) $\left(f'(x)=\right)\ 4x^3$ *(A1)* *(C1)*

(b) $4\times 2^3$ *(M1)*

| **Note:** Award *(M1)* for substituting 2 into their derivative. |
|---|

$=32$ *(A1)*(ft) *(C2)*

| **Note:** Follow through from their part (a). |
|---|

(c) $y-16=-\dfrac{1}{32}(x-2)$ **or** $y=-\dfrac{1}{32}x+\dfrac{257}{16}$ *(M1)(M1)*

| **Note:** Award *(M1)* for their gradient of the normal seen, *(M1)* for point substituted into equation of a straight line in only $x$ and $y$ (with any constant '$c$' eliminated). |
|---|

$x+32y-514=0$ **or** any integer multiple *(A1)*(ft) *(C3)*

| **Note:** Follow through from their part (b). |
|---|

***[6 marks]***

**11.** (a) 1.5 *(A1)* *(C1)*

(b) $C = 2.5$ (accept $y = 2.5$) *(A1)(A1)* *(C2)*

| |
|---|
| **Notes:** Award *(A1)* for $C$ (or $y$) = a positive constant, *(A1)* for the constant $= 2.5$. <br> Answer must be an equation. |

(c) $2.4 = 2.5 - 2^{-t}$ *(M1)*

| |
|---|
| **Note:** Award *(M1)* for setting the equation equal to 2.4 or for a horizontal line drawn at approximately $C = 2.4$. <br> Allow $x$ instead of $t$. |

**OR**

$-t\ln(2) = \ln(0.1)$ *(M1)*

$t = 3.32192...$ *(A1)*

$t = 3$ hours and 19 minutes (199 minutes) *(A1)*(ft) *(C3)*

| |
|---|
| **Note:** Award the final *(A1)***(ft)** for correct conversion of **their** time in hours to the nearest minute. |

***[6 marks]***

**12.** (a) 4 *(A1)* *(C1)*

(b) $320000\left(1+\frac{10}{2\times100}\right)^{2\times2}$ *(M1)(A1)*

> **Note:** Award *(M1)* for substituted compound interest formula, *(A1)* for correct substitutions.

**OR**

$N=2$
$I\%=10$
$PV=-320000$
$P/Y=1$
$C/Y=2$ *(A1)(M1)*

> **Note:** Award *(A1)* for $C/Y=2$ seen, *(M1)* for correctly substituted values from the question into the finance application.

**OR**

$N=4$
$I\%=10$
$PV=-320000$
$P/Y=2$
$C/Y=2$ *(A1)(M1)*

> **Note:** Award *(A1)* for $C/Y=2$ seen, *(M1)* for correctly substituted values from the question into the finance application.

amount to repay $=388\,962$ *(A1)* *(C3)*

> **Note:** Award *(C2)* for final answer 389 000 if 388 962 not seen previously.

(c) $\left|\frac{360000-388962}{388962}\right|\times100$ *(M1)*

> **Note:** Award *(M1)* for correctly substituted percentage error formula.

$=7.45(\%)\ (7.44597\ldots)$ *(A1)*(ft) *(C2)*

> **Notes:** Follow through from their answer to part (b).

***[6 marks]***

**13.** (a) 5 *(A1)* *(C1)*

(b) *at least one of the following equations required*

$a(4)^2 + 4b + 5 = 13$

$4 = -\frac{b}{2a}$

$a(8)^2 + 8b + 5 = 5$ *(A2)(A1)* *(C3)*

| | |
|---|---|
| **Note:** | Award ***(A2)(A0)*** for one correct equation, or its equivalent, and ***(C3)*** for any two correct equations.<br>Follow through from part (a).<br>The equation $a(0)^2 + b(0) = 5$ earns no marks. |

(c) $a = -\frac{1}{2}, b = 4$ ***(A1)*****(ft)*****(A1)*****(ft)*** ***(C2)***

| | |
|---|---|
| **Note:** | Follow through from their equations in part (b), but only if their equations lead to unique solutions for *a* and *b*. |

***[6 marks]***

**14.** (a) $q \Rightarrow p$ ***(A1)(A1)*** ***(C2)***

> **Note:** Award the first ***(A1)*** for seeing the implication sign, the second ***(A1)*** is for a correct answer only. Not using the implication earns **no** marks.

(b) $p \Rightarrow q$ ***(A1)*****(ft)** ***(C1)***

> **Note:** Award ***(A1)*****(ft)** where the propositions in the implication in part (a) are exchanged.

(c) Not equivalent; a kite or an isosceles trapezium (for example) can have diagonals that are equal in length. ***(A1)(R1)*** ***(C2)***

> **Notes:** Accept a valid sketch as reasoning.
> If the reason given is that *a square has diagonals of equal length, but is not a rectangle*, then award ***(R1)(A0)***.
> Do not award ***(A1)(R0)***.
> Do not accept solutions based on truth tables.

(d) Inverse ***(A1)*** ***(C1)***

> **Note:** Do not accept symbolic notation.

***[6 marks]***

**15.** (a) $3x^2 + k$ *(A1)* *(C1)*

(b) $3(2)^2 + k = 0$ *(A1)*(ft)*(M1)*

| |
|---|
| **Note:** Award *(A1)*(ft) for substituting 2 in their $\frac{dy}{dx}$, *(M1)* for setting their $\frac{dy}{dx} = 0$. |

$k = -12$ *(A1)*(ft) *(C3)*

| |
|---|
| **Note:** Follow through from their derivative in part (a). |

(c) $2^3 - 12 \times 2$ *(M1)*

| |
|---|
| **Note:** Award *(M1)* for substituting 2 and their $-12$ into equation of the curve. |

$= -16$ *(A1)*(ft) *(C2)*

| |
|---|
| **Note:** Follow through from their value of $k$ found in part (b). |

***[6 marks]***