Mock Exam 1 - Paper 1

M16/5/MATSD/SP1/ENG/TZ1/XX

SOLUTIONS

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematical studies
Standard level
Paper 1

Tuesday 10 May 2016 (afternoon)

1 hour 30 minutes

Candidate session number

Things to know about the format of Paper 1:

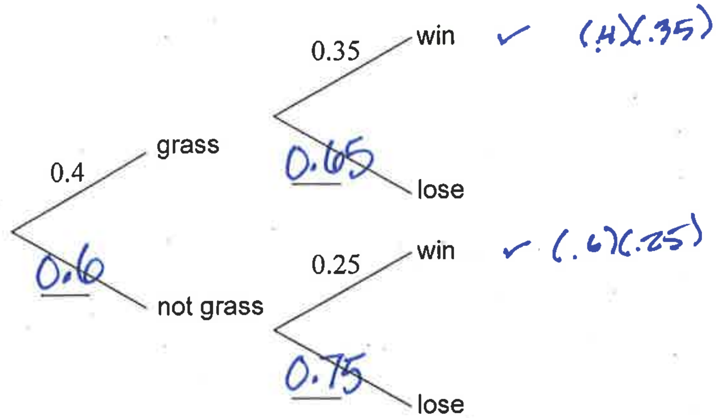
- ✓ You are given 5 minutes reading time at the start of the exam during which time you may not write, but may read the questions. This is not part of the 90 minutes and should be used well.
- ✓ 15 questions, 6 marks each
- ✓ 90 minutes total allowed, a mark a minute.
- ✓ Use your formula packet.
- ✓ Full marks awarded if the correct answer is shown on the answer blank, whether there is work or not. However, method marks are possible (showing work may get you points on if answer is incorrect, on some questions).
- ✓ Give answers to 3 significant figures unless it is otherwise stated or it is financial.
- ✓ Unit penalty applies at specific points in the mark scheme, but you won't know where. (therefore, write units on answers where appropriate!)
- ✓ On Paper 1 and 2 you don't have to show work for the following.
    - Mean and Standard deviation
    - Correlation coefficient, r .
    - LSRL
    - Chi-Square statistic

So, just use your GDC and calculate it quickly !!

Maximum marks will be given for correct answers. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. Write your answers in the answer boxes provided. Solutions found from a graphic display calculator should be supported by suitable working, for example, if graphs are used to find a solution, you should sketch these as part of your answer.

1. The probability that Nikita wins a tennis match depends on the surface of the tennis court on which she is playing. The probability that she plays on a grass court is 0.4.
The probability that Nikita wins on a grass court is 0.35. The probability that Nikita wins when the court is not grass is 0.25.

(a) Complete the following tree diagram. [3]



(b) Find the probability that Nikita wins a match. [3]

Working:

grass and win — Not grass and WIN

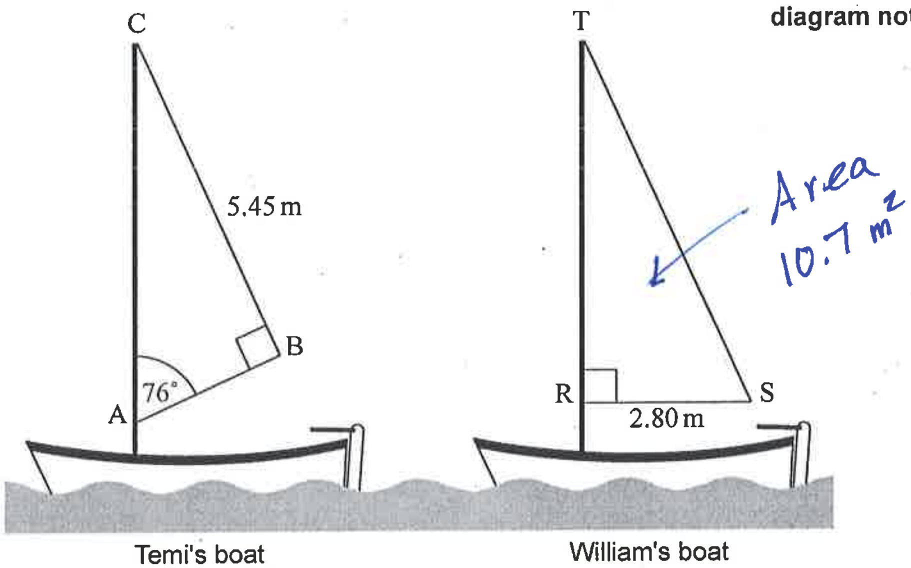
$$P(\text{wins a match}) = (.4)(.35) + (0.6)(0.25)$$
$$= 0.29$$

Answer:
(b) 0.29

2. Temi's sailing boat has a sail in the shape of a right-angled triangle, ABC. $BC = 5.45\,\text{m}$, angle $CAB = 76^\circ$ and angle $ABC = 90^\circ$.

(a) Calculate AC, the height of Temi's sail. [2]

diagram not to scale



William also has a sailing boat with a sail in the shape of a right-angled triangle, TRS. $RS = 2.80\,\text{m}$. The area of William's sail is $10.7\,\text{m}^2$.

(b) Calculate RT, the height of William's sail. [2]

(c) Calculate the size of angle RST. [2]

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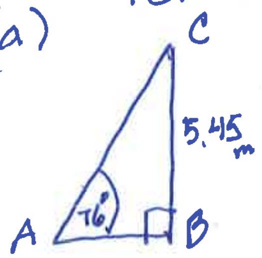
(Question 2 continued)

Working:

(a) Temi's boat

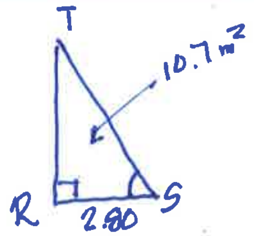


$\sin(76°) = \frac{5.45}{AC} \rightarrow AC = \frac{5.45}{\sin(76°)}$

$= 5.6168\ldots$

can also use Law of Sines

(b) William's boat



Area $= \frac{1}{2}bh$

$10.7 = \frac{1}{2}(2.80)h$

h (RT) $= 7.64285\ldots = \underline{7.64 \text{ m}}$

(c) $\tan(\hat{RST}) = \frac{7.64285\ldots}{2.80}$

$\hat{RST} = \tan^{-1}\left(\frac{7.64285\ldots}{2.80}\right) = 69.8794\ldots$

$= \underline{69.9°}$

can only 1 mark if premature rounding from (b) produces 69.8°

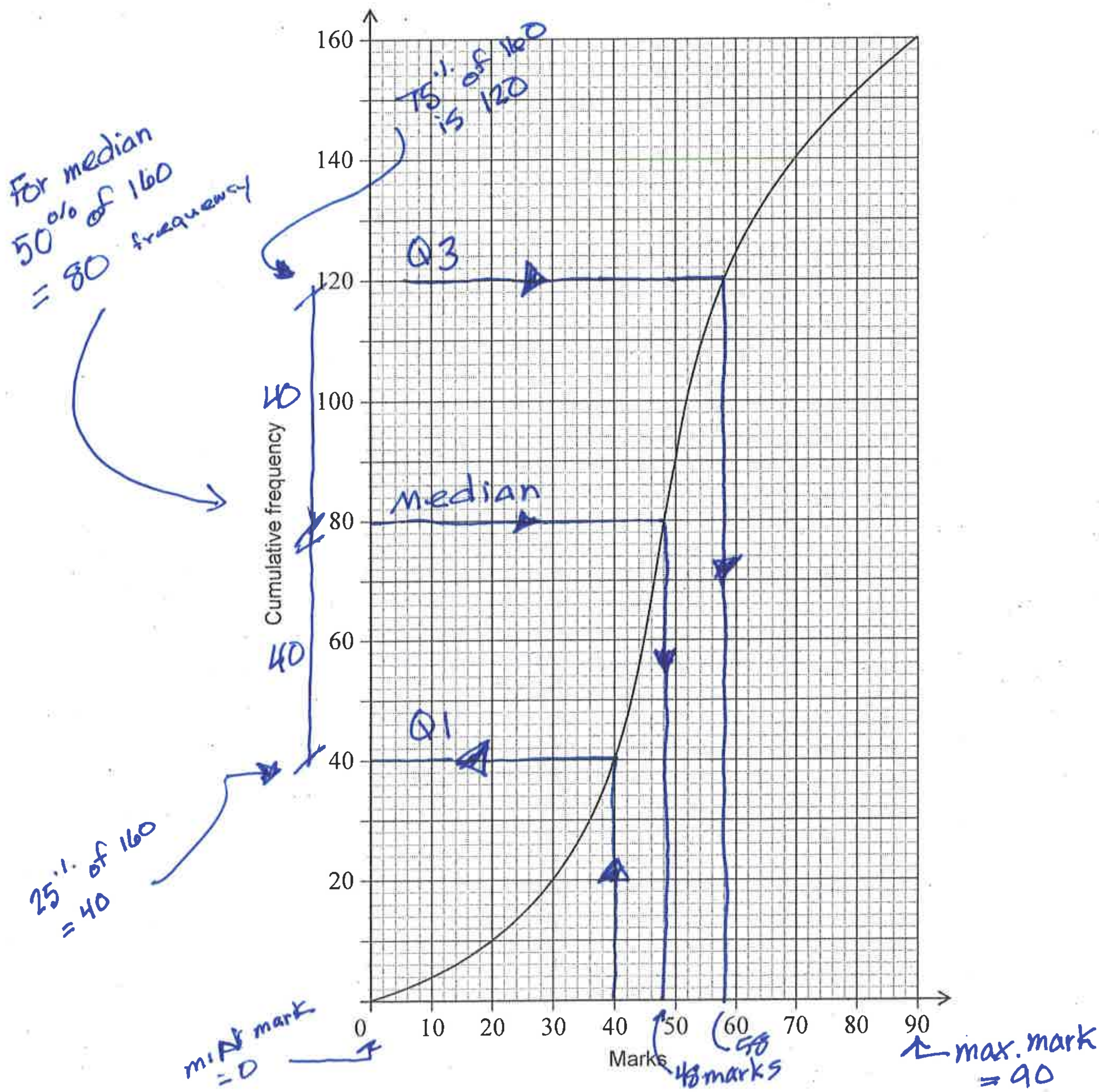
Answers:

(a) 5.62 m 2

(b) 7.64 m 2

(c) 69.9° 2

3. In a school 160 students sat a mathematics examination. Their scores, given as marks out of 90, are summarized on the cumulative frequency diagram.



(a) Write down the median score. [1]

The lower quartile of these scores is 40. .... So the Upper Quartile must use Cum. freq. of 120

(b) Find the interquartile range. $58 - 40 = 18$ marks [2]

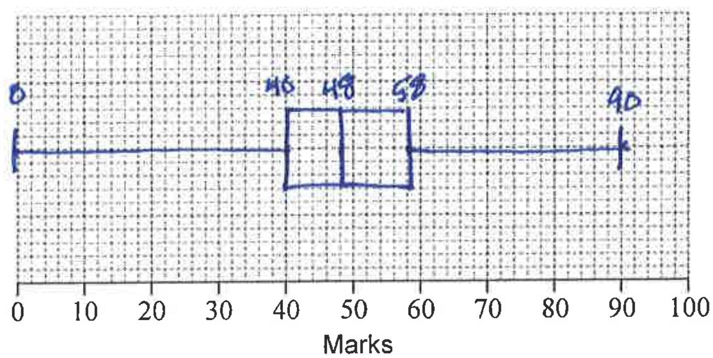
(This question continues on the following page)

(Question 3 continued)

The lowest score was 6 marks and the highest score was 90 marks.

(c) Draw a box-and-whisker diagram on the grid below to represent the students' examination scores. [3]

5 number summary



Working:

1 Point for correct maximum and min plotted

1 Point for correct median.

1 Point for 40 and upper quartile plotted (58).
(cannot earn this last point if a ruler is not used or if a horiz. line is extended beyond what is shown)

Answers:

(a) 48 marks

(b) 18 marks

4. FreshWave brand tuna is sold in cans that are in the shape of a cuboid with length 8 cm, width 5 cm and height 3.5 cm. HappyFin brand tuna is sold in cans that are cylindrical with diameter 7 cm and height 4 cm.

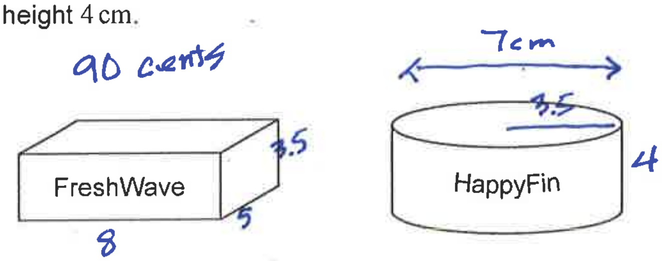


diagram not to scale

(a) Find the volume, in cm^3, of a can of

(i) FreshWave tuna;

(ii) HappyFin tuna. [4]

The price of tuna per cm^3 is the same for each brand. A can of FreshWave tuna costs 90 cents.

(b) Calculate the price, in cents, of a can of HappyFin tuna. [2]

Working:

a (i) $V = (8)(5)(3.5) = 140 \text{ cm}^3$

(ii) $V = \pi r^3 h = \pi (3.5)^3 \cdot 4 = 153.938\ldots = 154 \text{ cm}^3$

↑ 3 signif. figures

b Fresh Wave

$\dfrac{90 \text{ cents}}{140 \text{ cm}^3} = 0.643$ cents per cm^3 $(.642857\ldots)$

↑ 3 sig fig

cost of HappyFin can $154 \text{ cm}^3 \left(.643 \dfrac{\text{cents}}{\text{cm}^3}\right) = 99.022 \doteq 99$

Answers:

(a) (i) 140 cm^3

(ii) 154 cm^3

(b) 99

5. Consider the following statements

z: x is an integer
q: x is a rational number
r: x is a real number.

(a) (i) Write down, in words, $\neg q$.

(ii) Write down a value for x such that the statement $\neg q$ is true. [2]

(b) Write the following argument in symbolic form:
"If x is a real number and x is not a rational number, then x is not an integer". [3]

Phoebe states that the argument in part (b) can be shown to be valid, without the need of a truth table.

(c) Justify Phoebe's statement. [1]

Working:

c) All integers are rational numbers (and therefore x cannot be an integer if it is non-rational). (or something equivalent to this)

or

if x is an integer, then x is a rational #, therefore if x is not a rational #, then x is not an integor (contrapositive)

"x is an irrational number" also accepted

any non-rational number such as $\sqrt{2}$, $\sqrt{3}$, etc or π

Answers:

(a) (i) x is not a rational number. A1

(ii) A1

(b) $(r \wedge \neg q) \Rightarrow \neg z$ A1 A1 A1

(c) See above

6. One of the locations in the 2016 Olympic Games is an amphitheatre. The number of seats in the first row of the amphitheatre, u_1, is 240. The number of seats in each subsequent row forms an arithmetic sequence. The number of seats in the sixth row, u_6, is 270.

(a) Calculate the value of the common difference, d. [2]

There are 20 rows in the amphitheatre.

(b) Find the **total** number of seats in the amphitheatre. [2]

Anisha visits the amphitheatre. She estimates that the amphitheatre has 6500 seats.

(c) Calculate the percentage error in Anisha's estimate. [2]

remember that the formula sheet has two options for a sum of arithmetic sequence

Working:

first term   difference

a) $u_n = u_1 + d(n-1)$

$270 = 240 + d(6-1)$

$30 = d(5)$

$d = 6$

(b) $S_n = \frac{n}{2}\left[2u_1 + d(n-1)\right]$

$= \frac{20}{2}\left[2(240) + 6(20-1)\right]$

$= 5940$

c) % error

$\left|\frac{6500 - 5940}{5940}\right| \times 100 = 9.43\%$ $(9.42760\ldots)$

Answers:

(a) 6   M1 A1   or C2

(b) 5940   M1 A1   C2

(c) 9.43 (%)

7. The equation of line L_1 is $y = 2.5x + k$. Point $A(3, -2)$ lies on L_1.

(a) Find the value of k. [2]

The line L_2 is perpendicular to L_1 and intersects L_1 at point A.

(b) Write down the gradient of L_2. [1]

(c) Find the equation of L_2. Give your answer in the form $y = mx + c$. [2]

(d) Write your answer to part (c) in the form $ax + by + d = 0$ where a, b and $d \in \mathbb{Z}$. [1]

integers

Working:

a) $-2 = 2.5(3) + K$

$K = -9.5$

b) gradient of L_1 is 2.5

so gradient of L_2 is $-\frac{1}{2.5} = 0.4$

(c) $y = -0.4x + K$

$-2 = -0.4(3) + K$

$K = -0.8$

so... $y = -.4x - .8$

or $y = -\frac{2}{5}x - \frac{4}{5}$

or use point-slope form $y - y_1 = m(x - x_1)$

$y - (-2) = -.4(x-3)$

$y + 2 = -.4(x-3)$

(d) $y = -.4x - .8$

$.4x + y = -.8$

multiply by 10

$4x + 10y = -8$

or

$2x + 5y = -4$

$2x + 5y + 4 = 0$

Answers:

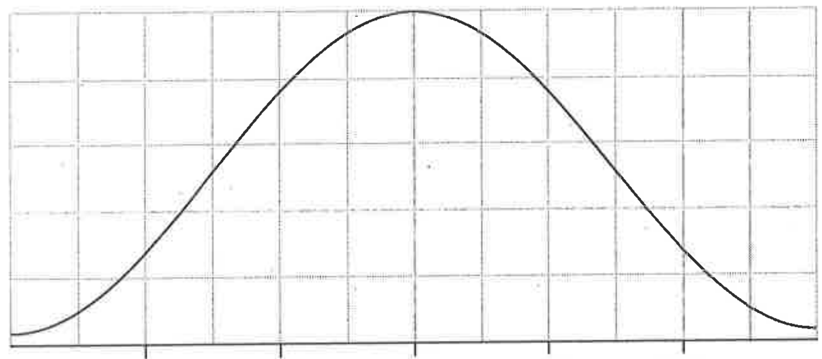
(a) -9.5

(b) -0.4

(c) $y = -.4x - .8$ or $y + 2 = -.4(x-3)$

(d) $2x + 5y + 4 = 0$

8. The lifetime, L, of light bulbs made by a company follows a normal distribution. L is measured in hours. The normal distribution curve of L is shown below.



(a) Write down the mean lifetime of the light bulbs. 5800 hours $= \mu$ [1]

The standard deviation of the lifetime of the light bulbs is 850 hours. $= \sigma$

(b) Find the probability that $5000 \le L \le 6000$, for a randomly chosen light bulb. [2]

The company states that 90% of the light bulbs have a lifetime of at least k hours.

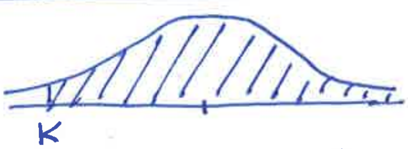
(c) Find the value of k. Give your answer correct to the nearest hundred. [3]

Working:

(b) $P(5000 < L < 6000) = 0.420 \quad (.419703\ldots\ldots)$

↑ normal cdf(5000, 6000, 5800, 850)

(c)



INVERSE NORMAL

$P(L > k) = 90\%$

so... $P(L < k) = 10\%$

$K = \text{invnorm}(.1, \underset{\mu}{5800}, \underset{\sigma}{850})$

probability to left

$= 4710.6811$

$= 4700$

3 signif. figures

Answers:

(a) 5800

(b) .420

(c) 4700

9. **In this question give all answers correct to the nearest whole number.**

Loic travelled from China to Brazil. At the airport he exchanged 3100 Chinese Yuan, CNY, to Brazilian Real, BRL, at an exchange rate of 1 CNY = 0.3871 BRL. No commission was charged.

(a) Calculate the amount of BRL he received. [2]

When he returned to China, Loic changed his remaining BRL at a bank. The exchange rate at the bank was 1 CNY = 0.3756 BRL and a commission of 5 % was charged. He received 285 CNY.

(b) (i) Calculate the amount of CNY Loic would have received if no commission was charged.

(ii) Calculate the amount of BRL Loic exchanged when he returned to China. [4]

a tough question!

Working:

a) $(3100 \text{ CNY})\left(\frac{.3871 \text{ BRL}}{1 \text{ CNY}}\right) = 1200.01 = 1200 \text{ BRL}$

b) Back to china with unknown about of Brazilian Real, X.

– with 5% commission taken away there is only 0.95 X left to exchange

– the exchange : $(.95X\ \cancel{BRL})\left(\frac{1 \text{ CNY}}{.3756\ \cancel{BRL}}\right) = 285$

so X = 112.68 BRL is the remaining amount brought back from Brazil.

(i) If there was no commission $(112.68)\left(\frac{1}{.3756}\right) = 300 \text{ CNY}$

(ii) 112.68 rounded to nearest whole number is 113

Answers:

(a) 1200 BRL

(b) (i) 300 CNY

(ii) 113 BRL

10. The manager of a travel agency surveyed 1200 travellers. She wanted to find out whether there was a relationship between a traveller's age and their preferred destination. The travellers were asked to complete the following survey.

Traveller survey

My age is:

25 or younger	26–40	41–60	61 or older

My preferred destination is:

New York	Tokyo	Melbourne	Dubai	Marrakech

A χ^2 test was carried out, at the 5 % significance level, on the data collected.

(a) Write down the null hypothesis. [1]

(b) Find the number of degrees of freedom. [2]

The critical value of this χ^2 test is 21.026.

(c) Use this information to write down the values of the χ^2 statistic for which the null hypothesis is rejected. [1]

From the travellers taking part in the survey, 285 were 61 years or older and 420 preferred Tokyo.

(d) Calculate the expected number of travellers who preferred Tokyo and were 61 years or older. [2]

(This question continues on the following page)

(Question 10 continued)

Working:

(b) (#rows − 1)(#columns − 1) = (4 − 1)(5 − 1) = 12   M1A1 C2

(c) $\chi^2_{\text{critical value}}$ given as 21.026   C1

$$\boxed{\chi^2_{calc} > 21.026}$$

must show value

d

	25 or younger	26-40	41-60	61 or older
NY				
Tokyo				
Melbourne				
Dubai				
Marrakesh				

420

285

So

$$\frac{285}{1200} \cdot \frac{420}{1200} \cdot 1200 = 99.75$$

or $\dfrac{285 \cdot 420}{1200} = 99.75$

you can always write your answers in this space

Answers:

(a) H_0: Age and preferred destination are independent

(b) 12

(c) $\chi^2_{calc} > 21.026$

(d) 99.8

11. Consider the function $f(x) = ax^2 + c$. a quadratic!

derivative

(a) Find $f'(x)$. [1]

Point $A(-2, 5)$ lies on the graph of $y = f(x)$. The gradient of the tangent to this graph at A is -6.

(b) Find the value of a. [3]

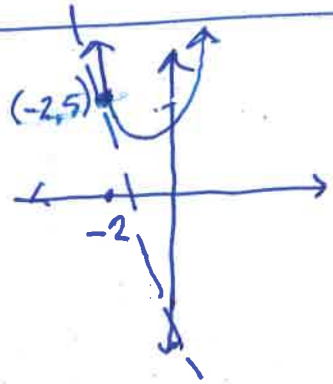
(c) Find the value of c. [2]

Working:

a) $f'(x) = a \cdot 2x^1 = 2ax$

the derivative always represents all gradients at every location of the function, $f(x)$

b)



Set $f'(x)$ equal to -6

$2ax = -6$ (-2,5)

$2a(-2) = -6$

$-4a = -6$

$a = \frac{3}{2}$ or 1.5

so $f(x) = 1.5x^2 + c$

c) since $(-2,5)$ is on the curve of

$f(x) = 1.5x^2 + c$

$5 = 1.5(-2)^2 + c$

$5 = 6 + c$

$c = -1$

Answers:

(a) $2ax$ C1

(b) 1.5 M1 M1 A1

(c) -1 M1 A1

12. In this question give all answers correct to two decimal places.

Diogo deposited 8000 Argentine pesos, ARS, in a bank account which pays a nominal annual interest rate of 15 %, **compounded monthly**.

(a) Find how much interest Diogo has earned after 2 years. [3]

Carmen also deposited ARS in a bank account. Her account pays a nominal annual interest rate of 17 %, **compounded yearly**. After three years, the total amount in Carmen's account is 10 000 ARS.

(b) Find the amount that Carmen deposited in the bank account. [3]

Working:

a) $FV = 8000\left(1 + \frac{15}{100(12)}\right)^{12\cdot 2} = 10,778.81$ ARS

Interest earned is the difference between FV and PV

$10,778.81 - 8000 = 2,778.81$

b) $10000 = PV\left(1 + \frac{17}{100(1)}\right)^{1\cdot 3}$

$10000 = PV(1.17)^3$

$PV = \frac{10000}{(1.17)^3} = 6243.70556\ldots$

Answers:

(a) 2778.81 ARS M1 A1 A1

(b) 6243.71 ARS M1 A1 A1

Turn over

13. The golden ratio, r, was considered by the Ancient Greeks to be the perfect ratio between the lengths of two adjacent sides of a rectangle. The exact value of r is $\dfrac{1+\sqrt{5}}{2}$.

(a) Write down the value of r

(i) correct to 5 significant figures;

(ii) correct to 2 decimal places. [2]

Phidias is designing rectangular windows with adjacent sides of length x metres and y metres. The area of each window is $1\ \text{m}^2$.

(b) Write down an equation to describe this information. [1]

Phidias designs the windows so that the ratio between the longer side, y, and the shorter side, x, is the golden ratio, r.

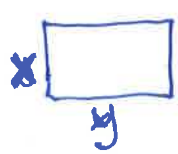
(c) Write down an equation in y, x and r to describe this information. [1]

(d) Find the value of x. [2]

Working:

a, (i) $\dfrac{1+\sqrt{5}}{2} = 1.618033.... = 1.6180$ 5 s.f.

b



$xy = 1$

c $\dfrac{y}{x} = r$ or $\dfrac{y}{x} = \dfrac{1+\sqrt{5}}{2}$

or $\dfrac{y}{x} = 1.6180$

several possible answers

d $\dfrac{y}{x} = 1.618033....$

$y = (x)(1.618033.....)$

$\boxed{xy = 1}$

$x(x)(1.61803....) = 1$

$x^2 = \dfrac{1}{1.61803...}$

$x = 0.786$

Answers:

(a) (i) 1.6180 A1

(ii) 1.62 A1

(b) $xy = 1$ A1

(c) A1

(d) .786 M1 A1

14. A population of 200 rabbits was introduced to an island. One week later the number of rabbits was 210. The number of rabbits, N, can be modelled by the function

(1, 210)

$$N(t) = 200 \times b^t, \ t \geq 0,$$

where t is the time, in weeks, since the rabbits were introduced to the island.

(a) Find the value of b. [2]

(b) Calculate the number of rabbits on the island after 10 weeks. [2]

An ecologist estimates that the island has enough food to support a maximum population of 1000 rabbits.

(c) Calculate the number of weeks it takes for the rabbit population to reach this maximum. [2]

Working:

(a)

$N(t) = 200(b)^t$ (1, 210)

$210 = 200(b)^1$

$b = 1.05$

(b)

$N(10) = 200(1.05)^{10}$

$= 325.7789$

$= 325$ rabbits (whole rabbits)

(c) max population is 1000 rabbits

$200(1.05)^t = 1000$

$1.05^t = 5$

$t = \log_{1.05}(5)$

$= \dfrac{\log(5)}{\log(1.05)}$

$= 32.9869...$

$= 33.0$ weeks 3. s.f.

Answers:

(a) 1.05 M1 A1

(b) 325 M1 A1

(c) 33 weeks M1 A1

Turn over

15. A company sells fruit juices in cylindrical cans, each of which has a volume of 340 cm^3. The surface area of a can is $A \text{ cm}^2$ and is given by the formula

$$A = 2\pi r^2 + \frac{680}{r},$$

where r is the radius of the can, in cm.

To reduce the cost of a can, its surface area must be minimized.

(a) Find $\frac{dA}{dr}$. ← means find the derivative **[3]**

(b) Calculate the value of r that minimizes the surface area of a can. **[3]**

Working:

$A = 2\pi r^2 + \frac{680}{r} = 2\pi r^2 + 680 r^{-1}$

(a)

so $\frac{dA}{dr} = 4\pi r^1 - 680 r^{-2}$

$= 4\pi r - \frac{680}{r^2}$

(b) maximum occurs when the tangent to the curve is flat (when gradient is equal to 0 in other words)

$4\pi r - \frac{680}{r^2} = 0$

multiply by r^2

$4\pi r^3 - 680 = 0$

$4\pi r^3 = 680$

$r^3 = \frac{170}{\pi}$

$r = \sqrt[3]{\frac{170}{\pi}}$

$= 3.78239....$

Answers:

(a) $4\pi - \frac{680}{r^2}$ A1 A1 A1

(b) 3.78 cm M1 A1 A1