

Worksheet 6.2.3

Double Substitution into $y = ab^x + c$
 ↑ 12 asymptote

1. An exponential function, with asymptote $y=12$, passes through (1, 18) and 4, 32.25). Determine the equation of the appropriate exponential function.

$$18 = ab^{-12} + 12 \quad 32.25 = ab^{-12} + 12$$

$$20.25 = \frac{6}{b} \cdot b^4$$

$$a = \frac{6}{1.5} = 4$$

$$6 = ab$$

$$20.25 = ab^4$$

$$6b^3 = 20.25$$

$$y = 4(1.5)^x + 12$$

$$\hookrightarrow a = \left(\frac{6}{b}\right)$$

$$b = \sqrt[3]{\frac{20.25}{6}} = 1.5$$

2. Use log properties to solve each equation.

a) $\log x + \log 8 = 2$

condense

$$\log(8x) = 2$$

$$10^2 = 8x$$

$$x = \frac{100}{8} = 12.5$$

b) $-6\log_3(x-3) = -24$

divide

$$\log_3(x-3) = 4$$

$$3^4 = x-3$$

$$x = 84$$

c) $\log x + \log 7 = \log 37$

$$\log(7x) = \log(37)$$

So $7x$ must be equal to 37

$$7x = 37$$

$$x = \frac{37}{7}$$

d) $\log(-2a+9) = \log(7-4a)$

↓

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$$\begin{matrix} -2a+9 & = & 7-4a \\ +4a & & +4a \end{matrix}$$

$$2a+9 = 7$$

$$2a = -2$$

$$a = -1$$

- 4) Convert the log expression $\log_2 30$ to one with base 8

$$\log_2(30) = \frac{\log(30)}{\log(2)} = \frac{\log_8(30)}{\log_8(2)}$$

↑
base 10

- 5) Solve each equation. Leave each answer exact in terms of base 10 AND round to 3 decimal places. can solve by converting to log form or by taking log of both sides.

a) $3^b = 17$

✓

$$b = \log_3(17)$$

$$= \frac{\log(17)}{\log(3)} \approx 2.579$$

b) $5 \cdot 18^{6x} = 26$

divide

$$18^{6x} = \frac{26}{5}$$

$$\sqrt{\log(18^{6x})} = \log\left(\frac{26}{5}\right)$$

$$6x \cdot \log(18) = \log\left(\frac{26}{5}\right)$$

$$x = \frac{1}{6} \cdot \frac{\log\left(\frac{26}{5}\right)}{\log(18)} \approx 0.095$$

c) $16^{n-7} + 5 = 24$

-5 -5

$$16^{n-7} = 19$$

✓

$$n-7 = \log_{16}(19)$$

$$n = \log_{16}(19) + 7$$

$$n = \frac{\log(19)}{\log(16)} + 7 \approx 7.562$$

6) Solve the quadratic equation, $x^2 - 4x + 1 = 0$, using:

a) The quadratic formula

$$\begin{aligned}
 a &= 1 \\
 b &= -4 \\
 c &= 1 \\
 x &= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(1)}}{2(1)} \\
 &= \frac{4 \pm \sqrt{12}}{2} \leftarrow \sqrt{12} = \sqrt{4 \cdot 3} \\
 &= \frac{4 \pm 2\sqrt{3}}{2} \\
 &= \boxed{2 \pm \sqrt{3}}
 \end{aligned}$$

b) Completing the Square

$$\begin{aligned}
 x^2 - 4x &= -1 \\
 x^2 - 4x + 4 &= -1 + 4 \\
 (x-2)^2 &= 3 \\
 \sqrt{\quad} \quad \sqrt{\quad} \\
 x-2 &= \pm\sqrt{3} \\
 x &= \boxed{2 \pm \sqrt{3}}
 \end{aligned}$$

same!

7) Simplify $\frac{a^2}{a+5} + \frac{10a+25}{a+5}$

$$\begin{aligned}
 &\frac{a^2 + 10a + 25}{a+5} \\
 &= \frac{(a+5)(a+5)}{a+5} \\
 &= \boxed{a+5}
 \end{aligned}$$

and

$$\begin{aligned}
 &\frac{x^2 - 2xy - y^2}{x-y} \\
 &\frac{x^2 - [2xy - y^2]}{x-y} \\
 &\downarrow \\
 &\frac{x^2 - 2xy + y^2}{x-y} \rightarrow \frac{(x-y)(x-y)}{x-y} \\
 &= \boxed{x-y}
 \end{aligned}$$

Diagram showing the expansion of $(x-y)^2$ in a grid:

	x	$-y$	
x	x^2	$-xy$	$\begin{matrix} \times 22 \\ \times y \\ -xy \\ -xy \end{matrix}$
$-y$	$-xy$	y^2	

8) Find the algebraic inverse

$$p(x) = 3(x^3 + 6) - 5$$

$$y = 3(x^3 + 6) - 5$$

reverse

$$x = \frac{3(y^3 + 6) - 5}{+5}$$

$$3(y^3 + 6) = x + 5$$

divide

$$y^3 + 6 = \frac{x+5}{3}$$

$$y^3 = \frac{x+5}{3} - 6$$

$$\sqrt[3]{\quad} \quad \sqrt[3]{\quad}$$

$$y = \sqrt[3]{\frac{x+5}{3} - 6}$$