Pick Up the Warm Up

Skip #6

HW Questions

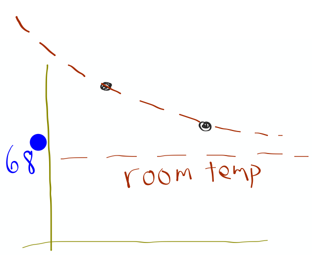
Warm Up

| 6.2.4

$y = ab^x$

1. Remember Kendra from yesterday? You know, the person that let the hot chocolate cool down to room temperature at 68°?

After 1 minute the temperature was down to 109.6° and after 3 minutes it dropped down to 94.624°. Create an exponential function that models the situation.



$(1, 109.6)$ $\quad$ $(3, 94.624)$

$y = ab^x + c$ $\quad$ $y = ab^x + c$

$109.6 = ab^1 + 68$ $\quad$ $94.624 = ab^3 + 68$

$-68 \quad -68$ $\quad$ $-68 \quad -68$

$41.6 = ab^1$ $\quad$ $26.624 = ab^3$

$41.6 = ab^1$ $\quad$ $26.624 = ab^3$

$$\frac{\cancel{a}b^3}{\cancel{a}b^1} = \frac{26.624}{41.6}$$

$$b^2 = \frac{26.624}{41.6}$$

$$b = 0.8$$

↑ must be positive

$41.6 = a(.8)$

$a = 52$

$y = 52(.8)^x + 68$

After 1 minute the temperature was down to 109.6° and after 3 minutes it dropped down to 94.624°. Create an exponential function that models the situation.

2. How long, in minutes, will it take to cool the drink down to almost room temperature, say 70° ?

$70 = 52(.8)^x + 68$

$-68 \qquad -68$

$2 = 52(.8)^x$

$2/52 = (.8)^x$

$$\frac{2}{52} = (.8)^x$$

$$x = \log_{0.8}(2/52)$$

$$x = \frac{\log(2/52)}{\log(.8)}$$

$$\approx 14.6$$

3. Solve the equation $\frac{5}{3}(1.5)^x = 100$

$$5(1.5)^x = 300$$

$$1.5^x = 60$$

convert to log form

take log of both sides

$$x = \log_{1.5}(60)$$

$$x = \frac{\log(60)}{\log(1.5)} \approx \underline{\underline{10.098}}$$

$$\log(1.5^x) = \log(60)$$

$$x \cdot \log(1.5) = \log(60)$$

$$x = \frac{\log(60)}{\log(1.5)} =$$

4. Use log properties (from your notes or your reference sheet) to expand the statements

$$\log\left(\frac{2x}{3}\right)$$

quotient prop.

$\log(\quad) - \log(\quad)$

$\log(2x) - \log(3)$

$\log(2) + \log(x) - \log(3)$

5. Use log properties to solve the equations

$\log(5) + 7\log(x) = 1000$

Condense

$\log(5 \cdot x^7) = 1000$

base 10

$10^{1000} = 5x^7$

$\cancel{x = \sqrt[7]{10^{1000}}}$

$x^7 = \frac{10^{1000}}{5}$

$x = \sqrt[7]{\frac{10^{1000}}{5}}$

$Log(x+1) = \log(4x+5)$

$x+1 = 4x+5$

$1 = 3x+5$

$-4 = 3x$

$x = -\frac{4}{3}$

extraneous because $\log(x+1)$ must be ≥ 0

6. **Triple Dog Dare You Challenge**

Use "Completing The Square" to solve the quadratic equation: $3x^2 - 4x - 2 = 0$
And attempt to keep things in fractional values.

HW Questions?

Worksheet 6.2.3

1. An exponential function, with asymptote y=12, passes
the appropriate exponential function.

$18 = ab^1 + 12$ $\quad$ $32.25 = ab^4 + 12$

$-12 \quad -12$ $\quad$ $-12 \quad -12$

$6 = ab$ $\quad$ $20.25 = ab^4$

$a = \frac{6}{b}$

2. **Use log properties to solve each equation.**

a) $\log x + \log 8 = 2$

condense

$\log(8x) = 2$

$10^2 = 8x$

$x = \frac{100}{8} = 12.5$

b) $-6\log_3(x-3) = -24$

divide

$\log_3(x-$

$3^4 =$

$x =$

c) $\underbrace{\log x + \log 7} = \log 37$

$\log(7x) = \log(37)$

so $7x$ must be equal to 37

$7x = 37$

$x = \frac{37}{7}$

d) $\log(-2a+9) = \log(7$

$-2a + 9 =$

$+4a$

$2a + 9 =$

-9

$2a =$

Convert the log expression $\log_2 30$ to one with base 8

$$\log_2(30) = \frac{\log(30)}{\log(2)} = \boxed{\frac{\log_8(30)}{\log_8(2)}}$$

base 10

5) Solve each equation. Leave each answer exact in terms of base 10 AND round to 3 d places. can solve by converting to log form or by tak of bo

log

a) $3^b = 17$

$b = \log_3(17)$

$= \boxed{\frac{\log(17)}{\log(3)}}$

≈ 2.579

b) $5 \cdot 18^{6x} = 26$

divide

$18^{6x} = \frac{26}{5}$

$\log(18^{6x}) = \log(26/5)$

$6x \cdot \log(18) = \log(26/5)$

$x = \frac{1}{6} \cdot \frac{\log(26/5)}{\log(18)} \approx .095$

c) $16^{n-7} + 5 = 24$

$-5 \quad -5$

$16^{n-7} = 19$

$n-7 = \log_{16}(19)$

$n = \log_{16}(19) + 7$

$\boxed{n = \frac{\log(19)}{\log(16)} + 7}$

a) The quadratic formula

b) Completing

$a = 1$
$b = -4$
$c = 1$

$$X = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(1)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{12}}{2} \leftarrow \sqrt{12} = \sqrt{4} \cdot \sqrt{3}$$

$$= \frac{4 \pm 2\sqrt{3}}{2}$$

$$= 2 \pm \sqrt{3}$$

same!

x

x

(

7) Simplify $\frac{a^2}{a+5} + \frac{10a+25}{a+5}$

$$\frac{a^2 + 10a + 25}{a+5}$$

$$= \frac{(a+5)\cancel{(a+5)}}{\cancel{a+5}}$$

$$= a+5$$

and $\frac{x^2}{x-y} - \frac{2xy-y^2}{x-y}$

	x	$-y$
x	x^2	$-xy$
$-y$	$-xy$	y^2

$x^2 y^2$ $-xy$ $-xy$

$-2xy$

$$\frac{x^2 - [2xy - y^2]}{x-y}$$

$\downarrow$

$$\frac{x^2 - 2xy + y^2}{x-y} \rightarrow \frac{(x-y)\cancel{(x-y)}}{\cancel{x-y}}$$

$$= \boxed{x-y}$$

8) Find the algebraic inverse

$p(x) = 3(x^3 + 6) - 5$

$y = 3(x^3+6) - 5$

reverse

$x = 3(y^3+6) - 5$

$+5$ $\quad$ $+5$

$3(y^3+6) = x+5$

divide

$y^3 + 6 = \frac{x+5}{3}$

$y^3 = \frac{x+5}{3} - 6$

$\sqrt[3]{}$ $\quad$ $\sqrt[3]{}$

$$y = \sqrt[3]{\frac{x+5}{3} - 6}$$

HW

questions

Schedule this

Week

M Naturual Logs

T
W
Th

Ch. Review for next Monday's Test

Grade 11 students report directly to A-1, A-2

[return here with Mr. Katsamura]

FRI Start Ch. 7

After school → M Naturual Logs

T
W
→ Th

Ch. Review for next Monday's Tes

Grade 11 students repo

to A-1, A-2

[return here with Mr.

I'll be here for help 7:30 → FRI Start Ch. 7

Natural Logarithms

last new topic of Ch. 6

Look at
Your reference sheet

Compound Interest Formula: ***Future value of money is*** $\text{FV} = \text{PV}(1 + \frac{r}{n})^{nt}$

Where r: *annual interest rate, as a decimal,* t: *# of years,*
n: *#times interest is compounded per year*

1) Use the compound interest formula:
Suppose you invest your $5,000 savings to save for a car. You find a bank that pays 5.8% *annual interest*. Find out how much you would be in your account 7 years from now if you bank pays you interest compounded quarterly. $(n = 4)$

$FV = PV(1 + \frac{r}{n})^{nt}$ or $F = P(1 + \frac{r}{n})^{nt}$

$$5000\left(1 + \frac{.058}{4}\right)^{4 \cdot 7}$$

$$\approx \$7482.^{\underline{17}}$$

2) Repeat the calculation, but assume monthly compounding ($n = 12$)

$$5000\left(1+\frac{.058}{12}\right)^{12\cdot 7}$$

```
5000(1+.058/12)^
(12*7)
          7496.677301
```

3) Repeat once more, but this time assume compounding daily ($n = 365$)

```
5000(1+.058/365)
^(365*7)
          7503.770731
```

Compound Interest Formula

Invest \$5,000

for 7 years

at 5.8%

compound interest quarterly

$K=4 \longrightarrow K=12 \longrightarrow K=365 \longrightarrow K=\infty$

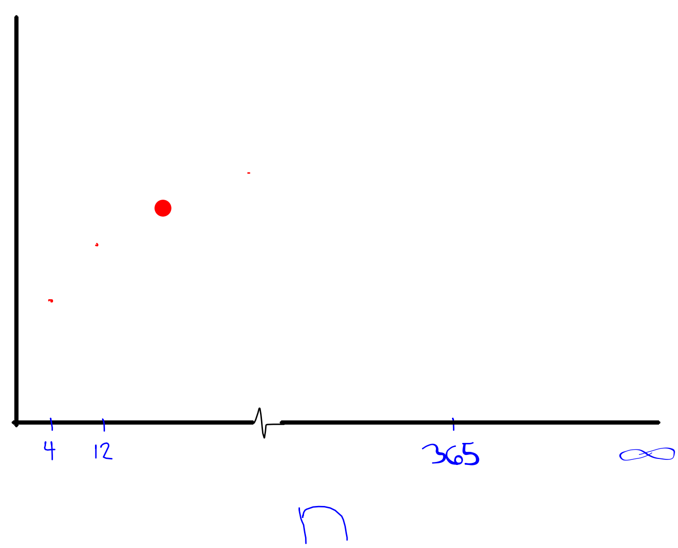
the higher the "n"

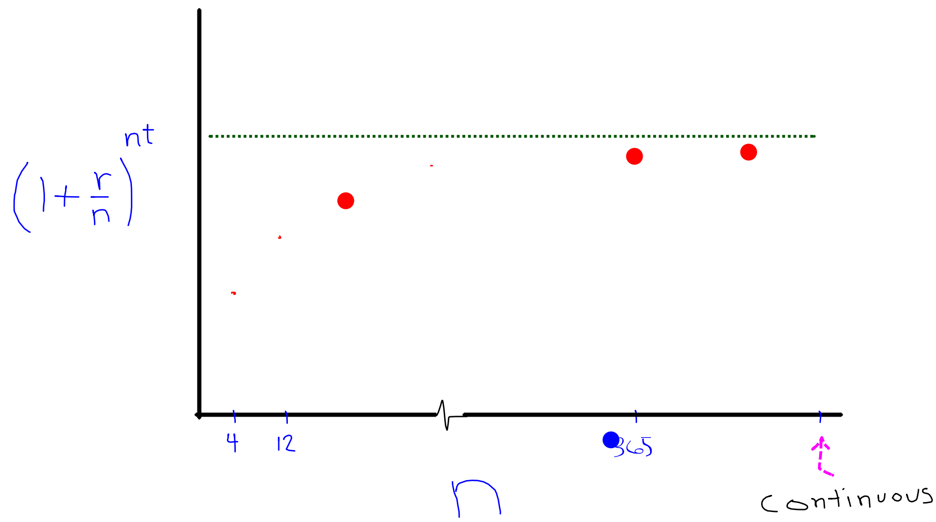
the larger the multiplier

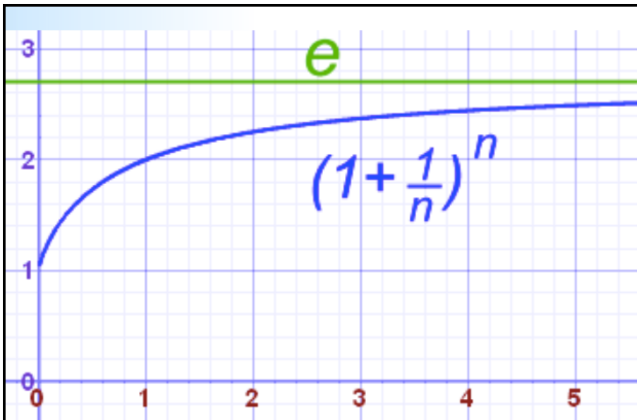
but.....

the increase starts to slow down.

$\left(1+\frac{r}{n}\right)^{nt}$





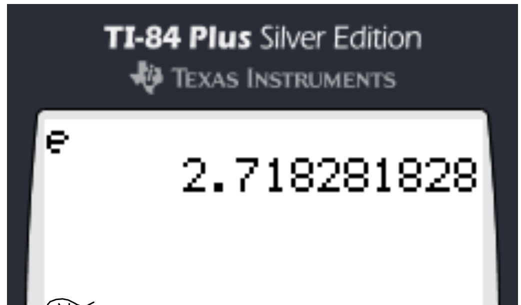


n	$(1 + 1/n)^n$
1	2.00000
2	2.25000
5	2.48832
10	2.59374
100	2.70481
1,000	2.71692
10,000	2.71815
100,000	2.71827

$$\left(1+\frac{1}{n}\right)^{n}$$

As $n \to \infty$

$$\left(1+\frac{1}{n}\right)^{n} \to e$$



π

3.14159....

Add to Your Reference Sheet

$$F = P\left(1+\frac{r}{\cancel{n}}\right)^{\cancel{n}t} \longrightarrow F = Pe^{rt}$$

for continuous compounding

F, P, r, t same

4) Now find the final balance if the bank uses *continuous* compounding.

$F = P \cdot e^{rt}$

$= 5000\, e^{.058 \cdot 7}$

e

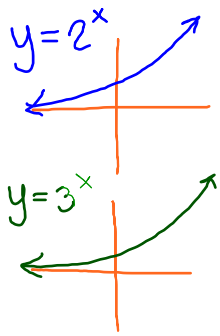
e^{x}

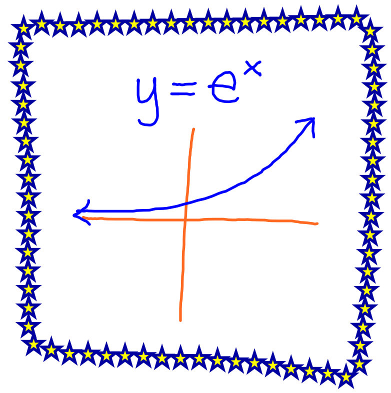
Aim

Natural Logarithms

• Solve exponential equations that have the natural base, e.

Notes when you see 🙂





The exponential function

etc.

e has many uses

finance, science, etc

Top 4 REASONS WHY $f(x)=e^x$ is KNOWN AS the Exponential Function

#4

$f(x) = e^x$ has special calculus properties that simplify many calculations

Using logarithms to solve Newton's Law of Cooling

$$\frac{T(t)-T_a}{T_0-T_a} = e^{-kt}$$

Radioactive half-life

$$P(t) = P_0 e^{-kt}$$

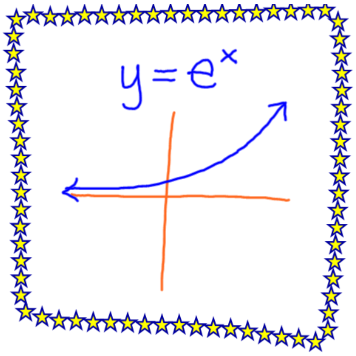
#3 e is considered to be the natural base.

#2 $e > 1$ so $f(x) = e^x$ is a growth function and the number one reason why $f(x) = e^x$

is THE natural exponential function ....

#1 Leonhard Euler introduced the notation and he could call it what he wanted to call it!

e



$y = e^x$

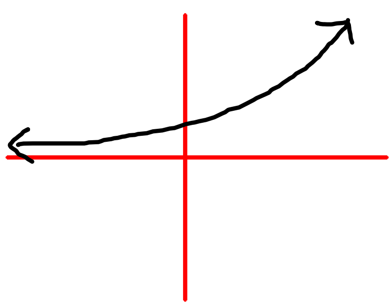
has an inverse which is called the natural log function.

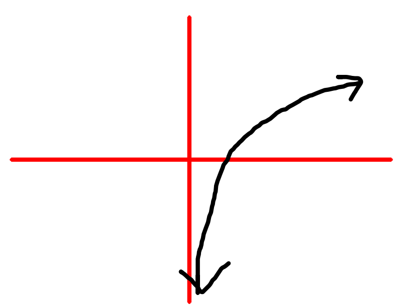
inverse

$f(x) = e^x$

$f^{-1}(x) = \cancel{\log_e} x$

$f^{-1}(x) = \ln(x)$





e is so prevalent out in the real world its logarithm gets its very own notation

$\cancel{\log_e x} \dashrightarrow \ln x \qquad \ln x$

$\log_e(6)$

$\ln(6) \qquad \ln(6)$

B.B.

Was mathematics invented or discovered?

π $\qquad$ i $\qquad$ e

200 B.C ± $\qquad$ 1500's $\qquad$ 1700's

$e^{\pi i} =$

Was mathematics invented or discovered?

π — 200 B.C $\pm$

i — 1500's

e — 1700's

$$e^{\pi i} =$$

Solve Natural Log Equations

Solve each equation. Check your answers.

$\ln(x) = 0.1$ think $\log_e(x) = 0.8$

$\downarrow$

convert to exponential form

$$x = e^{0.1}$$

≈ 1.11

$$\ln\left(\frac{x+2}{3}\right) = 12$$

↑ exponent

base is e

$$e^{12} = \frac{x+2}{3}$$

$$3e^{12} = x+2$$
$$-2 \qquad -2$$

$$x = 3e^{12} - 2$$

$$\underbrace{\ln 5 - \ln(2x)} = 1$$

$$\ln\left(\frac{5}{2x}\right) = 1$$

↖ exponent

base e

↓ convert to exp. form

$$e^{1} = \frac{5}{2x}$$

$$2x \cdot e = 5$$

$$x = \frac{5}{2e}$$

Shortcut

$\log(10)$ $\ln(e)$

and expon. equations with base *e*

Solve for x

$\cancel{e^x = 18}$

$e^{x+1} = 30$

method 1

convert

exponent is x+1
base is e

$x+1 = \ln(30)$

$x = \ln(30) - 1$

An initial investment of $200 is now valued at $245.25. The interest rate is 6% **compounded continuously**. How long has the money been invested?

$$A = Pe^{rt}$$

$$245.25 = 200e^{.06t}$$

Was mathematics invented or discovered?

π 200 B.C ±

i 1500's

e 1700's

$$e^{\pi i} =$$

Assignment

Worksheet 6242