Part III

Key

Key to Inference Multiple Choice

1. A We are trying to prove that $p_F > p_M$, and the pooled $\hat{p} = \frac{63+130}{211+651} = 0.224$.

2. B We are comparing two means, that of halogen bulbs and that of incandescent bulbs.

3. D The confidence interval is (0.2, 2.0). Zero is not in this interval, so we reject that claim. A 95% confidence level corresponds to an α-level of 0.05 for a two-sided test.

4. A Use a *z* test if σ is known; use a *t* test if we must approximate σ using *s* from the sample.

5. A We are comparing two groups on their answer to a categorical question.

6. C Standard error $= \sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}} = 0.0511$ (Answer choice B incorrectly uses the pooled $\hat{p}$, which would be correct in a significance test, but not a confidence interval.)

7. D When a test is significant, H_o was rejected. The claim (or hypothesized mean) was NOT in the interval.

8. C The *t* distribution approaches the normal distribution as *n* increases, but the χ^2 distribution is always skewed.

9. C We are looking at one group's answers to two questions with categorical answers.

10. C The expected value for each group is 20. The value of the χ^2 statistics is 3.1, and the P-value of the test is 0.212. This is higher than any α level, so we do not reject H_o, which says that the groups are the same. We can't say that the groups differ significantly.

11. E The $df = 15$, so the P value is 0.0271, which is less than 0.05, so we reject H_o.

12. A For a two-sided test, double the P value to 0.054, which is greater than 0.05, so we do not reject H_o.

Rubric for Inference for Free Response

1. **Solution**

Part 1: Identify the correct test by name or formula, subtract to get the improvement, check conditions.

Since the two lists are "before" and "after" results for the same 12 people, this is a matched pair t test. OR $t = \dfrac{\overline{x_{diff}} - 0}{s_{diff}/\sqrt{n}}$

Subtract "after" – "before" to get each participant's improvement:

Person	1	2	3	4	5	6	7	8	9	10	11	12
Before	15	12	21	22	17	19	10	25	12	17	8	19
After	17	15	22	22	21	24	11	28	14	16	12	21
Improvement	2	3	1	0	4	5	1	3	2	-1	4	2

Check conditions for one-sample t test:
The data are independent because the participants were randomly chosen and n is less than 10% of the population of adults over 55.
AND
The data is nearly normal because an examination of the dotplot of the differences shows a unimodal graph with no outliers:

```
-1 | X
 0 | X
 1 | XX
 2 | XXX
 3 | XX
 4 | XX
 5 | X
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Part 2: Write hypotheses, identifying the parameter of interest.

$H_o : \mu_{improvement} = 0$

$H_a : \mu_{improvement} > 0$

where $\mu_{improvement}$ = the true mean improvement in number of push-ups done

Part 3: Perform the test, using correct mechanics, including value of the t statistic, the degrees of freedom, and the P value.

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{2.167 - 0}{1.749/\sqrt{12}} = 4.29$$

With $df = 11$, $P(\bar{x}_{improvement} > 2.167) = P(t > 4.29) = 0.00064$

Part 4: Using the calculations, write a conclusion in the context of the problem.

Since the P value of 0.00064 is less than any reasonable α, we have evidence to reject H_o. We can conclude that the improvement is significantly above zero; the participants did improve the number of push-ups they could do.

Scoring

Each part is *essentially correct* (E), *partially correct* (P), or *incorrect* (I).

Part 1 is *essentially correct* if the student correctly identifies the test, subtracts to find the improvement, checks for independence, and graphs the improvements to show that they are unimodal and symmetric.
Part 1 is *partially correct* if the student correctly does two or three of those.
Part 1 is incorrect if the student does only zero or one of those.

Part 2 is *essentially correct* if the student correctly gives both hypotheses and identifies the parameter.
Part 2 is *partially correct* if the student does only one of those.

Part 3 is *essentially correct* if the student correctly gives the value of the t statistic, the degrees of freedom, and the P-value.
Part 3 is *partially correct* if the student gives only one or two of these.

Part 4 is *essentially correct* if the student correctly links the P-value to the alpha-level in order to reject H_O AND gives the conclusion (that the program does help) in context.
Part 4 is *partially correct* if the student gives only one of these conclusions.

To assign a score to this question let an E = 1point, a P = 0.5 points, and an I = 0 points. Sum the total points for the student's score. If a student has a half point, look at the question holistically to determine if the score should be rounded up or truncated.

4 Complete Response

3 Substantial Response

2 Developing Response

1 Minimal Response

2. **Solution**

Part 1: Identify the correct test by name or formula, check conditions necessary to do test.

Since we are comparing two different random samples on multiple categories, we will do a χ^2 test for homogeneity OR

Check the conditions for a χ^2 test:
The data are counts.
The two samples are independent.
Each expected value is at least 5.

Part 2: Write hypotheses.
H_O: The White Rock Lake dog park and the downtown dog park have the same distribution of dogs by size (are homogeneous).
H_A: The White Rock Lake dog park and the downtown dog park do not have the same distribution of dogs by size.

Part 3: Perform the test, using correct mechanics, including value of the χ^2 statistic, the degrees of freedom, the expected values, and the P value.

Expected values are in parentheses:

	Toy (< 10 lbs)	Small (11 – 20 lbs)	Medium (21-50 lbs)	Large (51-100 lbs)	Giant (over 100 lbs)
Downtown	39 (34)	72 (68)	101 (85)	89 (107)	12 (19)
White Rock Lake	77 (82)	158 (162)	188 (204)	275 (257)	51 (44)

$$df = (rows - 1)(columns - 1) = (2-1)(5-1) = 4$$

$$\chi^2 = \sum \frac{(O-E)^2}{E} = \frac{(39-34)^2}{34} + \frac{(72-68)^2}{68} + \frac{(101-85)^2}{85} + \ldots + \frac{(51-44)^2}{44} = 13.21$$

$$P(\chi^2 > 13.21) = 0.0103$$

Part 4: Using the calculations, write a conclusion in the context of the problem.
Since the P-value of 0.0103 is less than the α-level of 0.05, we can reject H_O. Based on these samples, the White Rock Lake dog park and the downtown dog park have a significantly different distribution of dogs by size.

Scoring

Parts 1, 3, and 4 can be *essentially correct* (E), *partially correct* (P), or *incorrect* (I). Part 2 can be *essentially correct* (E) or *incorrect* (I).

Part 1 is *essentially correct* if the student correctly identifies the test, mentions independence, and states that the expected values are over 5.
Part 1 is *partially correct* if the student correctly does one or two of those.

Part 2 is *essentially correct* if the student correctly gives both hypotheses in words.

Part 3 is *essentially correct* if the student correctly gives the value of the χ^2 statistic, the degrees of freedom, the expected values, and the P-value.
Part 3 is *partially correct* if the student gives only two or three of these.
Part 3 is *incorrect* if the student gives only zero or one of these.

Part 4 is *essentially correct* if the student correctly links the P-value to the alpha-level in order to reject H_O AND gives the conclusion (that the sizes of the dogs at each park differ) in context.
Part 4 is *partially correct* if the student gives only one of these conclusions.

To assign a score to this question let an E = 1point, a P = 0.5 points, and an I = 0 points. Sum the total points for the student's score. If a student has a half point, look at the question holistically to determine if the score should be rounded up or truncated.

4 Complete Response

3 Substantial Response

2 Developing Response

1 Minimal Response

3. **Solution**

Part 1: Identify the correct interval by name or formula, check conditions.

This is a two-sample z interval for the difference in two proportions OR

$$(\hat{p}_1 - \hat{p}_2) \pm z^* \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}.$$

Check conditions for two-proportion z interval:
The data are independent because the participants were randomly chosen and each n is less than 10% of the population of college-bound high school male or female seniors in the Dallas/Fort Worth area.
AND
The sample sizes are large enough because

$n\hat{p} = (450)(0.269) = 121 > 10$ *or* 5

$n(1-\hat{p}) = (450)(0.731) = 329 > 10$ *or* 5

$n\hat{p} = (306)(0.307) = 94 > 10$ *or* 5

$n(1-\hat{p}) = (306)(0.693) = 212 > 10$ *or* 5

Part 2: Calculate the interval. (This may be done in either order, male-female or female-male.)

$$\hat{p}_M = \frac{94}{306} = 0.307 \qquad \hat{p}_F = \frac{121}{450} = 0.269$$

$$(\hat{p}_M - \hat{p}_F) \pm z^* \sqrt{\frac{\hat{p}_M(1-\hat{p}_M)}{n_M} + \frac{\hat{p}_F(1-\hat{p}_F)}{n_F}}$$

$$= (0.307 - 0.269) \pm 1.96 \sqrt{\frac{0.307(0.693)}{306} + \frac{0.269(0.731)}{450}}$$

$$= 0.038 \pm 0.066 = (-0.028, 0.104) = (-2.8\%, 10.4\%)$$

Part 3: Interpret the interval.

Based on these samples, I am 95% confident that the interval (-2.8%, 10.4%) captures the true difference between the population proportion of male DFW students who attend college outside Texas and the population proportion of female DFW students who attend college outside Texas.

OR

Based on these samples, I am 95% confident that the true difference between the population proportion of male DFW students who attend college outside Texas and the population proportion of female DFW students who attend college outside Texas is between -2.8% and 10.4%.

Part 4:

Since 0 is in the 95% confidence interval, zero is a plausible value for the difference in proportions, $p_M - p_F$. The evidence shows no significant difference between the proportion of male students attending college outside of Texas and the proportion of female students attending college outside of Texas.

Scoring

Each of the four parts can be essentially correct (E) or incorrect (I).

Part 1 is essentially correct if the interval is identified and the student comments on both independence and large sample size. The minimum amount necessary is an indication that the number of successes and failures for both samples is over 10 (or 5) AND a mention of independence (or independence with a check mark). The student does not have to repeat the fact that the samples are random.

Part 2 can be essentially correct even if there is an identifiable minor arithmetic error.

Part 4 is essentially correct if the student states that zero is not in the interval and links this to either a 95% confidence level or a 5% significance level. Part 4 is incorrect if the student says no without justification or if the student says no because zero is not in the interval.

4 **Complete Response** (4E)
All four parts essentially correct

3 **Substantial Response** (3E)
Three parts essentially correct

2 **Developing Response** (2E)
Two parts essentially correct

1 **Minimal Response** (1E)
One part essentially correct

AP Statistics Exam Connections

The list below identifies free response questions that have been previously asked on the topic of Inference on the AP Statistics Exam. These questions are available from the CollegeBoard and can be downloaded free of charge from AP Central.

http://apcentral.collegeboard.com.

Free Response Questions	
2001	Question 5
2002	Question 6
2004	Question 6