Pick Up the Warm Up
and work through as a group.

A school counselor suspects that, on average, students at their school are sleeping less than 8 hours per night. They survey a random sample of students about how many hours they slept the previous night to see if their average sleep amount is significantly less than 8 hours.

Which of these inference procedures is most appropriate?

Choose 1 answer:

(A) A two-sample z-test for the difference of proportions

(B) A paired t-test for the mean difference

(C) A z-test for a proportion

(D) A t-test for a mean

(E) A two-sample t-test for the difference of means

I February 12, 2019

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- Proportion or Mean
- 2-Sample or 1 sample
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PROBABILITY 1

Review Book *A* advertises an average SAT gain of 40 points with a standard deviation of 12 points, and Review Book *B* claims an average SAT gain of 35 points with a standard deviation of 15 points. Assuming both assertions are correct and assuming normal distributions, which review book is more likely to result in an SAT gain of over 60 points?

(A) Review Book *A* because of its greater mean.

(B) Review Book *B* because of its greater standard deviation.

(C) For both plans, the probability of an SAT gain over 60 points is .04779.

(D) For both plans, the probability of an SAT gain over 60 points is .95221.

(E) The problem cannot be solved from the information given.

P1

Answer: (C) In both cases, 60 points is 5/3 standard deviations from the mean with a right tail probability of .04779.

PROBABILITY 2

Suppose that the probabilities that an answer can be found on Google is .95, on Answers.com is .92, and on both websites is .874. Are the possibilities of finding the answer on the two websites independent?

(A) Yes, because (.95)(.92) = .874.

(B) No, because (.95)(.92) = .874.

(C) Yes, because .95 > .92 > .874.

(D) No, because .5(.95 + .92) ≠ .874.

(E) There is insufficient information to answer this question.

P2

Answer: (A) If $P(E \cap F) = P(E)P(F)$, then E and F are independent.

PROBABILITY 11

The owner of a coffee shop, an amateur statistician, advertises that the price of coffee on any given day will be randomly picked using a normal distribution with mean \$1.35 and standard deviation \$0.10. If a customer buys a cup of coffee on 10 days, what is the probability that he will pay a total exceeding \$14.00?

(A) .0316 (B) .0568 (C) .3085 (D) .3160 (E) .9432

P11

Answer: (B) The sampling distribution of $\bar{x}$ is approximately normal with a mean of 1.35 and a standard deviation of $\frac{.10}{\sqrt{10}} = 0.0316$.

The probability that the average amount exceeds \$1.40 is `normalcdf(1.40,100,1.35,0.0316)` = .0568.

PROBABILITY 12

If $P(A) = .25$ and $P(B) = .34$, what is $P(A \cup B)$ if A and B are independent?

(A) .085

(B) .505

(C) .590

(D) .675

(E) There is insufficient information to answer this question.

P12

Answer: (B) If A and B are independent $P(A \cap B) = P(A)P(B)$ and thus $P(A \cup B) = .25 + .34 - (.25)(.34) = .505$.

Today

Perform a full Chi-Square
Test for Goodness of Fit

... and do a follow up
analysis

Have the handout
from yesterday
handy

Malcolm Gladwell
"Outliers"

Tries to explain strange things
~ Pennsylvania town (unusually healthy)
~ Eastern European National Soccer Team
with unusual birthdays
~ Similar in Canada

Pick Up Handout

- let's read about an observation in Canada

In his book Outliers, Malcolm Gladwell suggests that a hockey player's birth month has a big influence on his chance to make it to the highest levels of the game. Specifically, because January 1 is the cut-off date for youth leagues in Canada [where many National Hockey League (NHL) players come from], players born in January will be competing against players up to 12 months younger. The older players tend to be bigger, stronger, and more coordinated and hence get more playing time, more coaching, and have a better chance of being successful. To see if birth date is related to success (judged by whether a player makes it into the NHL), a random sample of 80 NHL players from a recent season was selected and their birthdays were recorded. The one-way table summarizes the data on birthdays for these 80 players.

Birthday	Jan–Mar	Apr–Jun	Jul–Sep	Oct–Dec
Number of players	32	20	16	12

Do these data provide convincing evidence that the birthdays of NHL players are not uniformly distributed across the four quarters of the year? If there is statistically significant evidence, perform a follow up analysis.

$\boxed{\text{STATE}}$

Use $\alpha = 0.05$

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State Hypotheses: Significance Leve

H_0: The birthdays of all NHL players are uniformly distributed across the four quarters of the year.

H_a: The birthdays of all NHL players are not uniformly distributed across the four quarters of the year.

$n = 80$

Observed frequencies

Birthday	Jan–Mar	Apr–Jun	Jul–Sep	Oct–Dec
Number of players	32	20	16	12
	20	20	20	20.

Expected frequencies to be Uniform

$80(\frac{1}{4}) = 20$

Plan Name of Procedure: Chi-square test for Goodness of fit

k conditions:

Random - data came from a random sample of NHL players. ✓

10% - Assuming that $80 < \frac{1}{10}$(of all NHL Players) ✓

Large Counts - All expected Counts $= 80(\frac{1}{4}) = 20 \geq 5$ ✓

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So we can generalize to <u>all</u> NHL Players

Note

Large Counts condition

- ensures that the probability distribution we use (chi-Square distrib. in this case) to Calculate P-Value is a good model.

Do

Specific Formula

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

Picture:

Test Statistic $\chi^2 = 11.2$

Work

$$\chi^2 = \frac{(32-20)^2}{20} + \frac{(20-20)^2}{20} + \ldots$$

P-Value

$$= 7.2 + .8 + .8 + 3.2 =$$

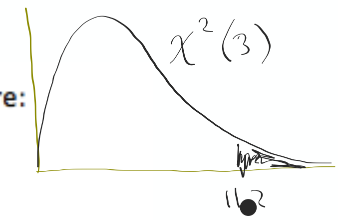
Do

$df = \text{categories} - 1 = 4 - 1 = 3$

Specific Formula

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

Picture:



Work

$$\chi^2 = \frac{(32-20)^2}{20} + \frac{(20-20)^2}{20} + \ldots\ldots$$

$$= 7.2 + .8 + .8 + 3.2 =$$

Test Statistic $\chi^2 = 11.2$

P-Value

Table C → P-Value is between 0.01 and .02

or

$\chi^2\text{cdf}[\underset{\text{Lower}}{11.2}, \underset{\text{upper}}{10000}, \underset{\text{df}}{3}]$

$= .011$

Conclude

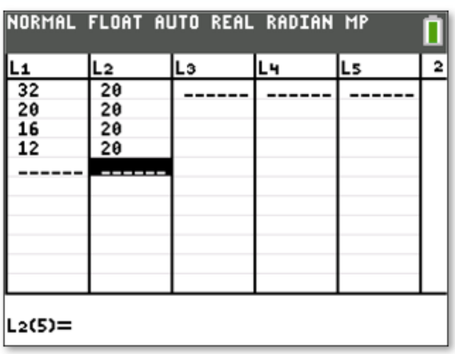
Conclude

Because the P-Value of 0.011 < α = 0.05, we reject H_0.

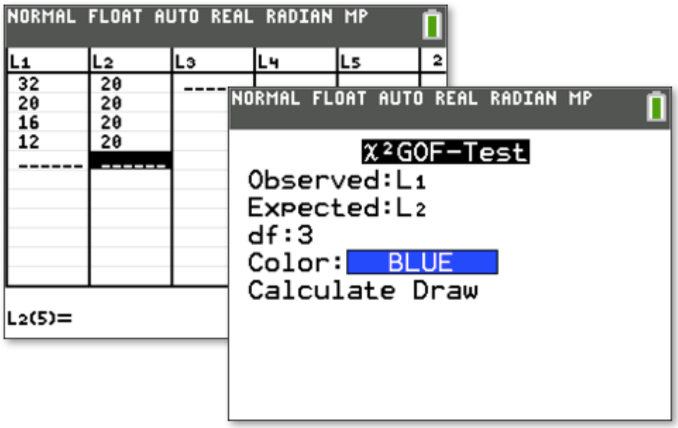
∴ We have convincing evidence that the birthdays of NHL players are not uniformly distributed across the four quarters of the year.

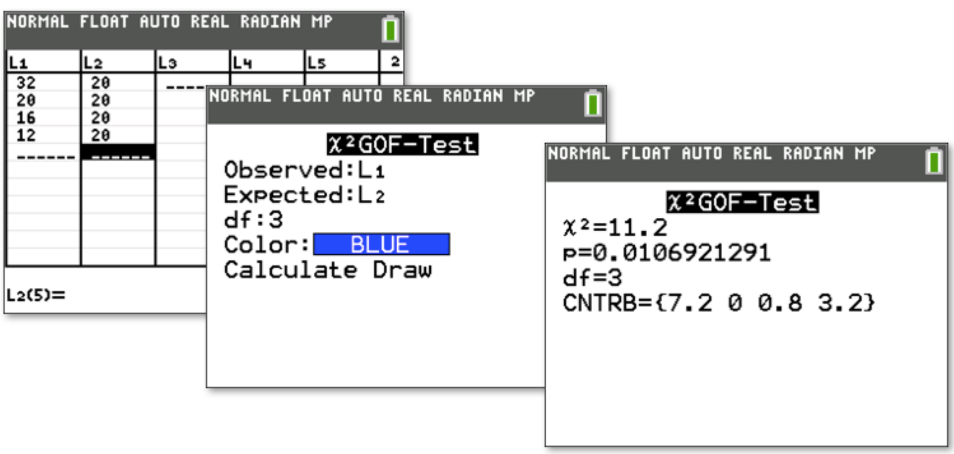
Never "accept H_0"

In a trial
A verdict of "NOT GUILTY"
doesn't mean the defendant
is necessarily innocent.



L_1	L_2	L_3	L_4	L_5
32	20	------	------	------
20	20			
16	20			
12	20			
------	------			


NORMAL FLOAT AUTO REAL RADIAN MP
L1 L2 L3 L4 L5 2
32 20
20 20
16 20
12 20
L2(5)=
NORMAL FLOAT AUTO REAL RADIAN MP
χ²GOF-Test
Observed:L1
Expected:L2
df:3
Color: BLUE
Calculate Draw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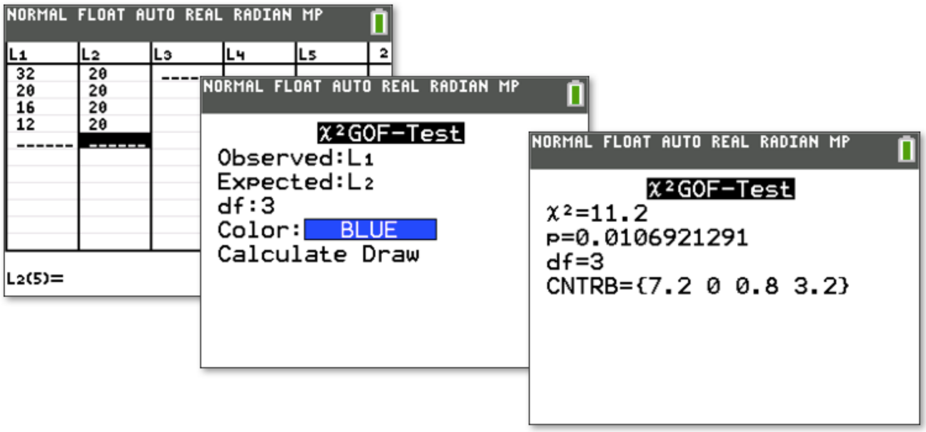
Calculate Draw
NORMAL FLOAT AUTO REAL RADIAN MP
χ²GOF-Test
χ²=11.2
p=0.0106921291
df=3
CNTRB={7.2 0 0.8 3.2}



Note: When you run the chi-square test for goodness of fit on the TI-84 calculator, a list of these individual components will be produced and stored in a list called CNTRB (for contribution).

AP® Exam Tip

You can use your calculator to carry out the mechanics of a significance test on the AP® Statistics exam.

But there's a risk involved. If you just give the calculator answer with no work, and one or more of your values is incorrect, you will likely get no credit for the "Do" step.

We recommend writing out the first few terms of the chi-square calculation followed by ". . .". This approach might help you earn partial credit if you enter a number incorrectly.

Be sure to name the procedure (chi-square test for goodness of fit) and to report the test statistic ($\chi^2 = 11.2$), degrees of freedom (df = 3), and *P*-value (0.011).

Follow Up Analysis

If the sample data lead to a statistically significant result, we can conclude that our variable has a distribution different from the one stated.

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Birthday	Observed	Expected	O − E	$(O - E)^2/E$
Jan–Mar	32	20	12	7.2
Apr–Jun	20	20	0	0.0
Jul–Sep	16	20	−4	0.8
Oct–Dec	12	20	−8	3.2

The two biggest contributions to the chi-square statistic came from Jan–Mar and Oct–Dec.

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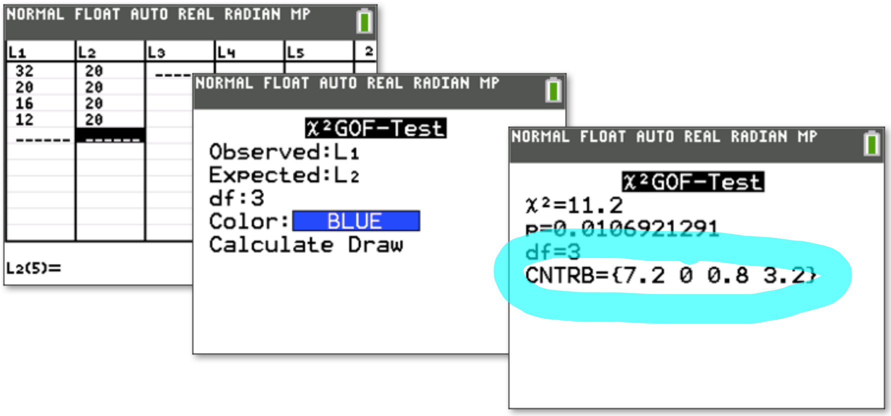
In January through March, 12 ***more*** players were born than expected.



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Lists CNTRB → L_3

Follow up Analysis

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B.B.

Car Colors
in Arizona

Car Colors in Arizona - Does the warm, sunny weather in Arizona affect a driver's choice of car color? Cass thinks that Arizona drivers might opt for a lighter color with the hope that it will reflect some of the heat from the sun. To see if the distribution of car colors in Oro Valley, near Tucson, is different from the distribution of car colors across North America, she selected a random sample of 300 cars in Oro Valley. The table shows the distribution of car color for Cass's sample in Oro Valley and the distribution of car color in North America, according to www.ppg.com.

Color	White	Black	Gray	Silver	Red	Blue	Green	Other	Total
Oro Valley sample	84	38	31	46	27	29	6	39	300
North America	23%	18%	16%	15%	10%	9%	2%	7%	100%

1. Do these data provide convincing evidence that the distribution of car color in Oro Valley differs from the North American distribution?

H_0: The distrib of car colors in Oro Valley is the same as the distrib of car colors across North America.

H_a: the distrib. of car colors in Oro Valley is not the same as the distribution of car colors across North America.

Use $\alpha = 0.05$

Observed

Color	White	Black	Gray	Silver	Red	Blue	Green	Other	Tot
Oro Valley sample	84	38	31	46	27	29	6	39	300
North America	23%	18%	16%	15%	10%	9%	2%	7%	100
	69	54	48	45	30	27	6	21	

Expected

$300(.23) = 69$

$300(.18) = 54$

etc

sample in Oro Valley and the distribution of car color in North America, according to www.ppg.com.

Color	White	Black	Gray	Silver	Red	Blue	Green	Other	Total
Observed → Oro Valley sample	84	38	31	46	27	29	6	39	300
North America	23%	18%	16%	15%	10%	9%	2%	7%	100%
Expected →	69	54	48	45	30	27	6	21	

1. Do these data provide convincing evidence that the distribution of car color in Oro Valley differs from

STATE: Chi - Square test for goodness of fit

Random - Random sample of 300 cars

10% - $n = 300 < \frac{1}{10}$(all cars in Oro Valley)

Large Counts - Expected Counts (69, 54, 48, 45, 30, 27, 6, 21) ≥ 5

PLAN:

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Large Counts - Expected counts (69, 54, 48, 45, 30, 27, 6, 21) ≥ 5

DO

$$\chi^2 = \frac{(84-69)^2}{69} + \frac{(38-54)^2}{54} + \ldots = 29.921 \qquad df = 8-1 = 7$$

P-value $\chi^2 cdf\,[29.921, 10000, 7] \approx 0$

If using: χ^2 GOF-Test

$\chi^2 =$ ~~17.54~~ 29.92

$P =$ ~~0.014~~ 0

$df = 7$

DO:

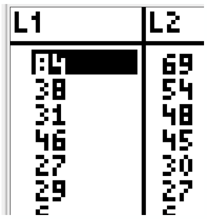
$$\chi^2 = \frac{(84-69)^2}{69} + \frac{(38-54)^2}{54} + \ldots = 29.921 \qquad df = 8-1 = 7$$

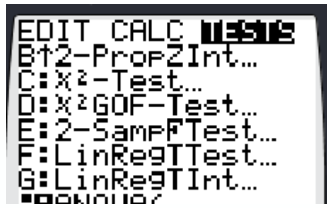
P-value $\chi^2 cdf\,[29.921, 10000, 7] \approx 0$

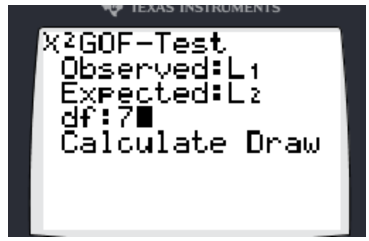
CONCLUDE: Because the P-value of approximately $0 < \alpha = 0.05$, we reject H_0

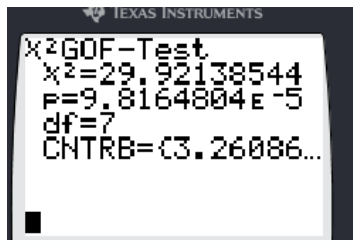
We have convincing evidence that the distrib. of car colors in Oro Valley is not the same as it is across North America.

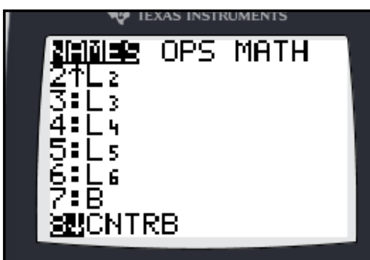
2. If there is convincing evidence of a difference in the distribution of car color, perform a follow-up analysis.

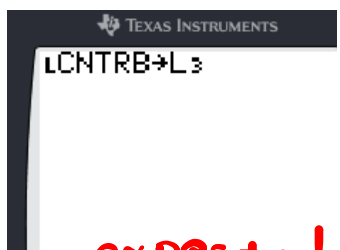






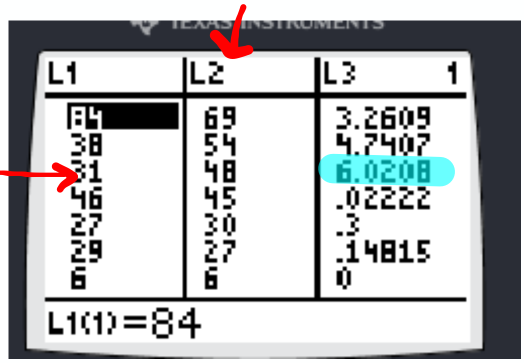






expected

gray



other

2. If there is convincing evidence of a difference in the distribution of car color, perform a follow-up analysis.

The two biggest contributions to the statistic came from gray and other colored cars. There were fewer grays than expected and more "other-colored" cars than expected.

11.1 .... 9, 13, 19-21

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