Today:

Testing Claims about Population Proportions

•

Pick up the Warm Up

Warm Up (9.3 Day 1)

Section 9.3 is about testing claims about a population mean, μ. Previously you learned how to construct a Conficence Interval (in Ch.8) for a population mean. Before you did, you had to check necessary conditions. It turns out that the conditions are exactly the same needed for testing claims about population means. Here are those conditions. Read through them carefully.

Conditions for Performing a Significance Test About a mean

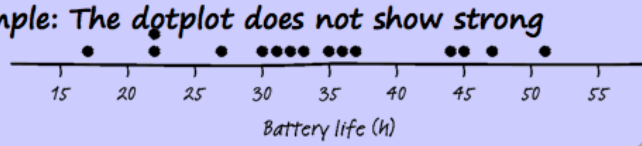
- **Random:** The data come from a random sample from the population of interest. *This condition is so we can generalize to the population.*
- **10%:** When sampling without replacement, $n < 0.10N$. *This conditioin is so sampling without replacement is ok.*
- **Normal/Large Sample:** The population has a Normal distribution or the sample size is large ($n \geq 30$). If the population distribution has unknown shape and $n < 30$, use a graph of the sample data to assess the Normality of the population. Do not use t procedures if the graph shows strong skewness or outliers. *Because we don't know the population mean most of the time we use a t-distribution instead of a z-distribution.*

Problem: Here are the lifetimes (in hours) of the 15 deluxe AAA batteries from the company's simple random sample:

17	32	22	45	30	36	51	27
37	47	35	33	44	22	31	

Check if the conditions for performing the significance test are met.

- Random: SRS of 15 deluxe AAA batteries. ✓
- 10%: Assume that 15 is less than 10% of all the company's deluxe AAA batteries. ✓
- Normal/Large Sample: The dotplot does not show strong skewness or outliers. ✓



Multiple Choice Answers

Answer: (C) The coefficient of determination r^2 gives the proportion of the y-variance that is predictable from a knowledge of x. In this case, $r^2 = (.27)^2 = .0729$ or 7.29 percent.

Answer: (A) Fifty percent of the data are on either side of the median. The interquartile range gives the distance between Q_1 and Q_3, but doesn't say where the median falls between Q_1 and Q_3. For example, in this example, it is possible that $Q_1 = 265$ and $Q_3 = 285$. Depending upon the shape of the distribution, the mean and standard deviation could be anything.

Significance Test for a Population Mean, μ.

Important ideas:

CONDITIONS

Random

10% $n < \frac{1}{10}(\text{Pop})$

Normal/Large Counts

- Pop. Normal
- $n \geq 30$ CLT
- No strong skew or outliers

$$\text{standardized test statistic} = \frac{\text{statistic} - \text{parameter}}{\text{standard deviation of statistic}}$$

In an ideal world, for a test of H_0: $\mu = \mu_0$, our standardized test statistic would be $z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$

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Because the population standard deviation σ is almost always unknown, we use the sample standard deviation s_x in its place. The resulting standardized test statistic has the *standard error* of $\bar{x}$ in the denominator and is denoted by *t*.

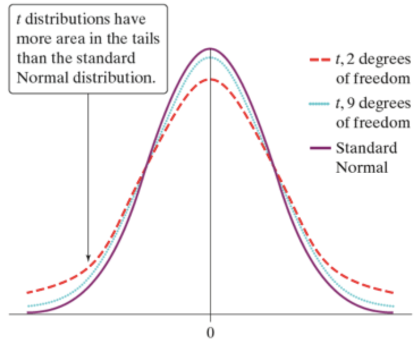
$$t = \frac{\bar{x}-\mu_0}{s_x/\sqrt{n}}$$

When we perform inference about a population mean μ using a t distribution, the appropriate degrees of freedom are df $= n - 1$.

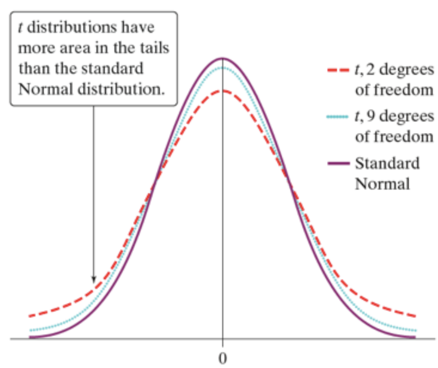
When we perform inference about a population mean μ using a t distribution, the appropriate degrees of freedom are df = $n - 1$.

n

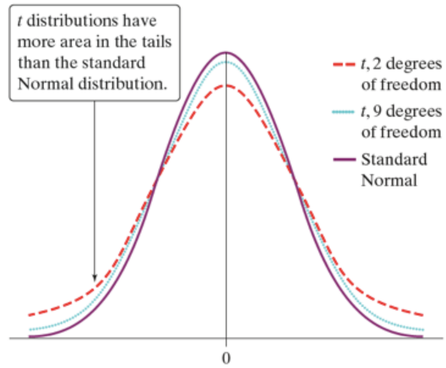
A ***t* distribution** is described by a symmetric, single-peaked, bell-shaped density curve. Any t distribution is completely specified by its degrees of freedom (df).
When performing inference about a population mean based on a random sample of size n when the population standard deviation σ is unknown, use a t distribution with df = $n - 1$.



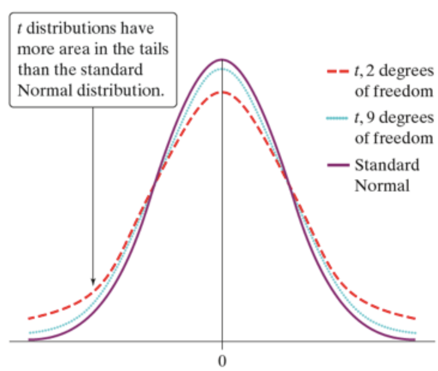
- The t distributions are similar in shape to the standard Normal distribution. They are symmetric about 0, single-peaked, and bell-shaped.



The *t* distributions have more variability than the standard Normal distribution, because the *t* distributions have more area in the tails.



As the degrees of freedom increase, the *t* distributions approach the standard Normal distribution.



Significance Test for a Population Mean, μ.

Important ideas:

CONDITIONS

Random

10%

Normal/Large Counts

- Pop Normal
- $n \geq 30$ CLT
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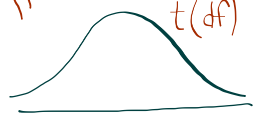
t - Statistic

$$t = \frac{\bar{x} - \mu}{s_x / \sqrt{n}}$$

TABLE B

t(df)



P-Value

Use Table B to find the P-Value

We can use Table B to find a *P*-value from the appropriate *t* distribution when performing a test about a population mean.

In the "Better batteries" example,
$H_0: \mu = 30$
$H_a: \mu > 30$

$$t = \frac{\bar{x} - \mu_0}{s_x / \sqrt{n}} = \frac{33.93 - 30}{9.82 / \sqrt{15}} = 1.55$$

df = $n - 1 = 15 - 1 = 14$

	Upper-tail probability *p*		
df	**0.10**	**0.05**	**0.025**
13	1.350	1.771	2.160
14	1.345	1.761	2.145
15	1.341	1.753	2.131
	80%	**90%**	**95%**
	Confidence level *C*		

$P(t \geq 1.55)$ is between 0.10 and 0.05.

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$P(t \geq 1.55)$ is between 0.10 and 0.05.

We'll report it as an interval of probabilities

Table B has limitations for finding *P*-values.

- It gives an interval of possible *P*-values for a significance test.
- The table includes probabilities only for t distributions with degrees of freedom from 1 to 30 and then skips to df = 40, 50, 60, 80, 100, 1000.

Upper-tail probability p			
df	0.10	0.05	0.025
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14	1.345	1.761	2.145
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Table B has limitations for finding P-values.

- It gives an interval of possible P-values for a signi...
- The table shows probabilities for only *posit*... value for a negative value of t, we use ... distributions.
- The table includes probab... with degrees of freedom from 1 to 30 ..., 60, 80, 100, 1000.

		0.025
		2.160
	1.761	2.145
	1.753	2.131
...0%	90%	95%

Confidence level C

caution !

CAUTION:

If the df you need isn't provided in Table B, use the next smaller df that is available.

Problem Without a Name

Suppose you want to perform a test of H_0: $\mu = 5$ versus H_a: $\mu \neq 5$ at the $\alpha = 0.01$ significance level. A random sample of size $n = 37$ from the population of interest yields $\bar{x} = 4.81$ and $s_x = 0.365$. Assume that the conditions for carrying out the test are met.

(a) Explain why the sample result gives some evidence for, H_a, the alternative hypothesis.

(b) Calculate the standardized test statistic and P-value.

a) $\bar{x} = 4.81 \neq 5$

b)

Problem Without a Name

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(a) Explain why the sample result gives some evidence for, H_a, the alternative hypothesis.

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(a) The sample mean is $\bar{x}$ = 4.81, which is not equal to 5 (as suggested by H_a).

Problem Without a Name

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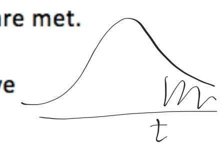


$P(t > 3.17) =$

$$t = \frac{4.81 - 5}{\frac{0.365}{\sqrt{37}}} = -3.17$$

$df = 37 - 1 = 36$

t(df)

$P(t < -3.17)$

(b)

$$t = \frac{4.81 - 5}{\frac{0.365}{\sqrt{37}}} = -3.17$$

$df = 37 - 1 = 36$

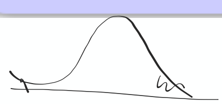
	Upper-tail probability p		
df	0.005	0.0025	0.001
29	2.756	3.038	3.396
30	2.750	3.030	3.385
40	2.704	2.971	3.307
	99%	99.5%	99.8%
	Confidence level C		

(b)

Using Table B: Because $df = 37 - 1 = 36$ is not available on the table, use $df = 30$.

$P(t \geq 3.17)$ is between 0.001 and 0.0025.

$P(t \leq -3.17$ or $t \geq 3.17)$ is between $2(0.001) = 0.002$ and $2(0.0025) = 0.005$.

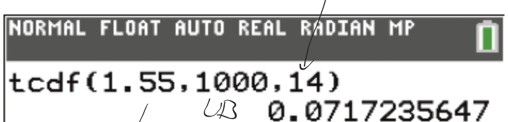


$H_a: \mu \neq 5$

Given the limitations of Table B, the advice is to use technology to find P-values when carrying out a significance test about a population mean.
(Just for the P-Value)

P(t ≥ 3.17)

df

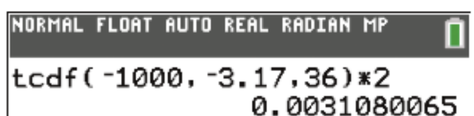


UB

LB

t_{cdf}

P(t ≤ –3.17 or t ≥ 3.17)



Putting It
All together

Carrying Out a Significance Test for μ

Significance Test: **One-Sample t Test for μ**

Problem: The level of dissolved oxygen (DO) in a stream or river is an important indicator of the water's ability to support aquatic life. A researcher measures the DO level at 15 randomly chosen locations along a stream. Here are the results in milligrams per liter (mg/l):

4.53	5.04	3.29	5.23	4.13	5.50	4.83	4.40
5.42	6.38	4.01	4.66	2.87	5.73	5.55	

An average dissolved oxygen level below 5 mg/l puts aquatic life at risk.

(a) Do the data provide convincing evidence at the α = 0.05 significance level that aquatic life in this stream is at risk?

(b) Given your conclusion in part (a), which kind of mistake—a Type I error or a Type II error—could you have made? Explain what this mistake would mean in context.

Space will be tight

State

[Define parameters, Show evidence for H_a(show the statistic), Hypotheses, and significance level, α]

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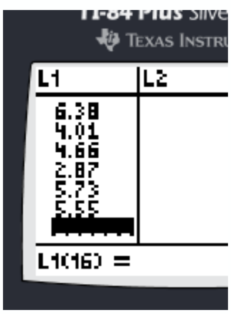
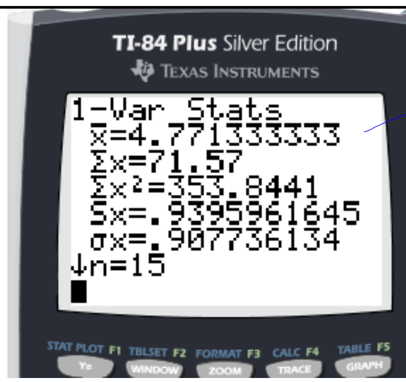
parameter μ = true mean DO level in the stream.

statistic $\bar{x}$ = ← Need to calculate

Hypothesis

α





$\bar{x} = 4.771$

State

[Define parameters, Show evidence for H_a (show the statistic), Hypotheses, and significance level, α]

parameter μ = true mean DO level in the stream (in mg/ℓ)

statistic $\bar{x} = 4.771$

Hypothesis $H_0: \mu = 5$

$H_a: \mu < 5$

α $\alpha = 0.05$

Plan

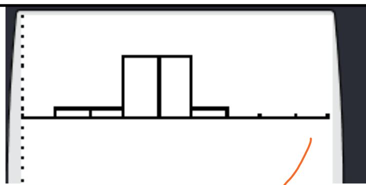
[Name the procedure and Check all conditions]

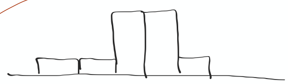
One sample t Test for μ

Random: Researcher selected 15 random locations in stream ✓

10%: ← there are an infinite number of locations in the stream so it is not necessary to check 10% condition.

Normal/Large Sample: ↖ Need to look at distribution (histogram, dot plot)





looks roughly symmetric ✓
with no strong outliers

Do [Give the sample statistic(s), Calculate the standardized test statistic, Find the P-Value]

For test statistic: General Formula, Specific Formula, then with numbers, then final answer

$\bar{x} = 4.771$

$S_x =$

GDC

Do [Give the sample statistic(s), Calculate the standardized test statistic, Find the P-Value]

For test statistic: General Formula, Specific Formula, then with numbers, then final answer

$\bar{x} = 4.771$

$S_x = 0.9396$

$$\text{Test Stat.} = \frac{\text{Stat} - \text{param}}{\text{SD}}$$

$$t = \frac{\bar{x} - \mu}{S_x / \sqrt{n}}$$

$$t = \frac{4.771 - 5}{.9396 / \sqrt{15}}$$

$$t = -0.94$$

Do [Give the sample statistic(s), Calculate the standardized test statistic, Find the P-Value]

For test statistic: General Formula, Specific Formula, then with numbers, then final answer

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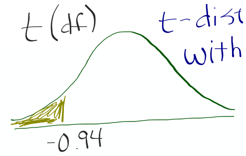

t-dist. with $df = 14$

P-Value

Table B Between ~~0.05~~ .15 and ~~0.10~~ .20

= ,

Alternative

tcd(-1000 (Lower), -.94 (Upper), 14 (df)) = ~~0.0186~~ .186

Conclude [Make a conclusion about the hypothesis in the context of the problem, two-sentence structure]

Because the P-Value range of 0.15 to .20 is > .05 We fail to reject H_0.

∴ We don't have convincing evidence that the true mean DO level in the stream is less than 5 mg/l not enough room for ?

(b) Given your conclusion in part (a), which kind of mistake—a Type I error or a Type II error—could you have made? Explain what this mistake would mean in context.

(b) Because we failed to reject H_0 in part (a), we could have made a Type II error (failing to reject H_0 when H_a is true). If we did, then the true mean dissolved oxygen level μ in the stream is less than 5 mg/l, but we didn't find convincing evidence with our significance test. That would imply aquatic life in this stream is at risk, but we weren't able to detect that fact.

Computer Output - will do t-test calculations

Session

One-Sample T: DO (mg/L)

Test of mu = 5 vs < 5

Variable	N	Mean	StDev	SE Mean	T	P
DO (mg/L)	15	4.771	0.940	0.243	-0.94	0.181

Read top of p.594

Session

One-Sample T: DO (mg/L)

Test of mu = 5 vs < 5

Variable	N	Mean	StDev	SE Mean	T	P
DO (mg/L)	15	4.771	0.940	0.243	-0.94	0.181

$$SE = \frac{S_x}{\sqrt{n}} = \frac{.940}{\sqrt{15}} = .243$$

$$t = \frac{4.771 - 5}{.243}$$

AP Exam Tip

another risk in using the Calculator for the "DO" step is :

LOSS OF UNDERSTANDING

- UNDERSTANDING THAT IS Required ON M/C QUESTIONS AND CAREFULLY CRAFTED FR QUESTIONS

7.......65, 67, 69, 73, 77, 79

study pp. 585-594

read about →

$$t = \frac{\bar{x} - \mu_0}{\frac{s_x}{\sqrt{n}}}$$

There are three conditions that must be met for the formula for the standardized test statistic to be valid—one for each of the three components in the formula.

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$$t = \frac{\bar{x} - \mu_0}{\frac{s_x}{\sqrt{n}}}$$

1. The Random condition

This condition helps ensure that $\bar{x} - \mu_0$ is a good estimate for the difference between the true value of μ and the null value of μ_0.

There are three conditions that must be met for the formula for the standardized test statistic to be valid—one for each of the three components in the formula.

$$t = \frac{\bar{x} - \mu_0}{\frac{s_x}{\sqrt{n}}}$$

2. The Normal/Large Sample condition

This condition allows us to model the distribution of the standardized test statistic *t* using a *t* distribution with n – 1 degrees of freedom.

There are three conditions that must be met for the formula for the standardized test statistic to be valid—one for each of the three components in the formula.

$$t = \frac{\bar{x} - \mu_0}{\frac{s_x}{\sqrt{n}}}$$

3. The 10% condition

This condition allows us to use the familiar formula for the standard deviation of the sampling distribution of $\bar{x}$ (with s_x replacing σ) when we are sampling without replacement from a finite population.