

# Michigan's next water crisis is PFAS - and you may already be affected

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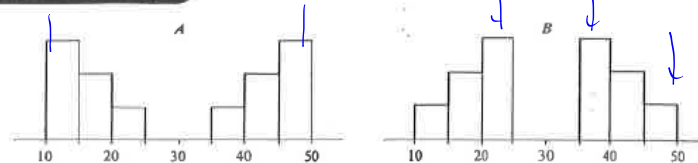
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Pick Up the Warm UP

## Warm Up

### AP Exam Review Question

#### DATA ANALYSIS 13



Given the above two histograms, which of the following statements is *incorrect*?

- (A) Both sets have the same mean.
- (B) Both sets have the same median.
- (C) Both sets have the same range.
- (D) Set A has a greater variance than does set B.
- (E) Each set has approximately 12 elements (6 under and 6 over the median).

S  
S<sup>2</sup>

*Answer:* (E) Both sets are symmetric about 30, so they have the same mean and median. Both sets have the range  $50 - 10 = 40$ . Set A has a higher percentage of values further from the mean than does set B, so set A has a greater variance. Histograms give relative frequencies, not actual numbers.

✓ INTERPRET a Type I error and a Type II error in context. GIVE a consequence of each error in a given setting.

**Should Rockford switch to bottled water?** 9.1 Day 2**WOLVERINE**

The Wolverine Worldwide (a shoe company in Rockford) improperly disposed of chemicals (PFAS), which have leaked into the ground water. The state's drinking water limit of 70 parts per trillion (ppt) is considered safe, while anything above 70 ppt is considered dangerous. Officials believe the water in Rockford may be unsafe. They take a random sample of 200 households in Rockford. They find the average lead level of the sample is 70.5 ppt.

1. State appropriate hypotheses for performing a significance test using words and symbols.
2. After conducting a significance test, a P-value of 0.045 is found. Interpret this value.
3. Based on the P-value, should Rockford keep the current water or switch to bottled water? Explain.

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1. State appropriate hypotheses for performing a significance test using words and symbols.

$H_0: \mu = 70 \text{ ppt.}$  ; The water is safe.  $\alpha = 0.05$

$H_a: \mu > 70 \text{ ppt.}$  : The water is unsafe.

2. After conducting a significance test, a P-value of 0.045 is found. Interpret this value.

Assuming that the water is safe ( $\mu = 70$ ) there is a 0.045 probability of getting a sample mean of 70.5 ppt. or more purely by chance.

3. Based on the P-value, should Rockford keep the current water or switch to bottled water? Explain.

4. Let's suppose this decision is wrong. What would be a consequence of this error?

5. Given the water is safe, how often will this error occur?

3. Based on the P-value, should Rockford keep the current water or switch to bottled water? Explain.

They should switch to bottled water since we have convincing evidence for the alternative hypothesis.

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They should switch to bottled water since we have convincing evidence for the alternative hypothesis.

4. Let's suppose this decision is wrong. What would be a consequence of this error?

They would waste money and resources

} TYPE I error

5. Given the water is safe, how often will this error occur?

5% of the time we get statistically significant results purely by chance.

} About 5% of time

6. Now suppose the P-value was 0.14. Should the town keep the current water or switch to bottled water?

They should keep the current water since they don't have convincing evidence against the null.

7. Let's suppose this decision is wrong. What would be a consequence of this error?

8. Are the consequences in question #4 or question #7 more serious? Explain.

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They should keep the current water since they don't have convincing evidence against the null.

7. Let's suppose this decision is wrong. What would be a consequence of this error?

People would drink unsafe water and could get sick / possibly die.

} Type II error

8. Are the consequences in question #4 or question #7 more serious? Explain.

6. Now suppose the P-value was 0.14. Should the town keep the current water or switch to bottled water?

They should keep the current water since they don't have convincing evidence against the null.

7. Let's suppose this decision is wrong. What would be a consequence of this error?

People would drink unsafe water and could get sick / possibly die.

8. Are the consequences in question #4 or question #7 more serious? Explain.

#7. People getting sick is much worse than wasting money.

## What it's like to live in the contaminated drinking water zones near Wolverine

Posted February 13, 2018 at 08:04 AM | Updated February 13, 2018 at 08:08 AM



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## Type 1 and Type 2 Errors

When we draw conclusions from a significance test we hope our conclusion will be correct. But sometimes it will be wrong. There are two types of errors we can make.

Important ideas:

Type 1 error :

Type 2 error :

Important ideas:

**TYPE I ERROR** : The null hypothesis is true but.... we make the wrong decision.

← occurs by chance  $\alpha$  % of the time

**TYPE II ERROR** : The alternate decision is true but.... we make the wrong decision.

we don't reject when we should

We reject  $H_0$  when we shouldn't.



3. Based on the P-value, should Rockford keep the current water or switch to bottled water? Explain.

They should switch to bottled water since we have convincing evidence for the alternative hypothesis.

4. Let's suppose this decision is wrong. What would be a consequence of this error?

They would waste money and resources

Type I error

$H_0$  is true

5. Given the water is safe, how often will this error occur?

5% of the time we get statistically significant results purely by chance.

Type I occurs 5% of the time

6. Now suppose the P-value was 0.14. Should the town keep the current water or switch to bottled water?

They should keep the current water since they don't have convincing evidence against the null.

7. Let's suppose this decision is wrong. What would be a consequence of this error?

People would drink unsafe water and could get sick / possibly die.

Type II error

$H_a$  is true

8. Are the consequences in question #4 or question #7 more serious? Explain.

#7. People getting sick is much worse than wasting money.

If a type I error has more serious consequences, consider using a smaller  $\alpha$  level

If a Type II error has more consequences, consider using a larger  $\alpha$

		Truth about the population	
		$H_0$ true	$H_a$ true
Conclusion based on sample	Reject $H_0$	Type I error	Correct conclusion
	Fail to reject $H_0$	Correct conclusion	Type II error

Don't use terminology "Accept  $H_0$ "

		Truth about the population	
		$H_0$ true	$H_a$ true
Conclusion based on sample	Reject $H_0$	Type I error	Correct conclusion
	Fail to reject $H_0$	Correct conclusion	Type II error

**If  $H_0$  is true:**

- Our conclusion is correct if we don't find convincing evidence that  $H_a$  is true.
- We make a type I error if we find convincing evidence that  $H_a$  is true.

**If  $H_0$  is false:**

- Our conclusion is correct if we find convincing evidence that  $H_a$  is true.
- We make a type II error if we don't find convincing evidence that  $H_a$  is true.

They are conditional probabilities:

$$P(\text{Type I error}) = P(\text{reject } H_0 \mid H_0 \text{ is true})$$

$$P(\text{Type II error}) = P(\text{fail to reject } H_0 \mid H_a \text{ is true})$$

## Check Your Understanding

The manager of a fast-food restaurant wants to reduce the proportion of drivethru customers who have to wait longer than 2 minutes to receive their food after placing an order. Based on store records, the proportion of customers who had to wait longer than 2 minutes was  $p = 0.63$ . To reduce this proportion, the manager assigns an additional employee to drive-thru orders. During the next month, the manager collects a random sample of 250 drive-thru times and finds that  $\hat{p} = \frac{144}{250} = 0.576$ .

The manager then performs a test of the following hypotheses at the  $\alpha = 0.10$  significance level:

$$\begin{aligned} H_0: p &= 0.63 && \text{63\% have to wait} \\ H_a: p &< 0.63 && \text{less than 63\% have to wait} \end{aligned}$$

where  $p$  = the true proportion of drive-thru customers who have to wait longer than 2 minutes to receive their food.

1. Describe a Type I error and a Type II error in this setting.

1. Describe a Type I error and a Type II error in this setting.

TYPE I :

TYPE II :

2. Which type of error is more serious in this case? Justify your answer.

1. Describe a Type I error and a Type II error in this setting.

TYPE I : 63% of the customers wait longer than 2 min, but the manager thinks there is less than that.

TYPE II : Less than 63% wait but the manager thinks 63% wait.

2. Which type of error is more serious in this case? Justify your answer.

Type I because the manager believes the extra employee reduces the proportion of customers who wait but it does not?

Anyone feel different about this?

3. Based on your answer to Question 2, do you agree with the company's choice of  $\alpha = 0.10$ ? Why or why not?

4. The  $P$ -value of the manager's test is 0.0385. Interpret the  $P$ -value.

3. Based on your answer to Question 2, do you agree with the company's choice of  $\alpha = 0.10$ ? Why or why not?

Perhaps not. If the null is true,  $\alpha = 0.10$  will result in a Type I error 10% of the time just by chance. They should use a smaller p-value.

4. The P-value of the manager's test is 0.0385. Interpret the P-value.

Assuming. ●●

3. Based on your answer to Question 2, do you agree with the company's choice of  $\alpha = 0.10$ ? Why or why not?

Perhaps not. If the null is true,  $\alpha = 0.10$  will result in a Type I error 10% of the time just by chance. They should use a smaller p-value.

4. The P-value of the manager's test is 0.0385. Interpret the P-value.

Assuming the proportion of customers who have to wait is  $p = 0.63$ , there is a 0.0385 prob. of getting a sample prop of 0.576 or less purely by chance.

We can decrease the probability of making a Type I error in a significance test by using a smaller significance level.

But there is a trade-off between  $P(\text{Type I error})$  and  $P(\text{Type II error})$ : as one increases, the other decreases.

If we make it more difficult to reject  $H_0$  by decreasing  $\alpha$ , we increase the probability that we will not find convincing evidence for  $H_a$  when it is true.

HW #15 on last night's assignment

Two  
Sentence  
Structure

Because the P-value of .1265 is greater than  $\alpha = 0.05$ , we fail to reject  $H_0$ .

We do not have convincing evidence that the true proportion of . . . .

## Reminder Time

- 7 Complete .. with appropriate detail
- 5 Complete but late (and complete with appropriate detail)
- 3 Partially complete

You get good  
at what you practice





(c) Construct and interpret a 90% confidence interval for the true mean weight of cereal in the boxes filled by this machine. Show details as done in class.

State: parameter: true mean  $\mu$  weight of cereal in boxes filled by this machine.  
confidence level: 90% confidence

Plan: one sample t interval for  $\mu$ .

conditions:

Random: "SRG of 30 boxes" *Careful* E

10%:  $n=30$  ~~10%~~ of all cereal boxes

Normal:  $n \geq 30$  CLT

DO: General formula for  $CI_n$ :  $CI_n = PE \pm MOE$

Specific formula for  $CI_n$ :  $CI_n = PE \pm t^* \cdot \frac{Sx}{\sqrt{n}}$   $t^* = 1.699$   
area: 0.05  
df: 29

E  $CI_n = 17.92 \pm 1.699 \cdot \frac{0.2}{\sqrt{30}}$

E  $CI_n = 17.92 \pm 0.062$

$CI_n = (17.858, 17.982)$

4

conclude: we are 90% confident that the interval from 17.858oz to 17.982oz captures the true mean weight  $\mu$  of the cereal in the boxes filled by this machine.

See your  
tests

(a) Construct and interpret a 90% confidence interval for the true mean weight of cereal in the boxes filled by this machine. Show details as done in class.

State :

Parameter:  $\mu$  = the true mean weight of cereal in the boxes filled by this machine

confidence level: 90%

4

Plan

one sample t-interval for  $\mu$

random: SRS of 30 boxes produced by the machine ✓ E

10%:  $30 < \frac{1}{10}$  (all boxes produced by this machine) ✓

normal:  $n \geq 30 = 30 \geq 30$  ✓

Do

Point est.  $\pm$  margin of error

$$\bar{x} \pm t^* \frac{s_x}{\sqrt{n}} \quad t^* = 1.649 \quad df = 29$$

InvT (area: 0.05, df: 29)

$$17.92 \pm 1.649 \frac{0.2}{\sqrt{30}}$$

$$17.92 \pm 0.062 = ( \quad )$$

$$= 17.858$$

$$= 17.982$$

Conclude

We are 90% confident that the interval 17.858<sup>ounces</sup> to 17.982 ounces captures the true mean weight of cereal in the boxes filled by this machine. E

9.1 .... 21-29 (odds), 30-32