Part 1 - Two-Sided Tests and Confidence Intervals

The link between two-sided tests and confidence intervals for a population mean allows us to make a conclusion directly from a confidence interval.

- If a 95% confidence interval for μ does not capture the null value μ_0, we can reject H_0: $\mu = \mu_0$ in a two-sided test at the $\alpha = 0.05$ significance level.
- If a 95% confidence interval for μ captures the null value μ_0, then we should fail to reject H_0: $\mu = \mu_0$ in a two-sided test at the $\alpha = 0.05$ significance level.

The same logic applies for other confidence levels, but *only* for a two-sided test.

Are radio stations honest?

A classic-rock radio station claims to play an average of 50 minutes of music every hour. To investigate the station's claim, you randomly select 12 different hours during the next week and record what the radio station plays in each of the 12 hours. Here is how much music (in minutes) was played during each of these hours:

48	49	50	51	49	53
49	47	47	50	46	48

(a) State an appropriate pair of hypotheses for a significance test in this setting. Be sure to define the parameter of interest.

Assume that the conditions have been checked and you are all clear to perform a test.

(b) A 95% confidence interval for the mean play time (in minutes) of all hours this week is (47.691, 50.142). Based on this interval, what conclusion would you make for a test of the hypotheses in part (a) at the $\alpha = 0.05$ significance level? [Ask: According to the confidence interval, is the null hypothesis value a plausible value?]

(c) Can we generalize our conclusion for this radio station for the whole year? Explain your answer.

The Power of A Significance Test?

The national mean score on the math portion of the SAT is 511 with a standard deviation of 120. Suppose we believe the students at SHS have a higher mean than the national average. To find out, we take a random sample of 8 students and find their average. We will then use the data to conduct a significance test with $\alpha = 0.05$.

1. Write the appropriate hypotheses for the significance test. Be sure to define the parameter of interest.

Suppose the mean math SAT score at SHS is 535 (*alt. μ*). Go to our textbook website and open the "Statistical Power" applet. Enter all of this information into the fields on the left of the applet. You'll notice a value called "Power". This is the probability that the significance test will find convincing evidence against the null with the information you've entered.

2. What is the **Power** (*or probability*) that the test will find convincing evidence against the null hypothesis?

Interpret this value in context.

3. We want to **increase** the power of our test. How could we adjust each of the following factors to increase our power? Use the applet to explore each, one at a time.

a. Sample size:

b. Alpha level:

c. Alternative μ:

Power of a Test

Power = probability of avoiding a type II error

= probability of finding convincing evidence that H_a is true when H_a is really true

= P(reject H_0 when parameter = some alternative value)

= 1 – P(Type II error)

Important ideas:

Increasing Power

Increasing the sample size

Increase the alpha level

Increasing the difference between the null and alternative hypothesis values

Preventing ADHD

The Centers for Disease Control and Prevention claims that 11% of American children, ages 4–17, have attention deficit/hyperactivity disorder (ADHD). A company claims that it has developed a new vitamin tablet that will lower a child's risk for ADHD. Researchers will administer the vitamin tablet to 200 volunteer children under the age of 4 (with parental consent). The subjects will be tracked through childhood, and the researchers will record the proportion of the subjects who develop ADHD. The researchers will perform a test at the $\alpha = 0.05$ significance level of

$$H_0 : p = 0.11$$

$$H_\alpha : p < 0.11$$

where p = the true proportion of all children like those in the study who would develop ADHD when given the new vitamin tablet. The new vitamin tablet is expensive to produce, so researchers would like to be convinced that it really does reduce the risk of ADHD. The power of the test to detect that $p = 0.05$ is 0.937.

1. **Interpret this value in context.**

2. **Find the probability of a type I error and the probability of a Type II error for the test.**

3. **Determine whether each of the following changes would increase or decrease the power of the test. Explain your answers.**

   (a) Use $\alpha = 0.10$ instead of $\alpha = 0.05$

   (b) If the true proportion is $p = 0.08$ instead of $p = 0.05$

   (c) Use $n = 500$ instead of $n = 200$