Where we've been

8.1 Confidence Interals

8.2 Estimating population proportions using Confidence Intervals

Where were going

8.3 Estimating population means using Confid. Intervals

How Much Does An Oreo weigh?

Work on 1 to 7 with partner/group

Get as much done as you can in 10 minutes.

Mrs. Gallas wanted to estimate the average weight of an Oreo cookie to determine if the average weight was less than advertised. She selected a random sample of 30 cookies and found the weight of each cookie (in grams). The mean weight was $\bar{x}$ = 11.1921 grams with a standard deviation of s_x = 0.0817 grams. Make a 95% confidence interval to estimate the true mean weight of an Oreo.

1. What is the **point estimate** for the true mean? ________

2. Identify the population, parameter, sample and statistic.

Population: ____________________ Parameter: ____________________

Sample: ____________________ Statistic: ____________________

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1. What is the **point estimate** for the true mean? $\bar{x} = 11.1921$

2. Identify the population, parameter, sample and statistic.

Population: All oreos Parameter: μ = true mean

Sample: 30 oreos Statistic: $\bar{x} = 11.1921$

3. Was the sample a random sample? Why is this important?

Yes. Important so we can generalize.

4. What is the formula for calculating the standard deviation of the sampling distribution of $\bar{x}$?

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

5. What condition must be met to use this formula? Has it been met?

10% CONDITION

$30 < \frac{1}{10}(\text{all oreos})$

6. In the formula for the standard deviation of the sampling distribution of $\bar{x}$, we don't know the value of σ (if we did, we would have known μ) so we will use s_x instead. Find the **standard error.**

$$SE_{\bar{x}} = \frac{S_x}{\sqrt{n}} = \frac{0.0817}{\sqrt{30}} = 0.0149$$

7. Would it be appropriate to use a normal distribution to model the sampling distribution of $\bar{x}$? Justify your answer.

yes. CLT $30 \geq 30$

Central Limit Theorem

which can be used when sampling means

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Condition 1

Random

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Condition 2

10%

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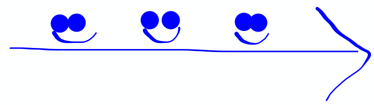
Condition 3

Normal

8. When finding the margin of error for a confidence interval for a **proportion** we use z*. For a **mean**, however, we will use ______ as the critical value (especially if the sample size is not really big). Why???



proportions $\quad \hat{p} + z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \quad$ one sample z interval for pop. proportion

$\downarrow$

means $\quad \bar{x} + z^* \cdot \frac{\sigma}{\sqrt{n}} \quad$ one sample z interval for popul mean proportions $\hat{p} + z^*\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ one sample z interval for pop. proportion

↓

means $\bar{x} + z^* \cdot \frac{\sigma}{\sqrt{n}}$ one sample z interval for popul mean

but we don't know pop. SD most of the time

So we use sample SD s

---

proportions $\hat{p} + z^*\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ one sample z interval for pop. proportion

↓

means $\bar{x} + \cancel{z^* \cdot \frac{\sigma}{\sqrt{n}}}$ one sample z interval for popul mean

$$x + z^* \cdot \frac{s_x}{\sqrt{n}}$$

but.....

$$x + z^* \cdot \frac{S_x}{\sqrt{n}}$$ ← has less variability than σ

$$x + z^* \cdot \frac{S_x}{\sqrt{n}}$$ ← has less variability than σ

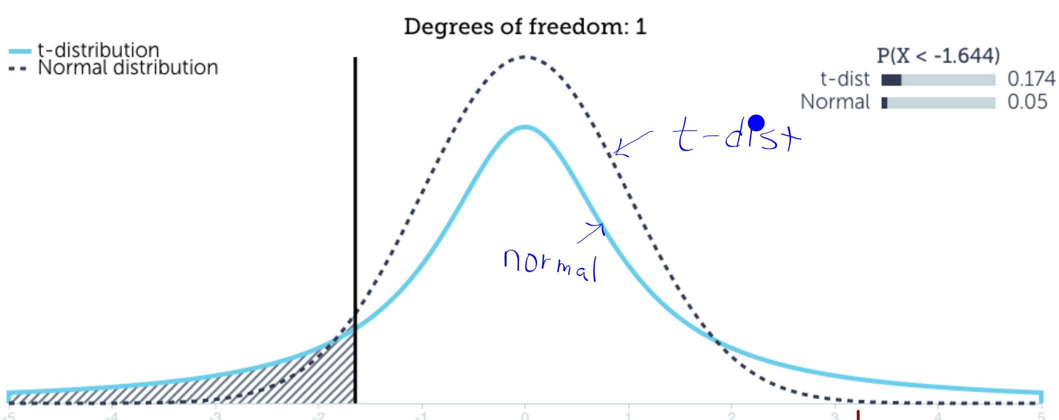
which cause the confidence intervals to be too short and we capture the mean less ☹

So, to lengthen them...

We use a new distribution ☺
(one very similiar to Normal Distrib)

t-distribution

[it provides us with a critical value that is larger]



will give us t^* rather than z^*

Video
about t-distribution

8. When finding the margin of error for a confidence interval for a **proportion** we use z*. For a **mean**, however, we will use __t*__ as the critical value (especially if the sample size is not really big). Why???

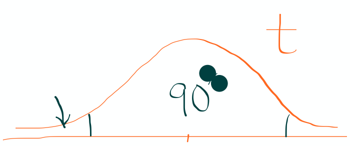
gives us a larger critical value.

student's t

Go to
TABLE B

Problem: What critical value t^* from Table B should be used in constructing a confidence interval for the population mean with a 90% confidence interval from a random sample of 48 observations

degrees of freedom $df = 48 - 1 = 47$

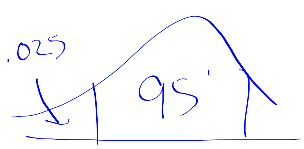


	Tail probability p			
df	**.10**	**.05**	**.025**	**.02**
30	1.310	1.697	2.042	2.147
40	1.303	1.684	2.021	2.123
50	1.299	1.676	2.009	2.109
∞	1.282	1.645	1.960	2.054
	80%	**90%**	**95%**	**96%**
	Confidence level C			

Now do question 9

9. What t^* is needed for this confidence interval? Use **Table B** and the **degrees of freedom =** $30-1$ $=29$
n - 1 to find it.

$t^* = 2.045$



10. Calculate the **margin of error** using *t** and the standard error.

11. Calculate the 95% confidence interval using **point estimate +/- margin of error**.

12. Interpret the interval.

10. Calculate the **margin of error** using *t** and the standard error.

$SE_{\bar{x}}$

So M.O.E. = $\underbrace{2.045}_{t^*} \times \underbrace{0.0149}_{SE_{\bar{x}}} = .0305$

11. Calculate the 95% confidence interval using **point estimate +/- margin of error**.

$\underbrace{11.19}_{\bar{x}} \pm \underbrace{0.0305}_{\text{moe}} \rightarrow (11.1616, 11.2226)$

12. Interpret the interval.

We are

10. Calculate the **margin of error** using *t** and the standard error.

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So $M.O.E. = 2.045 \times 0.0149$

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We are 95% confident that

10. Calculate the **margin of error** using *t** and the standard error.

$SE_{\bar{x}}$

So $M.O.E. = 2.045 \times 0.0149$

11. Calculate the 95% confidence interval using **point estimate +/- margin of error**.

$11.19 \pm 0.0305 \rightarrow (11.1616, 11.2226)$

12. Interpret the interval.

We are 95% confident that the interval from 11.16g to 11.22g captures the true mean weight of oreos

12. Interpret the interval.

13. Write a general formula for a confidence interval for a **population** mean.

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$$\bar{x} \pm t^* \cdot \frac{S_x}{\sqrt{n}}$$

means = mean more work

14. According to Nabisco, an Oreo weighs 11.3 grams. Does our confidence interval provide convincing evidence that the true average weight is less than 11.3 grams? Explain.

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yes.

Our entire interval is below 11.3 g

Conditions

CONDITIONS FOR ESTIMATING

1. The ***Random condition*** is crucial for doing inference. If the data don't come from a random sample, you can't draw conclusions about a larger population.

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1. The ***Random condition*** is crucial for doing inference. If the data don't come from a random sample, you can't draw conclusions about a larger population.

2. When sampling without replacement, the ***10% condition*** ensures that our formula for the standard deviation is approximately correct.

3. The ***Normal/Large Sample*** *condition* to ensure it is appropriate to use a *t* distribution to calculate the t^* critical value.

Constructing a Confidence Interval for μ *(mean)*

Important ideas:

Conditions

Critical Values

Constructing a Confidence Interval for μ *(mean)*

Important ideas:

Conditions

1. Random
2. 10% condition
3. Normal / Large Sample

Popul is normal

or $n \geq 30$ CLT

or Sample data shows no strong skew or outliers

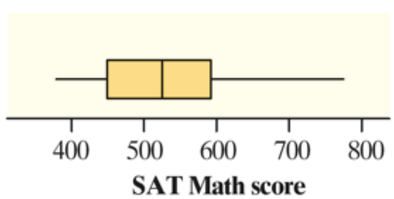
Critical Values

NORMAL / LARGE SAMPLE ?

23	0
24	0
25	
26	5
27	
28	7
29	
30	259
31	399
32	033677
33	0236

Key: 31|3 = 313 pounds of force required to pull a piece of Douglas fir apart

(b) No; the stemplot is strongly skewed to the left with possible low outliers and $n = 20 < 30$.



Yes; even though $n = 20 < 30$, the boxplot is only moderately skewed to the right and there are no outliers.

If a question on the AP® Statistics exam asks you to construct and interpret a confidence interval, all the conditions should be met. However, you are still required to state the conditions and show evidence that they are met—including a graph if the sample size is small and the data are provided.

Constructing a Confidence Interval for *μ (mean)*

Important ideas:

Conditions

(1) Random
(2) 10% Condition
(3) Normal / Large Sample

Popul is normal
or $n \geq 30$ CLT
or Sample data shows no strong skew or outliers

Critical Values

Use t^* for critical value for means

degrees of freedom
$df = n-1$

Use TABLE B
(with Confid Level and df)

$$\bar{x} \pm t^* \cdot \frac{s}{\sqrt{n}}$$

Your Understanding....
Check it !

Check Your Understanding

1. Use Table B to find the critical value t* that you would use for a confidence interval for a population mean μ in each of the following settings. If possible, check your answer with technology.

   (a) A 96% confidence interval based on a random sample of 22 observations

   (b) A 99% confidence interval from an SRS of 71 observations

Check Your Understanding

1. Use Table B to find the critical value t* that you would use for a confidence interval for a population mean μ in each of the following settings. If possible, check your answer with technology.

(a) A 96% confidence interval based on a random sample of 22 observations

TABLE B → $t^* = 2.189$

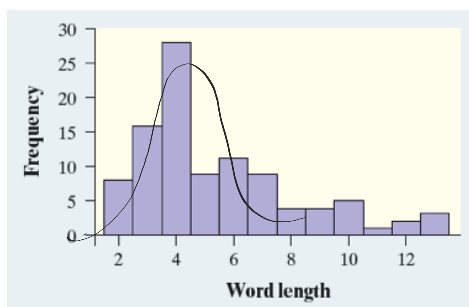
df = 21 96%

(b) A 99% confidence interval from an SRS of 71 observations

df = 70 99%

$t^* = 2.660$ ← Lower

2. Judy is interested in the reading level of a medical journal. She records the length of a random sample of 100 words. The histogram displays the distribution of word length for her sample. Determine if the conditions for constructing a confidence interval for a mean have been met in this context.



① Random :

② 10%

③ Normal

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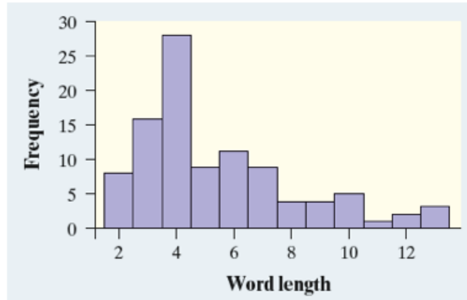

(1) Random: Rand. sample of 100 words

(2) 10%: $100 < \frac{1}{10}$(all words)

(3) Normal: yes, because $100 \geq 30$ CLT

Check Your Understanding

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(a) A 96% confidence interval based on a random sample of 22 observations

TABLE B → $t^* = 2.189$ invT(.02, 21) = 2.189

df = 21 96%

tail df

(b) A 99% confidence interval from an SRS of 71 observations

df = 70 99% $t^* = 2.660$ invT(.005, 70) = 2.648

8.3.....61, 63, 65, 67

and study pp. 525-534