

Learning Target 1

CONSTRUCT and INTERPRET a confidence interval for a population proportion using the 4-step process.

Lesson 8.2: Day 2: How much of the Earth is covered by water?



↑
Lots

What proportion of the Earth is covered by water? We will investigate this question by taking a random sample of locations on the globe.

- A. How many locations did your class sample? 50 How many locations were water? 29
- B. Calculate the proportion of locations from your sample that are water. $\hat{p} = \frac{29}{50} = 0.58$

C. Construct a 95% confidence interval to estimate the proportion of the Earth that is water.

1. STATE: State the parameter you want to estimate and the confidence level.

Parameter: _____

Confidence level: _____

2. PLAN: Identify the appropriate inference method and check conditions.

Name of procedure:

Check conditions:



C. Construct a 95% confidence interval to estimate the proportion of the Earth that is water.

1. STATE: State the parameter you want to estimate and the confidence level.

Parameter: $p = \text{true proportion of the earth covered by water}$

Confidence level: 95%

2. PLAN: Identify the appropriate inference method and check conditions.

Name of procedure: $\text{One sample } z \text{ interval for } p$

Check conditions:

C. Construct a 95% confidence interval to estimate the proportion of the Earth that is water.

1. STATE: State the parameter you want to estimate and the confidence level.

Parameter: $p =$ true proportion of the earth covered by water

Confidence level: 95%

2. PLAN: Identify the appropriate inference method and check conditions.

Name of procedure: One sample z interval for p ✓

Check conditions: random - took a random sample of locations ✓

10% - $50 < \frac{1}{10}$ of all locations ✓

Large Counts - $n\hat{p} = 50(.58) = 29$ ✓
 $n(1-\hat{p}) = 50(.42) = 21$

AP Exam Tip

Not likely that you will be asked to construct a Confidence Interval where the conditions are not met

[but its possible]

If you think a condition is not met should still perform all calculations.

In the conclusion, note any concern

3. DO: If the conditions are met, perform the calculations.

General Formula for any confidence interval:

Specific Formula for this confidence interval:

Plug numbers into the formula:

Answer:

4. CONCLUDE: Interpret your interval in the context of the problem.

Interpret:

3. DO: If the conditions are met, perform the calculations.

General Formula for any confidence interval:

Point Estim \pm margin of error

Specific Formula for this confidence interval:

$$\hat{p} \pm z^* \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

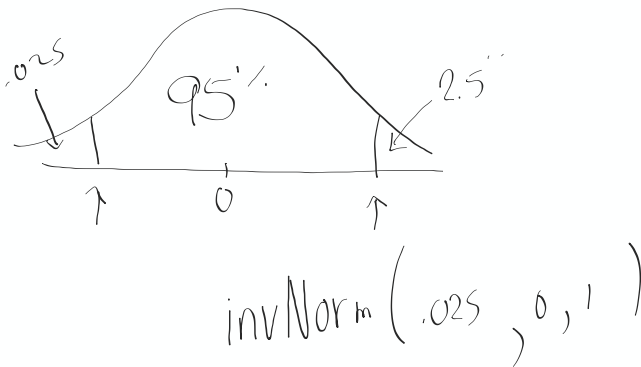
Plug numbers into the formula: $.58 \pm 1.96 \cdot \sqrt{\frac{(.58)(.42)}{50}}$

Answer: $.58 \pm .1368 = (.443, .717)$

4. CONCLUDE: Interpret your interval in the context of the problem.

Interpret:

We are 95% confident that the interval from .443 to .717 captures the true proportion of the earth covered by H₂O.



In ch. 9 we will do significance testing. There we will use Z (without the asterisk) when we calculate the test statistic.

→ Make sure you show the asterisk with Z (Z^*) when constructing confidence intervals.

The Four Step Process (for estimating the mean, μ) ^{proportion p}

Important ideas:

STATE ●

PLAN ●

DO:

CONCLUDE ●

The four-step process will be used repeatedly in ch. 8-12

S_{tate}

Plan

Do

Conclude

↑

These steps were carefully designed based on the scoring rubric for C.I. questions on the AP exam.

← each Free Response question is worth 4 points (hint hint)

S_{tate}P_{lan}D_oC_{onclude}

↑

These steps were carefully designed
based on the scoring rubric
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S_{tatistics}P_{roblems}D_{emand}C_{onsistency!}

hint hint
Each Free Response question is 4 points !!!



The Four Step Process (for estimating the mean, μ)

Important ideas:

STATE • Parameter and level

PLAN • Name of procedure
Conditions

DO: General + Specific
formulas + final interval

CONCLUDE • Interpret in context

C.I.
"We are 95% confident."

The Conclude step includes interpretation of the confidence interval, not the confidence level.

CAUTION IF you also include an interpretation of confidence level.

Learning Target 2

Choose the required sample size in a study to obtain a $C\%$ confidence interval for a population proportion with a specified margin of error.

Choosing the Sample Size

We won't know the value of \hat{p} until after the study has been conducted. This means we have to guess the value of \hat{p} when choosing n .

There are two ways to do this:

1. Use a guess for \hat{p} based on a pilot (preliminary) study or past experience with similar studies.
2. Use $\hat{p} = 0.5$ as the guess. The margin of error ME is largest when $\hat{p} = 0.5$, so this guess is conservative. If we get any other \hat{p} when we do our study, the margin of error will be smaller than planned.

$$\begin{array}{l}
 P \\
 \downarrow \\
 (.2)(.8) = 0.16 \\
 (.4)(.6) = 0.24 \\
 (.5)(.5) = 0.25 \\
 (.6)(.4) = 0.24 \\
 (.8)(.2) = 0.16
 \end{array}$$

the maximum
occurs when
 $\hat{p} = 0.5$

The Four Step Process (for estimating the mean, μ)

Important ideas:

STATE: Parameter and level

PLAN: Name of procedure
Conditions

DO: General + Specific
formulas + final interval

CONCLUDE: Interpret in context

C.I.
"We are 95% confident."

Choosing a Sample Size

$$M.O.E. = z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

Solve for n.

* Always round up

* If \hat{p} is unknown, use

$\hat{p} = 0.5$, that's conservative

Check Your Understanding

Check Your Understanding - A company has received complaints about its customer service. The managers intend to hire a consultant to carry out a survey of customers. Before contacting the consultant, the company president wants some idea of the sample size that she will be required to pay for. One value of interest is the proportion p of customers who are satisfied with the company's customer service. She decides that she wants the estimate to be within 3 percentage points (0.03) at a 95% confidence level.

1. Using a conservative estimate for \hat{p} , how large of a sample is needed?
2. In the company's prior-year survey, 80% of customers surveyed said they were satisfied. Using this value as a guess for \hat{p} , find the sample size needed for a margin of error of at most 3 percentage points with 95% confidence. How does this compare with the required sample size from question #1?

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1. Using a conservative estimate for \hat{p} , how large of a sample is needed?

$$.03 = 1.96 \sqrt{\frac{(.5)(.5)}{n}} \rightarrow \frac{.03}{1.96} = \sqrt{\frac{(.5)(.5)}{n}} \rightarrow \left(\frac{.03}{1.96}\right)^2 = \frac{(.5)(.5)}{n}$$

2. In the company's prior-year survey, 80% of customers surveyed said they were satisfied. Using this value as a guess for \hat{p} , find the sample size needed for a margin of error of at most 3 percentage points with 95% confidence. How does this compare with the required sample size from question #1?

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$$n = \frac{(.5)(.5)}{\left(\frac{.03}{1.96}\right)^2}$$

2. In the company's prior-year survey, 80% of customers surveyed said they were satisfied. Using this value as a guess for \hat{p} , find the sample size needed for a margin of error of at most 3 percentage points with 95% confidence. How does this compare with the required sample size from question #1?

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$$n = \frac{(.5)(.5)}{\left(\frac{.03}{1.96}\right)^2} = 1067.11 \rightarrow$$

2. In the company's prior-year survey, 80% of customers surveyed said they were satisfied. Using this value as a guess for \hat{p} , find the sample size needed for a margin of error of at most 3 percentage points with 95% confidence. How does this compare with the required sample size from question #1?

$$.03 = 1.96 \sqrt{\frac{(.8)(.2)}{n}} \Rightarrow n = 682.95$$

$n = 683$

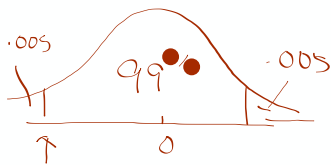
Why not round to the nearest whole number? (1067)

Because a smaller sample size will result in a larger margin of error, possibly more than the 3 percentage points for the poll.

3. What if the company president demands 99% confidence instead of 95% confidence? Would this require a smaller or larger sample size, assuming everything else remains the same? Explain your answer.

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$$.03 = 2.576 \sqrt{\frac{(.80)(.2)}{n}}$$



$$\text{invNorm}(.005, 0, 1)$$

$$n = 1179.7$$

↓

$$\text{so } n = 1180 \text{ people}$$

A larger sample will make us right about p more often so % goes up.

We'll start to use calculators to construct confidence intervals at the end of the chapter. It will look like:

1-Prop Z Int

I insist you don't use it in place of your work until I give you the green light.

☺ → This will help you understand the structure of confidence intervals for the rest of the course and.....

☺ → Prepare you for multiple-choice questions that focus on the way confidence intervals are constructed

LCQ

8.2.....41, 45, 49, 55-58

and study pp. 517-520