

Use a Confidence Intervals applet, as a class, to learn what it means to say that we are

"95% confident"

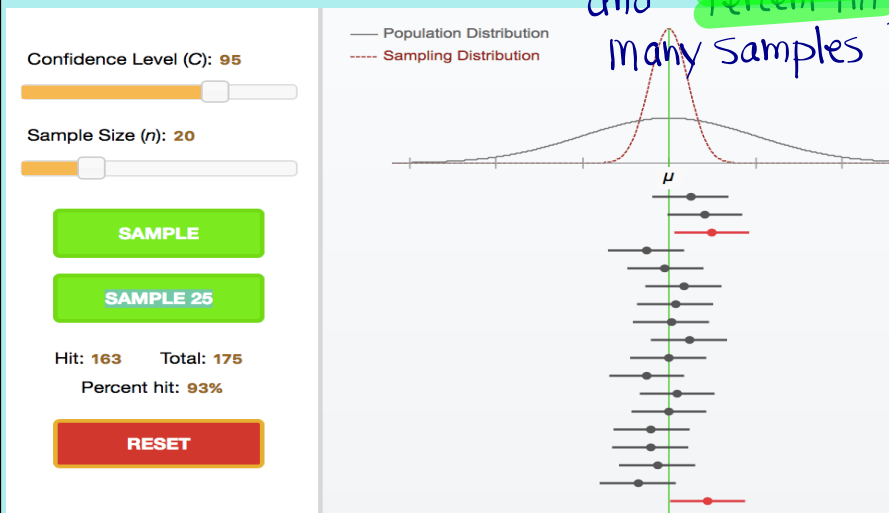
later each of you will use the same applet with laptops

by the way, your learning targets today are:

- Interpret a Confidence Level
- Describe how the sample size and confidence level affect Margin of Error.
- How do practical issues like non-response, undercoverage, and response bias affect the interpretation of confidence interval.

Our trusty volunteer  
will now guide us.

What's the relationship  
between confidence level  
and "Percent Hit" after taking  
many samples?



The activity confirms that the **confidence level** is the overall capture rate if the method is used many times.

If many samples are taken and 95% confidence intervals are constructed for each of these samples, then about 95% of the intervals will capture the true parameter being estimated.

[in real life we usually calculate just one interval].

AP Stats

**8.1 Day 2 - Classwork 1 – Interpreting Confidence Levels**

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**Interpreting a Confidence Levels**

Two weeks before a presidential election, a polling organization asked a random sample of registered voters the following question: "If the presidential election were held today, would you vote for Candidate A or Candidate B?" Based on this poll, the 95% confidence interval for the population proportion who favor Candidate A is (0.48, 0.54).

- a) Interpret the confidence interval. (like we did in the last lesson. Remember a confidence interval gives a set of plausible values for the parameter)

AP Stats

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**Interpreting a Confidence Levels**

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- a) Interpret the confidence interval. (like we did in the last lesson. Remember a confidence interval gives a set of plausible values for the parameter)

We are 95% confident that the interval from 0.48 to 0.54 captures the true proportion of all registered voters who favor candidate A.

## When interpreting a confidence interval:

Say "You are '95%' confident"  
the interval captures the parameter.

don't say "There is a 95% probability"  
"there is a 95% chance"

AP  
Exam  
Tip

Ok, the part b question is a little backwards. You get to see the response. Then decide what the question should be. Good luck.

b) Question: \_\_\_\_\_

Response: *If we were to select many random samples of registered voters and construct a 95% **confidence interval** using each sample, about 95% of the intervals would capture the true proportion of all registered voters who favor Candidate A in the election.*

Ok, the part b question is a little backwards. You get to see the response. Then decide what the question should be. Good luck.

b) Question: Interpret the Confidence Level.

Response: *If we were to select many random samples of registered voters and construct a 95% **confidence interval** using each sample, about 95% of the intervals would capture the true proportion of all registered voters who favor Candidate A in the election.*

#### Interpreting a Confidence Interval

To interpret a confidence level  $C$ , say, "If we were to select many random samples from a population and construct a  $C\%$  confidence interval using each sample, about  $C\%$  of the intervals would capture the [parameter in context]."

#### Note

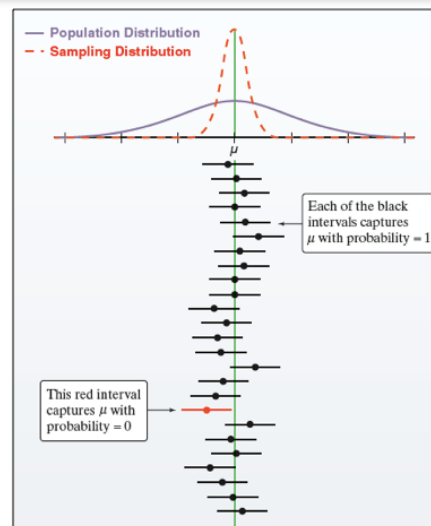
We don't need an actual sample to interpret the confidence level. Interpreting the confid. level is about describing the method for calculating it; not about interpreting a specific confidence interval calculated from an actual sample

3 min.  
Silent Reading

page 501  
half-way down

## Interpreting Confidence Level

Once a particular confidence interval is calculated, its endpoints are fixed. And because the value of a parameter is also a constant, a particular confidence interval either includes the parameter (probability = 1) or doesn't include the parameter (probability = 0).

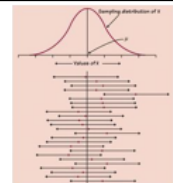


# What affects the Margin of Error

You will need a laptop



## What Affects the Margin of Error



1. Go to textbook statistical applets and select *Confidence Intervals* or find the url: ([http://digitalfirst.bfwpub.com/stats\\_applet/stats\\_applet\\_4\\_ci.html](http://digitalfirst.bfwpub.com/stats_applet/stats_applet_4_ci.html))



2. Click "Sample" to choose an SRS and display the resulting confidence interval. The *confidence interval* is displayed as a horizontal line segment with a dot representing the sample mean in the middle of the interval. The true mean ( $\mu$ ) is the green vertical line.

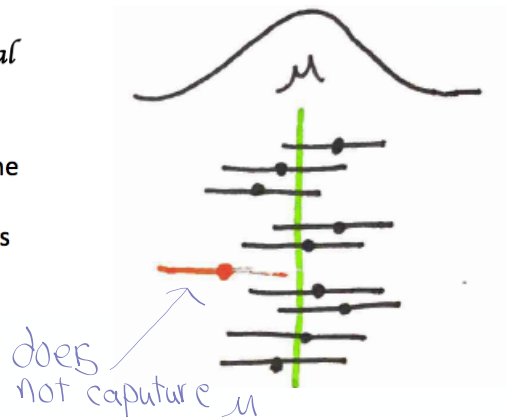
Did the first *confidence interval* capture the true mean?

Repeat this 10 times and, in the open space, sketch what you see. How many of the intervals capture the true mean?

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Repeat this 10 times and, in the open space, sketch what you see. How many of the intervals capture the true mean?



3. "Reset" and then take a total of 100 *confidence interval* (sample 25 four times). How many out of 100 captured the true mean? Is this surprising? Why?
  
4. Watch your *confidence intervals* as you drag the **confidence level** from 95% to 99% (don't "Reset"). What happens to the intervals when the **confidence level** is increased? Why does this make sense?

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↑ confidence  
↑ width

5. "Reset", then sample 100 times at an 80% confidence interval. How many of the intervals capture the true mean? Do your best to Interpret the **confidence LEVEL**:

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~ About 80% ~

If we take many 80% confident intervals, we expect about 80% to capture the true mean.

6. Now we will see what happens when we adjust the sample size. Change the sample size from 5 to 50 and sample for 1 interval. Then change it to 250 and sample for 1 interval. What happens to the interval when the sample size is increased? Why?

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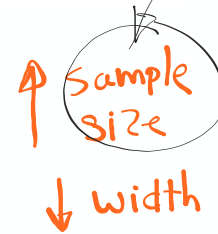
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↑ sample size  
↓ width

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costs money and time

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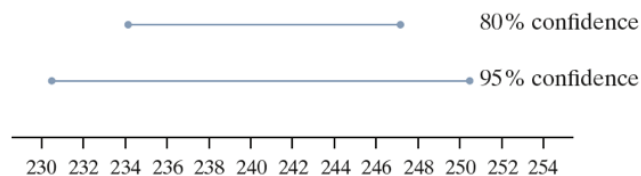


The price we pay for greater confidence is a wider interval.

### Decreasing the Margin of Error

In general, we prefer an estimate with a small margin of error. The margin of error gets smaller when:

- *The confidence level decreases.* To obtain a smaller margin of error from the same data, you must be willing to accept less confidence.



- *The sample size  $n$  increases.* In general, increasing the sample size  $n$  reduces the margin of error for any fixed confidence level.

The margin of error also depends on the standard deviation of the statistic. As you learned in Chapter 7, the sampling distribution of a statistic will have a smaller standard deviation when the sample size is larger. This is why the margin of error decreases as you increase the sample size.

## Other Tidbits

For a normal sampling distribution, extending about **2** standard deviations in each direction from the statistic produces an approximate **95%** confidence interval.

For a normal sampling distribution, extending about **3** standard deviations in each direction from the statistic produces and approximate **99.7%** confidence interval.

The more general formula for a confidence interval is:  
**statistic  $\pm$  (critical value)  $\cdot$  (standard deviation of statistic)**

The **critical value** is a multiplier that makes the interval wide enough to have the stated capture rate.

point estimate  $\pm$  margin of error

example  $\bar{X} \pm 2 \sigma_x$

$$240.80 \pm 2 \cdot \frac{20}{\sqrt{n}}$$

$$\text{statistic} \pm \left( \text{critical value} \right) \cdot \left( \text{std. dev. of the statistic} \right)$$

**CAUTION:**

The margin of error does not account for any sources of bias in the data collection process.

↖  
doesn't account for  
undercoverage  
non-response  
response bias  
etc



**Check Your Understanding:** As part of a project about response bias, Ellery surveyed a random sample of 25 students from her school. One of the questions in the survey required students to state their GPA aloud. Based on the responses, Ellery said she was 90% confident that the interval from 3.14 to 3.52 captures the mean GPA for all students at her school.

(a) Interpret the confidence level.

If we were to select many random samples from a pop. and construct a 90% confident interval using each sample, about 90% of the intervals would capture the mean GPA for all the students at Ellery's school.

(b) Explain what would happen to the length of the interval if the confidence level were increased to 99%.

as the confidence level increases, so does the length of the interval

**Check Your Understanding:** As part of a project about response bias, Ellery surveyed a random sample of 25 students from her school. One of the questions in the survey required students to state their GPA aloud. Based on the responses, Ellery said she was 90% confident that the interval from 3.14 to 3.52 captures the mean GPA for all students at her school.

(a) Interpret the confidence level.

If we make many 90% confidence intervals, about 90% will capture the true mean GPA

(b) Explain what would happen to the length of the interval if the confidence level were increased to 99%.

The interval would widen because the margin of error increases.

(c) How would a 90% confidence interval based on a sample of size 200 compare to the original 90% interval?

THE INTERVAL WOULD NARROW A LOT  
BC WE EXPECT LESS VARIABILITY  
(SMALLER S.D.)

(d) Describe one potential source of bias in Ellery's study that is not accounted for by the margin of error.

The kids lied.

(c) How would a 90% confidence interval based on a sample of size 200 compare to the original 90% interval?

It would be more narrow because  
an increase in sample decrease the  
margin of error.

(d) Describe one potential source of bias in Ellery's study that is not accounted for by the margin of error.

Students might not tell the truth  
about their GPA.

**8.1 ... 11, 15, 17, 17, 19, 21, 23-26**

**study pp. 499-505**