

Correlation Coefficient

Try our energy drink —  
it's highly correlated with  
performance!

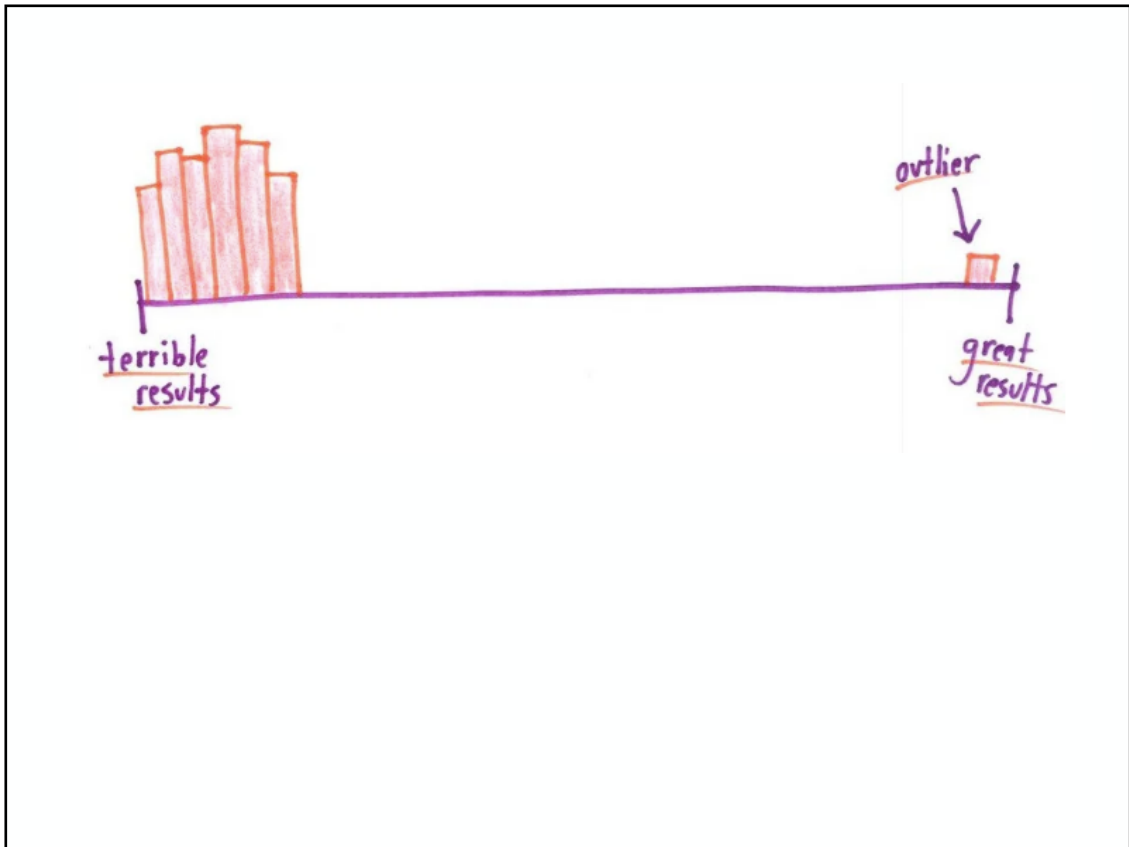
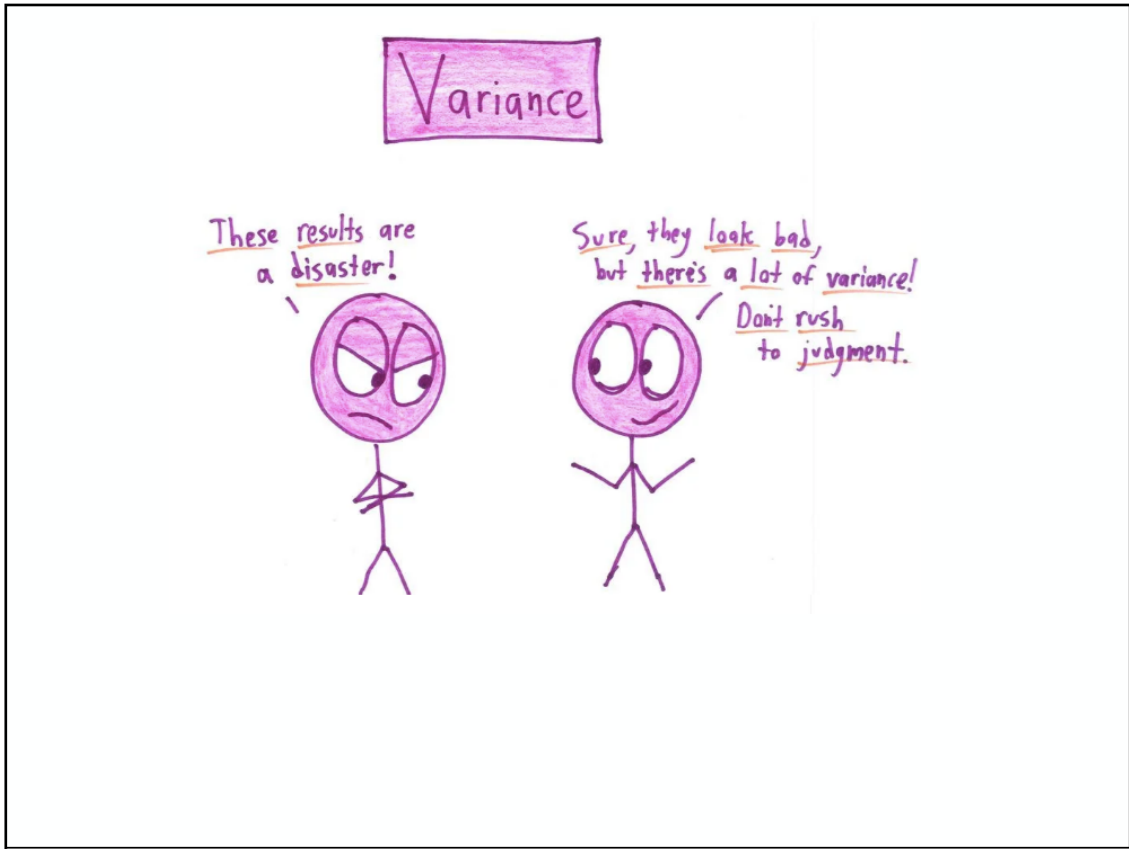


athletic performance

professional athletes we  
paid to guzzle the stuff  
...

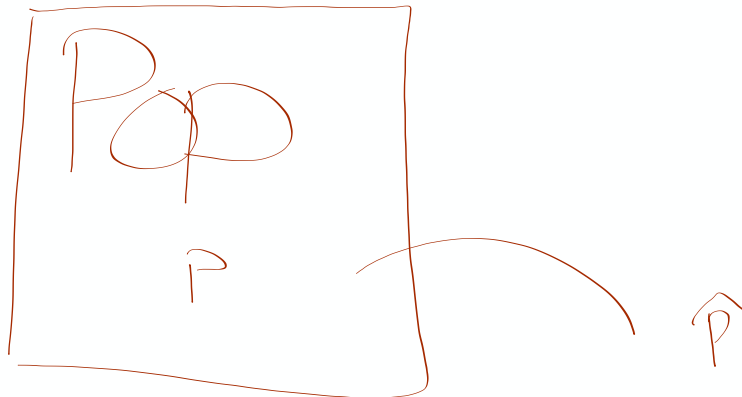


amount of  
drink consumed



*By the end of this section, you should be able to:*

- ✓ CALCULATE the mean and standard deviation of the sampling distribution of a sample proportion  $\hat{p}$  and INTERPRET the standard deviation.
- ✓ DETERMINE if the sampling distribution of  $\hat{p}$  is approximately Normal.
- ✓ If appropriate, USE a Normal distribution to CALCULATE probabilities involving  $\hat{p}$ .



## Lesson 7.2: What's the proportion of orange Reese's Pieces?



**If we take a sample of Reese's Pieces, what proportion of the candies will be orange?**

Suppose a large bag of Reese's Pieces has 1000 pieces. The manufacturer says that exactly 40% of the candies are orange. If we select a sample of 50 pieces, how many will be orange?

**Let  $X$  = the number of orange candies in the sample.**

1. What type of probability distribution does  $X$  have? Justify.

B I N S

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**B** Binary      **I** 10% condition      **N**  $n=50$       **S**  $p=.40$   
 ↳ success - orange       $50 < 100$        $Z \frac{1}{10} \cdot 1000$   
 ↳ failure - not orange

2. Draw a sample of 50 Reese's Pieces using the **Rossman/Chance Applet Collection (Google it)** applet. How many pieces were orange? Repeat [this](#) 5 times. Write the values below.

$X =$                        $X =$                        $X =$                        $X =$                        $X =$

Google  
 Rossman/Chance Applet

- [Least Squares Regression](#) (js)

### Sampling Distribution Simulations

- [Reeses Pieces](#) (js)
- [Sampling Words](#) (js)
- [Sampling from a Finite Population/Model/Bootstrap](#) (js)
- [Simulating Confidence Intervals for Population Parameter](#) (js)
- [Improved Batting Averages \(Power\)](#) (js)
- [ANOVA simulation](#) (js)
- **NEW:** [Guess the p-value](#) (js)

Classical (A)

## Reeses Pieces

Probability of orange

Number of candies

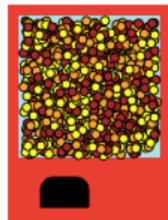
Number of samples

Animate

Total = 0

Number of orange  
 Proportion of orange

As extreme as



Summary Stats

3. Write the values on sticker dots and add it to the dotplot on the board. Sketch the dotplot below.

4. What does each dot represent?

# orange pieces in a sample of 50

5. What is the mean and the standard deviation for this binomial distribution of X? Show work.

$$\mu_x = np \underset{\text{(binomial)}}{=} (50)(.4) =$$

$$\sigma_x = \sqrt{np(1-p)} \underset{\text{(binomial)}}{=}$$

4. What does each dot represent?

The number of orange pieces from a sample of 50

5. What is the mean and the standard deviation for this binomial distribution of X? Show work.

$$\mu_x = np = 50 \times 0.4 = \underline{20} \quad \sigma_x = \sqrt{np(1-p)} = \sqrt{50 \cdot 0.4 \cdot (.6)} = \underline{3.46}$$

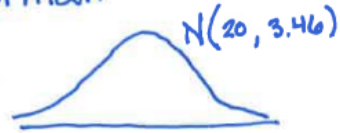
6. What is the approximate shape of the sampling distribution for  $X$ ? Explain and sketch it below.

Large Counts  
Condition

$$- np \geq 10 \quad 50(.4) = 20 \checkmark$$

$$n(1-p) \geq 10 \quad 50(.6) = 30 \checkmark$$

So distribution is approx.  
Normal.

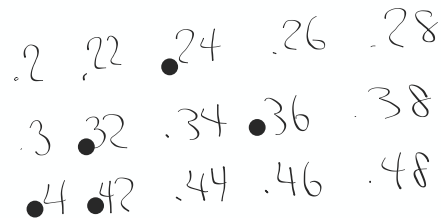


**Instead of finding the number of candies that are orange, we will now find the proportion of candies that are orange.**

7. Use your samples from #2 and turn each number of orange candies into the **proportion of orange candies** in the sample ( $\hat{p}$ ). Write the proportions below and add them to the second dotplot on the board.

$$\hat{p} = \frac{29}{50} = .58 \quad \hat{p} = \quad \hat{p} = \quad \hat{p} =$$

$$X = 29$$



8. Sketch the dotplot using your  $\hat{p}$  (sample proportion values) below.

9. What does each dot represent?

The proportion of orange pieces from a sample of 50

10. Find the new mean and standard deviation. Show work.

divide by 50

$$\mu_{\hat{p}} = \frac{20}{50} = 0.40$$

$$\Rightarrow \sigma_{\hat{p}} = \frac{\sqrt{np(1-p)}}{50} = \frac{\sqrt{50(.4)(.6)}}{50} = .069$$

$$\mu_{\hat{p}} = \frac{np}{n} = p$$

$$\sigma_{\hat{p}} =$$

$$\sqrt{25x}$$
$$= 5\sqrt{x}$$

$$7\sqrt{x}$$
$$= \sqrt{49x}$$

$$\sqrt{\frac{x}{n}}$$
$$= \sqrt{\frac{x}{4n}}$$
$$= \frac{1}{2}\sqrt{\frac{x}{n}}$$

VARIABILITY

$$\frac{1000}{n} \rightarrow \frac{9000}{n}$$

$$\sqrt{\frac{P(1-P)}{n}}$$

$$= \sqrt{\frac{P(1-P)}{9n}}$$

$$= \frac{1}{3} \sqrt{\frac{P(1-P)}{n}}$$

10. Find the new mean and standard deviation. Show work.

divide by 50

$$\mu_{\hat{p}} = \frac{20}{50} = 0.40$$

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$$\mu_{\hat{p}} = \frac{np}{n} = p$$

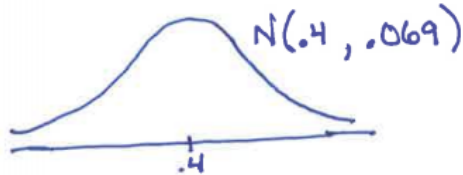
$$\sigma_{\hat{p}} = \frac{\sqrt{np(1-p)}}{n} = \sqrt{\frac{np(1-p)}{n^2}} = \sqrt{\frac{p(1-p)}{n}}$$

↓

$$\frac{1}{n} \sqrt{np(1-p)}$$

11. What is the approximate shape of the sampling distribution for  $\hat{p}$ ? Explain and sketch it below.

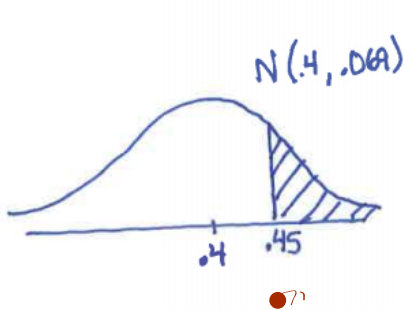
Normal  
because of  
Large Counts



$$50(.4) = 20 \checkmark$$

$$50(.6) = 30 \checkmark$$

12. We know that bags of Reese's Pieces contain exactly 40% that are orange. If we select a random sample of 50 candies, what is the probability that the sample proportion will be <sup>45</sup>50% or greater?



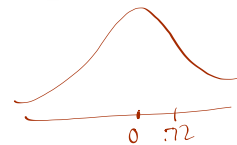
$$Z = \frac{\hat{p} - p}{\sigma}$$

$$= \frac{.45 - .40}{.069}$$

$$= .72$$

TABLE A

$$= .2358$$



$$P(Z > .72) = \text{normal cdf} \left[ \begin{matrix} L & U & \mu & \sigma \\ .72, & 1000, & 0, & 1 \end{matrix} \right]$$



## The Sampling Distribution of $\hat{p}$ (sample proportions)

Important ideas:

## The Sampling Distribution of $\hat{p}$ (sample proportions)

Important ideas:

**MEAN**

$$\mu_{\hat{p}} = P$$

**Std. Dev.**

$$\sigma_{\hat{p}} = \sqrt{\frac{P(1-P)}{n}}$$

if 10% condition met  
 $n < .10N$

**Normal  
Approx.**

if large counts

$$np \geq 10$$

$$n(1-p) \geq 10$$

for Probability  
if sampling distrib.  
of  $\hat{p}$  is approx.  
normal.

$$Z = \frac{\hat{p} - P}{\sqrt{\frac{P(1-P)}{n}}}$$

on-line  
videos

Suppose that 75% of young adult Internet users (ages 18 to 29) watch online videos. A polling organization contacts an SRS of 1000 young adult Internet users and calculates the proportion  $\hat{p}$  in this sample who watch online videos.

1. Identify the mean of the *sampling distribution* of  $\hat{p}$ .
2. Calculate and interpret the standard deviation of the sampling distribution of  $\hat{p}$ . Check that the 10% condition is met.

1. Identify the mean of the *sampling distribution* of  $\hat{p}$ .  $\mu_{\hat{p}} = p = 0.75$

2. Calculate and interpret the standard deviation of the sampling distribution of  $\hat{p}$ . Check that the 10% condition is met.

1000 < 10% of all internet users, (13-29)

$$\begin{aligned}\sigma_{\hat{p}} &= \\ &= \sqrt{\frac{.75(.25)}{1000}} \\ &= .014\end{aligned}$$

The proportion of young adults who watch online videos in a sample of 1000 typically varies by 0.014 from the true proportion of 0.75

3. Is the sampling distribution of  $\hat{p}$  approximately Normal? Check that the Large Counts condition is met.

4. Find the probability that the random sample of 1000 young adults will give a result within 2 percentage points of the true value.

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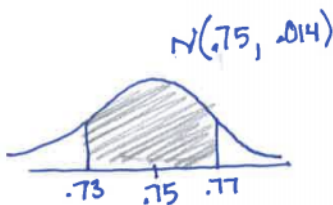
$$\begin{aligned} (.75)(1000) &= 750 \geq 10 \\ (.25)(1000) &= 250 \geq 10 \end{aligned} \quad \text{so, yes, it is approx. normal.}$$

4. Find the probability that the random sample of 1000 young adults will give a result within 2 percentage points of the true value.

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4. Find the probability that the random sample of 1000 young adults will give a result within 2 percentage points of the true value.



$$Z = \frac{.73 - .75}{.014} = -1.43 \quad Z = \frac{.77 - .75}{.014} = 1.43$$

$$P(-1.43 < Z < 1.43)$$

$$= \text{normalcdf} \left[ \underset{\text{Low}}{-1.43}, \underset{\text{Upper}}{1.43}, \underset{\mu}{0}, \underset{\sigma}{1} \right] = 0.8472$$

5. If the sample size were 9000 rather than 1000, how would this change the sampling distribution of  $\hat{p}$ ?

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- The shape would remain approx. normal.
- The center would stay the same (.75)
- the variability would decrease by  $\frac{1}{3}$

LCQ

**7.2**.....35, 37, 41, 43, 47–50

and study pp. 458–465