Solutions to Logic Practice

IB Style Solutions – Logic, Sets and Probability

Logic

Question 1 - Answers/Solutions

a. (i)   It is snowing if and only if I do not wear a hat.   [1 mark]

(ii)   It is not snowing, and the boys will walk to school.   [1 mark]

b. *He does not see or he does not run.*   [1 mark]

(ii)   Converse: If I visit the Louvre, then I go to Paris   [3 marks]

Inverse: If I do not go to Paris, then I do not visit the Louvre.

Contrapositive: If I do not visit the Louvre, then I do not go to Paris.

c. $\neg q \Rightarrow p$

Question 2 - Answers/Solutions

a. (i)   Converse: If $x^2 = 25$, then $x = 5$.   [1 mark]

(ii)   No   [1 mark]

(iii)   Contrapositive: If $x^2 \neq 25$, then $x \neq 5$.   [1 mark]

(iv)   Yes   [1 mark]

b. (i)   ~~True~~   [1 mark]

(ii)   False   [1 mark]

Question 3 - Answers/Solutions

a. (i) $T \underline{\vee} T \equiv F$ [1 mark]

(ii) $F \Rightarrow T \equiv T$ [1 mark]

(iii) $F \Leftrightarrow F \equiv T$ [1 mark]

b. continued on next page . . .

b.

p	q	$\neg p$	$\neg p \vee q$	$(\neg p \vee q) \Rightarrow p$
T	T	F	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	T	F

[3 marks, 1 mark per column]

Question 4 - Answers/Solutions

a. (i)   Conclusion: The team did not win.   [2 marks]

  (ii)   Conclusion: Carlos studies for his exam.   [2 marks]

b. (i) $\neg p \wedge \neg q$ [1 mark]

  (ii) $p \vee q$ (This is the negation of part (b) (i) above).   [1 mark]

Question 5 - Answers/Solutions

a. If he plays the game and he hates to lose, then we cannot party after the game.   [2 marks]

b.

p	q	r	$\neg r$	$p \wedge q$	$(p \wedge q) \Rightarrow \neg r$
T	T	T	F	T	F
T	T	F	T	T	T
T	F	T	F	F	T
T	F	F	T	F	T
F	T	T	F	F	T
F	T	F	T	F	T
F	F	T	F	F	T
F	F	F	T	F	T

[3 marks, 1 mark per column]

Question 6 - Answers/Solutions

a. If it is not Tuesday, then Marie does not play the violin.   *[1 mark]*

b. (i)

p	q	$p \Rightarrow q$	$\neg p$	$\neg p \vee q$	$(p \Rightarrow q) \Rightarrow (\neg p \vee q)$
T	T	**T**	F	**T**	T
T	F	**F**	F	**F**	T
F	T	**T**	T	**T**	T
F	F	**T**	T	**T**	T

[3 marks, 1 mark per column]

(ii) Yes, the statement is a tautology because it is true for all possible cases of p and q.   *[2 marks]*

Question 7 - Answers/Solutions

a.

p	q	$\neg p$	$\neg q$	$p \wedge \neg q$	$\neg q \Rightarrow \neg p$
T	T	**F**	F	**F**	T
T	F	**F**	T	**T**	F
F	T	**T**	F	**F**	T
F	F	**T**	T	**F**	T

[4 marks, 1 mark per column]

b. The two statements are not logically equivalent because corresponding (i.e. each row) truth values are not identical. In fact one is the negation of the other.   *[2 marks]*

Question 8 - Answers/Solutions

a. (i) $\neg p \Leftrightarrow r$ *[2 marks]*

    (ii) $\neg q \Rightarrow (\neg p \wedge \neg r)$ *[3 marks]*

b. (i)

[2 marks, 1 mark per column]

p	q	r	$p \wedge q$	$(p \wedge q) \Rightarrow r$
T	T	T	**T**	**T**
T	T	F	**T**	**F**
T	F	T	**F**	**T**
T	F	F	**F**	**T**
F	T	T	**F**	**T**
F	T	F	**F**	**T**
F	F	T	**F**	**T**
F	F	F	**F**	**T**

    (ii)

[2 marks, 1 mark per column]

p	q	r	$\neg r$	$p \wedge q$	$(p \wedge q) \vee \neg r$
T	T	T	F	**T**	**T**
T	T	F	T	**T**	**T**
T	F	T	F	**F**	**F**
T	F	F	T	**F**	**T**
F	T	T	F	**F**	**F**
F	T	F	T	**F**	**T**
F	F	T	F	**F**	**F**
F	F	F	T	**F**	**T**

    (iii)   No, the statements are not equivalent as the corresponding truth values are not identical (see last column of parts (i) and (ii) shown above.   *[3 marks]*