Recommended Review Assignment for the Test on Sets, Venn Diagrams, and Probability

1) Probability p. 490 Review Set 14A
(Check Answers in book)

p. 492 Review Set 14B
(answers and solutions attached!)

Review Set 14C solutions attached
if you want more!

2) Sets & Venn Diagrams

p. 82 Review Set A (answers attached)

p. 83 Review Set B
(1, 3, 5)

3) Probability Distributions

FST TEXT p. 485
61, 62, 65 (answers attached)

Probability

1 ABCD, ABDC, ACBD, ACDB, ADBC, ADCB, BACD, BADC, BCAD, BCDA, BDAC, BDCA, CABD, CADB, CBAD, CBDA, CDAB, CDBA, DABC, DACB, DBAC, DBCA, DCAB, DCBA

a There are 24 possible orderings.

$\therefore$ P(A is next to C)

$= \frac{12}{24}$ {12 have A next to C}

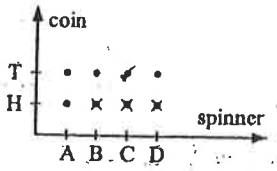
$= \frac{1}{2}$

b P(exactly one person between A and C)

$= \frac{8}{24}$ {8 have one person between A and C}

$= \frac{1}{3}$

2



a Consonants are B, C and D

$\therefore$ P(H and a consonant)

$= \frac{3}{8}$ {those with $\times$}

b P(T and C)

$= \frac{1}{8}$ {those with a $\checkmark$}

c P(T or vowel)

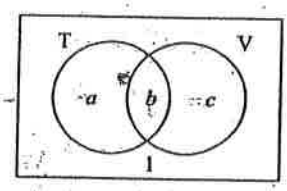
$=$ P(T or A)

$=$ P(T) + P(A) $-$ P(T and A)

$= \frac{4}{8} + \frac{2}{8} - \frac{1}{8}$

$= \frac{5}{8}$

3



$a + b + c = 24$

$a + b = 13$

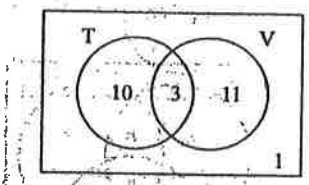
$b + c = 14$

Also $b = 13 - a$

$= 3$

$\therefore$ $13 + c = 24$ and $a + 14 = 24$

$\therefore$ $c = 11$ and $a = 10$



a P(T and V)

$= \frac{3}{25}$

b P(at least one)

$= 1 -$ P(neither)

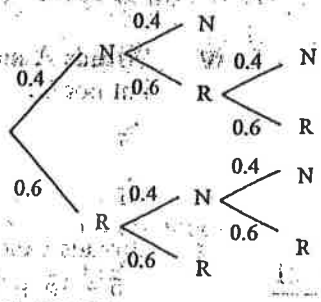
$= 1 - \frac{1}{25}$

$= \frac{24}{25}$

c $P(V \mid T') = \frac{11}{11+1}$

$= \frac{11}{12}$

4



P(Niklas wins)

$= (0.4)(0.4) + (0.4)(0.6)(0.4) + (0.6)(0.4)(0.4)$

$= 0.352$

5 P(M) $= \frac{3}{5}$, P(W) $= \frac{2}{3}$

a P(M and W)

$= \frac{3}{5} \times \frac{2}{3}$ {assuming independence}

$= \frac{2}{5}$

b P(at least one)

$=$ P(M or W)

$=$ P(M) + P(W) $-$ P(M and W)

$= \frac{3}{5} + \frac{2}{3} - \frac{2}{5}$

$= \frac{13}{15}$

c P(M′ and W)

$= (1 - \frac{3}{5}) \times \frac{2}{3}$

$= \frac{2}{5} \times \frac{2}{3}$

$= \frac{4}{15}$

6 **a** P(wins first 3 prizes)

$= \text{P(WWW)}$

$= \frac{4}{500} \times \frac{3}{499} \times \frac{2}{498}$

$\doteqdot 1.93 \times 10^{-7}$

b P(wins at least one of the 3 prizes)

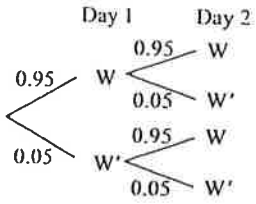
$= 1 - \text{P(wins none of them)}$

$= 1 - \text{P(W'W'W')}$

$= 1 - \frac{496}{500} \times \frac{495}{499} \times \frac{494}{498}$

$\doteqdot 0.0239$

7

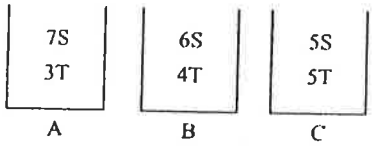


P(works on at least one day)

$= 0.95 \times 0.95 + 0.95 \times 0.05 + 0.05 \times 0.95$

$= 0.9975$

8



$\text{P(A)} = \frac{3}{6}, \quad \text{P(B)} = \frac{2}{6}, \quad \text{P(C)} = \frac{1}{6}$

a $\text{P(S)} = \text{P(A and S or B and S or C and S)}$

$= \frac{3}{6} \times \frac{7}{10} + \frac{2}{6} \times \frac{6}{10} + \frac{1}{6} \times \frac{5}{10}$

$= \frac{38}{60}$

$= \frac{19}{30}$

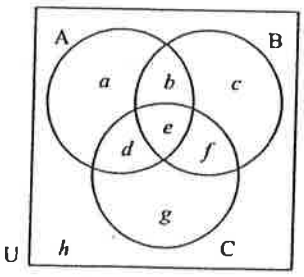
b $\text{P(B} \mid \text{S)} = \frac{\text{P(B} \cap \text{S)}}{\text{P(S)}}$

$= \frac{\frac{2}{6} \times \frac{6}{10}}{\frac{38}{60}}$

$= \frac{12}{38}$

$= \frac{6}{19}$

9



$a + b + e + d = 35$

$b + c + e + f = 31$

$d + e + f + g = 35$

$b + e = 7$

$e + f = 11$

$d + e = 8$

$e = 5$ and $h = 2$

As $e = 5$,

$b = 2, \quad f = 6, \quad d = 3$

$\therefore \quad a + 2 + 5 + 3 = 35$

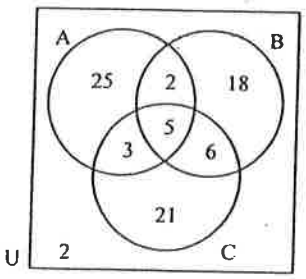
i.e., $a = 25$

$2 + c + 5 + 6 = 31$

i.e., $c = 18$

$3 + 5 + 6 + g = 35$

$\therefore \quad g = 21$



b Number in the club

$= 25 + 2 + 5 + 3 + 18 + 6 + 21 + 2$

$= 82$

c **i** P(reads A only)

$= \frac{25}{82}$

ii $\text{P(reads all)} = \frac{5}{82}$

iii P(reads B or C)

$= \frac{18 + 2 + 5 + 6 + 3 + 21}{82}$

$= \frac{55}{82}$

iv P(reads A and B, but not C)

$= \frac{2}{82}$

$= \frac{1}{41}$

v P(reads exactly one)

$= \frac{25 + 18 + 21}{82}$

$= \frac{64}{82}$

$= \frac{32}{41}$

vi P(reads at least one)

$= 1 - \frac{2}{82}$

$= \frac{80}{82}$

$= \frac{40}{41}$

REVIEW SET 14C

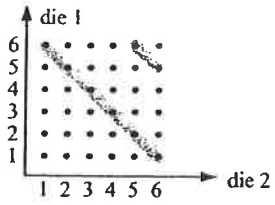
1 BBBB, BBBG, BBGB, BGBB, GBBB, BBGG, BGBG, BGGB, GGBB, GBBG, GBGB, BGGG, GBGG, GGBG, GGGB, GGGG

P(2B and 2G)

$= \frac{6}{16}$ ← 6 have 2B and 2G

$= \frac{3}{8}$

2



There are 36 possible outcomes.

a P(sum of 7 or 11)

$= \frac{8}{36}$ {those shaded}

$= \frac{2}{9}$

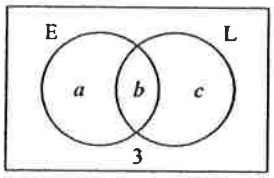
b P(sum of at least 8)

$= \frac{1+2+3+4+5}{36}$

$= \frac{15}{36}$

$= \frac{5}{12}$

3



$a+b+c = 37$

$a+b = 22$

$b+c = 25$

$\therefore \quad 22+c = 37$ and $a+25 = 37$

$\therefore \quad c = 15$ and $a = 12$

Hence, $b = 22-a = 10$

a P(E and L)

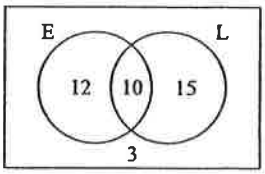
$= \frac{10}{40}$

$= \frac{1}{4}$

b P(at least one)

$= \frac{12+10+15}{40}$

$= \frac{37}{40}$



c $P(E \mid L) = \frac{10}{15+10} = \frac{10}{25} = \frac{2}{5}$

4 Multiples of 6 are: 6, 12, 18, 24, ..., 96 ← 6×16 i.e., 16 of them

Multiples of 8 are: 8, 16, 24, 32, ..., 88, 96 ← 8×12 i.e., 12 of them

Multiples of 6 and 8 are: 24, 48, 72, 96 i.e., 4 of them

The integers could be 1, 2, 3, 4, ..., 99 {between 0 and 100} i.e., 99 of them

$\therefore \quad P(M_6 \text{ or } M_8) = P(M_6) + P(M_8) - P(M_6 \text{ and } M_8)$

$= \frac{16}{99} + \frac{12}{99} - \frac{4}{99}$

$= \frac{24}{99}$

$= \frac{8}{33}$

5 **a** P(both blue)

$= P(BB)$

$= \frac{5}{12} \times \frac{4}{11}$

$= \frac{5}{33}$

b P(both same colour)

$= P(BB \text{ or } RR \text{ or } YY)$

$= \frac{5}{12} \times \frac{4}{11} + \frac{3}{12} \times \frac{2}{11} + \frac{4}{12} \times \frac{3}{11}$

$= \frac{19}{66}$

c P(at least one R)

$= 1 - P(\text{no reds})$

$= 1 - P(R'R')$

$= 1 - \frac{9}{12} \times \frac{8}{11}$

$= 1 - \frac{6}{11}$

$= \frac{5}{11}$

d P(exactly one Y)

$= P(YY' \text{ or } Y'Y)$

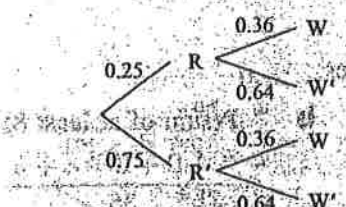
$= \frac{4}{12} \times \frac{8}{11} + \frac{8}{12} \times \frac{4}{11}$

$= \frac{16}{33}$

6 **a** Two events are independent if the occurrence of one does not influence the occurrence of the other. For A and B independent, $P(A) \times P(B) = P(A \text{ and } B)$.

b Two events, A and B, are disjoint if they have no common outcomes,
i.e., $P(A \text{ and } B) = 0$ and so $P(A \text{ or } B) = P(A) + P(B)$.

7



a $P(W \text{ and } R) = 0.25 \times 0.36$
$= 0.09$

b $P(W \text{ or } R) = P(W) + P(R) - P(W \text{ and } R)$
$= 0.36 + 0.25 - 0.09$
$= 0.52$

or $P(W \text{ or } R) = 1 - P(W'R')$
$= 1 - 0.64 \times 0.75$
$= 0.52$

8 $P(A) = 0.1$, $P(B) = 0.2$, $P(C) = 0.3$

$P(\text{group solves it}) = P(\text{at least one solves it})$
$= 1 - P(\text{no-one solves it})$
$= 1 - P(A' \text{ and } B' \text{ and } C')$
$= 1 - (0.9 \times 0.8 \times 0.7)$
$= 0.496$

9

a $P(E) = \frac{3}{7} \times \frac{7}{10} + \frac{4}{7} \times \frac{1}{4}$
$= \frac{3}{10} + \frac{1}{7}$
$= \frac{31}{70}$

b $P(C \mid E) = \frac{P(C \text{ and } E)}{P(E)}$
$= \frac{\frac{3}{7} \times \frac{7}{10}}{\frac{31}{70}}$
$= \frac{21}{31}$

p. 82 / 83 Sets / Venn Diag.

REVIEW SET 3A

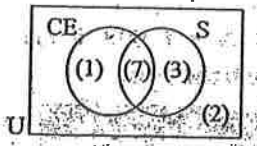
1 a $P = \{2, 3, 4, 5, 6, 7, 8\}$ b $n(P) = 7$

2 a i $\{x \mid x \in Z,\ x > 3\}$ ii $\{x \mid x \in Q,\ -5 \leqslant x \leqslant 5\}$
b i infinite ii infinite

3 a yes, $Q \subseteq P$ b yes, $\{\} \subseteq Q$

4 a $A = \{4, 8, 12\}$ b $B = \{2, 3, 5, 7, 11, 13\}$
c $B' = \{0, 1, 4, 6, 8, 9, 10, 12, 14, 15\}$
d $A \cup B = \{2, 3, 4, 5, 7, 8, 11, 12, 13\}$
e $A \cap B' = \{4, 8, 12\}$

5 a i $A = \{p, q, r, s\}$ ii $B = \{s, t, v, w\}$
iii $A \cap B = \{s\}$ iv $(A \cup B)' = \{a, e, i, o, u\}$
v $A \cup B' = \{a, e, i, o, p, q, r, s, u\}$
vi $A' \cap B = \{t, v, w\}$
b i $n(A \cup B) = 7$ ii $n(A \cap B') = 3$

6 a 

where CE represents company executive and S represents wearing suit
b i 3 ii 2

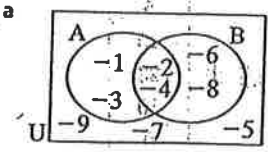
7 5 students 8 a 20 b 4 c 10

REVIEW SET 3B

1 a The set of all x such that x is a negative number between -4 and 0.
b $A = \{-3, -2, -1\}$ c A is finite

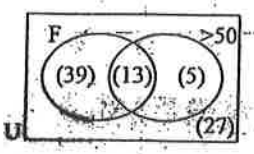
2 yes, $T \subseteq S$

3 a $P = \{2, 4, 6, 8, 10, 12, 14, 16, 18\}$; $Q = \{4, 8, 12, 16\}$
b $P \cap Q = \{4, 8, 12, 16\}$
c $P \cup Q = \{2, 4, 6, 8, 10, 12, 14, 16, 18\}$ d yes, $Q \subseteq P$
e $n(P \cup Q) = 9$; $n(P) + n(Q) - n(P \cap Q) = 9 + 4 - 4 = 9$

4 a 

b i $R = \{2, 4, 6, 8, 10, 12, 14, 16, 18\}$; $S = \{1, 4, 9, 16\}$
ii $n(S) = 4$
iii $S' = \{2, 3, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 17, 18, 19, 20\}$
iv no, since not all members of S are in R

5 a 8 b 9

6 a 

where F represents female and > 50 represents more than 50 years old
b i 27 ii 66

7 11 children 8 a 16 b 4 c 36

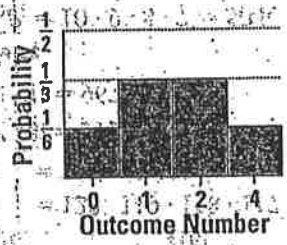
FST p. 485 Solutions

61. a. $P(0) = \frac{1}{6}$

$P(1) = \frac{2}{6} = \frac{1}{3}$

$P(2) = \frac{2}{6} = \frac{1}{3}$

$P(4) = \frac{1}{6}$

b. 

c. $\mu = 0 \cdot \frac{1}{6} + 1 \cdot \frac{1}{3} + 2 \cdot \frac{1}{3} + 4 \cdot \frac{1}{6}$

$= 0 + \frac{1}{3} + \frac{2}{3} + \frac{2}{3}$

$= \frac{5}{3}$

62. The sum of the $P(x)$ values is

$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{5} = \frac{43}{40} \neq 1.$

65. $1(3) + 2(10) + 3(10) + 4(12) + 5(20) + 6(30)$
$= 381$

Total destroyed $= 3 + 10 + 10 + 12 + 20 + 30$
$= 85$

Expected life span $= \frac{381}{85} \approx 4\frac{1}{2}$ years