Review 2

Name Solutions

1. Use long division:

$24x^4+31x^3+7x^2+4x+10 \div 3x+2$

$8x^3+5x^2-x+2$

$3x+2 \overline{)\,24x^4+31x^3+7x^2+4x+10}$

$-(24x^4+16x^3)$

$15x^3+7x^2$

$-(15x^3+10x^2)$

$-3x^2+4x$

$-(-3x^2-2x)$

$6x+10$

$-(6x+4)$

6

$\frac{24x^4}{3x}=8x^3$ $\quad \frac{15x^3}{3x}=5x^2$ $\quad \frac{-3x^2}{3x}=-x$ $\quad \frac{6x}{3x}=2$

ANSWER $8x^3+5x^2-x+2+\frac{6}{3x+2}$

2. Use synthetic division and the Remainder Theorem to find P(a).

$P(x)=-2x^4+14x^2+6\,;P(-3)$

$P(-3)=-30$ Same!

-3	-2	0	14	0	6
		6	-18	12	-36
	-2	6	-4	12	-30

$P(-3)=$ -30

Hint: Find P(-3) some other way to see if you got it right!

Put each polynomial in standard form, state its degree, leading term and whether it is a monomial, binomial, trinomial or polynomial (more than 4 terms).

	standard form	degree	leading term	classify # of terms
3. $10+3x^2-8x^3$	$-8x^3+3x^2+10$	3	$-8x^3$	trinomial
4. $5-8x-2x^3+2x^5+9x$	$2x^5-2x^3+x+5$	5	$2x^5$	quartic

Perform the indicated operations. Put your answers in standard form.

Hint: Avoid the common mistake!

5. $(6x^3-7x+8)-(3x^3-2)$

$6x^3-7x+8-3x^3+2$

$3x^3-7x+10$

6. $x(2x)(x+3)$

$2x^2(x+3)$

$2x^3+6x^2$

7. $(x^2+2)^2$

$(x^2+2)(x^2+2)$

$x^4+2x^2+2x^2+4$

x^4+4x^2+4

Find the zeros algebraically, showing work if it is needed. Include the multiplicity of any multiple zeros. For example, if the zeros are 4, 4, 5, 6, then write "4 (mult. of 2), 5, 6."

8. $f(x)=(x+1)^2(x+7)$ $\quad$ -1 (double root) -7 (single root)

9. $f(x)=4x^3-4x$ $\quad$ $0, -1, 1$ (all single roots)

$0=4x(x^2-1)$

$4x=0 \quad x^2-1=0 \quad x^2=1 \quad x=\pm 1$

Write a polynomial having the given zeros, first in factored form, then multiply it out and put it in standard form. Show your multiplication work.

10. zeros: −2, and 3 with a multiplicity of 2 (means double root)

f(x)= $(x+2)(x-3)^2$ (factored form) f(x)= $x^3-5x^2-3x+18$ (standard form)

$(x+2)(x-3)(x-3)$

$(x+2)(x^2-6x+9)$

$x^3-7x^2+9x+2x^2-12x+18$

11. **The volume of a box has a width of $(x-2)$ inches. The volume is expressed as a product of the length of its dimensions and is expressed by $V(x)=x^3+2x^2-5x-6$. Use synthetic division and the given width to completely factor $V(x)$. Put the dimensions in the blanks.**

$(x-2)(x^2+4x+3)$ can factor

$(x-2)(x+1)(x+3)$

2	1	2	−5	−6
		2	8	6
	1	4	3	0

x^2+4x+3

The dimensions of the box are $(x-2)$, $(x+1)$, and $(x+3$ inches.

Solve the following polynomial equations with factoring and the Zero Product Property. Show your work. Find all complex solutions. Find exact answers, using simplified radical form and/or the standard form for complex numbers when necessary.

diff of cubes!! (or you can find root and divide polynomials)

12. $x^4-4x^2-45=0$

$x=-3$
$x=3$

−3	1	0	−4	0	−45
		−3	9	−15	45
	1	−3	5	−15	0

3	1	−3	5	−15
		3	0	15
	1	0	5	0

$x^2+5=0$

$x^2=-5$

$x=\pm i\sqrt{5}$

$x=-3,3,-i\sqrt{5},i\sqrt{5}$

13. $8x^3-125=0$

$(2x-5)(4x^2+10x+25)=0$

$2x-5=0$

$2x=5$

$x=2.5$

$a=4$ $b=10$ $c=25$

$x=\dfrac{-(10)\pm\sqrt{(10)^2-4(4)(25)}}{2(4)}$

$=\dfrac{-10\pm\sqrt{-300}}{8}=\dfrac{-10\pm 10i\sqrt{3}}{8}$

$=\dfrac{-10}{8}\pm\dfrac{10i\sqrt{3}}{8}$

$x=2.5,\ \dfrac{-5}{4}\pm\dfrac{5i\sqrt{3}}{4}$

14. Perform the indicated operations. Put your answers in standard form.

a. $(2+5x)^2$

$(2+5x)(2+5x)$

$4+10x+10x+25x^2$

$25x^2+20x+4$

b. $(2y-3)(y^2+2y+1)$

	y^2	$+2y$	1
$2y$	$2y^3$	$4y^2$	$2y$
-3	$-3y^2$	$-6y$	-3

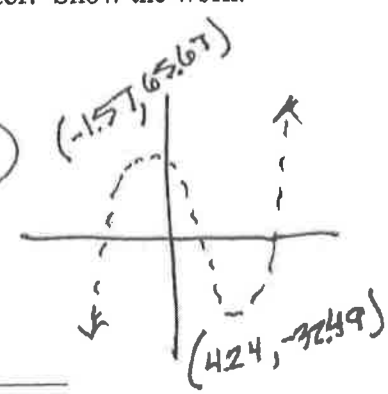
$2y^3+y^2-4y-3$

15. Completely factor $f(x)=x^3-4x^2-20x+48$ given that $(x+4)$ is a factor. Show the work.

-4	1	-4	-20	48
		-4	32	-48
	1	-8	12	0

$f(x)=(x+4)(x^2-8x+12)$

$f(x)=(x+4)(x-2)(x-6)$



Write f(x) in factored form. $f(x)=$ ________________

List the zeros of f(x). The zeros are: $-4,\ 2,\ 6$

Confirm your zeros by checking the graph on your calculator. Then use the calculator to find the relative minimum and relative maximum values of the function (remember - y-values!), rounding to the nearest hundredth.

relative minimum value: -32.49 relative maximum value: 65.67

local y-max

16. If $-7-8i$ is a root of a polynomial equation, what does the Imaginary Root Theorem tell you?

that $-7+8i$ is also a root

17. If $4+\sqrt{7}$ is a root of a polynomial equation, what does the Irrational Root Theorem tell you?

that $4-\sqrt{7}$ is also a root.

We talked about this idea but we never used this terminology

18. Given the polynomial equation $4x^6 + rx^5 + sx^4 + tx^3 + ux^2 + vx + 12 = 0$ ...

a) How many complex roots will it have? 6 (some may be repeated)

b) List the possible combinations of how many real and imaginary roots it could have.

could have 6 real, 2 real + 4 imaginary, or 0 real + 6 imaginary. (NOTE: imaginary roots come in pairs)

c) Use the Rational Root Theorem to list the set of all possible rational roots it could have.

constant is 12

-1 -12 -3 -4
1 12 3 4
-2 -6
2 6

Solve the following polynomial equations with factoring and the Zero Product Property. Show your work. Find all complex solutions. Find exact answers, using simplified radical form and/or the standard form for complex numbers when necessary.

19. a) $3x^3 - 5x^2 + 24x - 40 = 0$

from GDC $x = 1.666 \ldots 1\frac{2}{3} \ldots \frac{5}{3}$

5/3	3	-5	24	-40
		5	0	40
	3	0	24	0

$3x^2 + 24$

$3x^3 - 5x^2 + 24x - 40 = 0$

$(x - 5/3)(3x^2 + 24) = 0$

$3x^2 + 24 = 0$

$3x^2 = -24$

$x^2 = -8$

$x = \pm i\sqrt{8}$

ANSWERS
$x = 5/3$
$x = i\sqrt{8}$
$x = -i\sqrt{8}$

b) $64x^3 + 1 = 0$ ← Sum of Cubes

$(4x+1)(16x^2 - 4x + 1) = 0$ (or you can find 1 root and use division)

$a \cdot b = 0$

$4x + 1 = 0$

$x = -\frac{1}{4}$

$16x^2 - 4x + 1 = 0$

$a = 16$
$b = -4$
$c = 1$

$x = \dfrac{-(-4) \pm \sqrt{(-4)^2 - 4(16)(1)}}{2(16)}$

$x = \dfrac{4 \pm \sqrt{-48}}{32} = \dfrac{4 \pm 4i\sqrt{3}}{32}$

$x = -\frac{1}{4}$

$x = \frac{1}{8} \pm \frac{i\sqrt{3}}{8}$

Solve the following polynomial equations for all complex solutions. The following steps must be part of your work:

Use the Rational Root Theorem to list the set of potential rational zeros.
Use your graphing calculator to find at least one actual rational zero.
Use synthetic division to confirm at least one rational zero.
Use any other steps that are needed to find all complex zeros, and put them all in a box.

20. $x^3 + 4x^2 + 15x + 22 = 0$ Possible roots $-1, -22, 1, 22, -2, -11, 2, 11$

GDC $\rightarrow x = -2$

$(x+2)(x^2+2x+11) = 0$

$$\begin{array}{r|rrrr} -2 & 1 & 4 & 15 & 22 \\ & & -2 & -4 & -22 \\ \hline & 1 & 2 & 11 & 0 \end{array}$$

$\underbrace{1 \quad 2 \quad 11}_{x^2+2x+11}$

$x^2+2x+11 = 0$

$a = 1$
$b = 2$
$c = 11$

$x = \dfrac{-(2) \pm \sqrt{(2)^2 - 4(1)(11)}}{2(1)}$

$= \dfrac{-2 \pm \sqrt{-40}}{2}$

$= \dfrac{-2 \pm 2i\sqrt{10}}{2} = -1 \pm i\sqrt{10}$

$\boxed{\begin{array}{l} x = -2 \\ x = -1 \pm i\sqrt{10} \end{array}}$

21. $x^4 + 4x^3 - 17x^2 - 20x + 60 = 0$

Possible roots

$\pm 1 \quad \pm 60$
$\pm 2 \quad \pm 30$
$\pm 3 \quad \pm 20$
$\pm 4 \quad \pm 15$
$\pm 5 \quad \pm 12$
$\pm 6 \quad \pm 10$

GDC
$x = -6$
$x = 2$

$$\begin{array}{r|rrrrr} -6 & 1 & 4 & -17 & -20 & 60 \\ & & -6 & 12 & 30 & -60 \\ \hline & 1 & -2 & -5 & 10 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 2 & 1 & -2 & -5 & 10 \\ & & 2 & 0 & -10 \\ \hline & 1 & 0 & -5 & 0 \end{array}$$

$\underbrace{1 \quad 0 \quad -5}_{x^2-5=0}$

$x^2 = 5$

$\sqrt{\ } \quad \sqrt{\ }$

$\boxed{\begin{array}{l} x = -6 \\ x = 2 \\ x = \sqrt{5} \\ x = -\sqrt{5} \end{array}}$