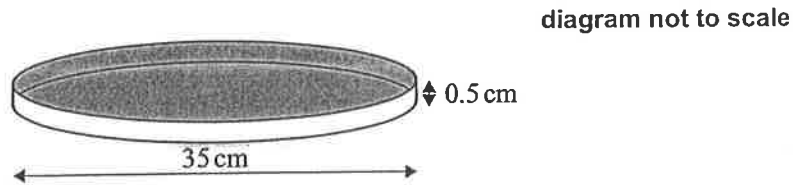4.   [Maximum mark: 15]

A pan, in which to cook a pizza, is in the shape of a cylinder. The pan has a diameter of 35 cm and a height of 0.5 cm.

diagram not to scale

$\updownarrow$ 0.5 cm

35 cm

(a)   Calculate the volume of this pan.   [3]

A chef had enough pizza dough to exactly fill the pan. The dough was in the shape of a sphere.

(b)   Find the radius of the sphere in cm, correct to one decimal place.   [4]

The pizza was cooked in a hot oven. Once taken out of the oven, the pizza was placed in a dining room.

The temperature, P, of the pizza, in degrees Celsius, °C, can be modelled by

$$P(t) = a(2.06)^{-t} + 19, \, t \geq 0$$

where a is a constant and t is the time, in minutes, since the pizza was taken out of the oven.

When the pizza was taken out of the oven its temperature was 230 °C.

(c)   Find the value of a.   [2]

(d)   Find the temperature that the pizza will be 5 minutes after it is taken out of the oven.   [2]

The pizza can be eaten once its temperature drops to 45 °C.

(e)   Calculate, to the nearest second, the time since the pizza was taken out of the oven until it can be eaten.   [3]

(f)   In the context of this model, state what the value of 19 represents.   [1]