be sure to have your Formula packet out on your desk today

The solutions for yesterday's HW have been posted for you to check tonight.

Today:

A

New Unit on

Sequences & Series

Here's a casual definition:

A sequence is a list of numbers (or other things) that changes according to some sort of pattern.

There are finite sequences that just stop after a certain number of terms.

Like this guy:

$-3, 1, 5, 9, 13, 17, 21$

And there are infinite sequences that keep on going forever and ever.

Like:

$0, 2, 4, 6, 8, 10, 12, \ldots$

These three dots means that it keeps going.

Pick up the
Notes packet

Here's a casual definition:

A sequence is a list of numbers (or other things) that changes according to some sort of pattern.

B

- **Sequence** (A second definition)

A list of numbers, called *terms*, written in a specific order. Each term has second number associated with it that relates to its position in the sequence.

it's all about the symbols

<u>Sequence</u> <u>(A second definition)</u>

A list of numbers, called *<u>terms</u>*, written in a specific order. Each term has second number associated with it that relates to its position in the sequence.

with numbers, we usually assign each spot with a special symbol:

$a_1, a_2, a_3, a_4, a_5, \ldots, a_n, \ldots$

Each of these is called a "term."

This is called the n^{th} term.

C

n^{th} term

$a_1, a_2, a_3, a_4, \ldots, a_n, \ldots$

The n^{th} term is given by a formula. We can use this formula to build the sequence.

a_n is known as the

explicit formula

Let's build the sequence whose n^{th} term is given by

$$a_n = 2^n - 3n$$

If we let $n=1$, we'll get the first term of the sequence:

$n=1 \rightarrow \quad a_1 = 2^1 - 3(1) = -1$

If we let $n=2$, we'll get the second term:

$$a_2 = 2^2 - 3(2) = -2$$

If we let $n=3$, we'll get the third term:

$$a_3 = 2^3 - 3(3) = -1$$

and so on...

$$a_4 = 2^4 - 3(4) = 4$$

$$a_5 = 2^5 - 3(5) = 17$$

$$a_6 = 2^6 - 3(6) = 46$$

$$\vdots$$

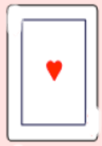
So, our sequence is

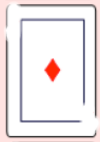
$$-1, -2, -1, 4, 17, 46, \ldots$$

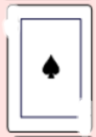
For this class, you will be responsible for two types of sequences for the most part.

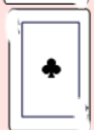
a) Arithmetic Sequences
b) Geometric Sequences

Arithmetic Sequences


2, 5, 8, 11, ... $d = 3$


2,6,10,14,18,... $d = 4$


15,11,7,3,... $d = -4$


19,16,13,10,... $d = -3$

Geometic Sequences

5, 10, 20, 40, ... $r = 2$

80, 20, 5, $\frac{5}{4}$, $\frac{5}{16}$, ... $r = \frac{1}{4}$

100, 1, .01, .0001, ...

$r = \frac{1}{100}$

$r = .01$

Aim today

Create and use an explicit formula for
ARITHMETIC SEQUENCES

Find the Sum of an ARITHMETIC SEQUENCE

Finding the Explicit Formula
(nth term)

for Arithmetic Sequences

5, 15, 25, 35, ...

(1) (2) (3) (4)

What is the common difference ?

Therefore, d = 10

Starting from 5, how many differences do we need to get to the 5th term?

6th term?

7th term?

50th term?

$5 + 10(n-1)$

example Find the explicit formula, a_n

(P)

6, 10, 14, 18, ...

$a_n = 6 + 4(n-1)$

t_n

U_n

Given an arithmetic sequence

$a_1, a_2, a_3, \ldots, a_n, \ldots$

that has a difference of d,

the n^{th} term is

$$\boxed{a_n = a_1 + (n-1)d}$$

or as shown in the Formula Packet

$$U_n = U_1 + (n-1)d$$

2.5	The n^{th} term of an arithmetic sequence	$u_n = u_1 + (n-1)d$
	The sum of n terms of an arithmetic sequence	$S_n = \frac{n}{2}(2u_1 + (n-1)d) = \frac{n}{2}(u_1 + u_n)$

-3

$$U_n = U_1 + (n-1)d$$
$$= 10 + (n-1)(-3)$$
$$= 10 - 3(n-1)$$

from Algebra $t(n) =$

$t_1 =$

5

Find the 85th term of the sequence
3, 8, 13, 18, ...

$$U_n = 3 + 5(n-1)$$
$$U_{85} = 3 + 5(85-1)$$
$$= 423$$

example from a different angle

The 32nd term of a Arithmetic sequence is 349.

The first term is 8. What is the common difference ?

it depends

arithmetic ?

geometric ?

The 32nd term of an **ARITHMETIC** sequence is 349. The first term is 8. What is the common difference ?

B.B.

Series

A series is the sum of a sequence.

Here's a sequence:

1, 2, 3, 4, 5

Here's the corresponding series:

1 + 2 + 3 + 4 + 5

A true story about

Carl Friedrich Gauss

Here was the day's problem:

Add the integers from 1 to 100.

They got out their slate boards and chalk and started hammering away!

$1+2=3$ $3+3=6$ $6+4=10$

$10+5=15$ $15+6=21$

$21+7=28$ $28+8=36$ $36+9=45$

$1+2+3+4+\ldots+50+51+\ldots+97+98+99+100$

There's a pattern here!

Check this out:

$1+2+3+4+\ldots+50+51+\ldots+97+98+99+100$

101

101

101

101

101

There are 50 pairs of 101...

That's $50(101)=5050$

• Pick Up Notes on SERIES

(W) Find the sum of the integers from 1 to 40

1 2 39 40

41

41

$(41)(20)$

$= 820$

$U_1 \ldots\ldots\ldots U_n$

Generalize

Find the sum of the terms in the sequence from $\mathbf{a_1}$ to $\mathbf{a_n}$ using the same method if there are $\boldsymbol{n}$ terms.

$$S_n = a_1 + a_n + \frac{n}{2} \qquad \left(\frac{a_n}{2}\right)(a_1 + a_n)$$

$$1 + 40 + \frac{40}{2} \qquad \left(\frac{40}{2}\right)(1 + 40)$$

$$20(41)$$

$$820$$

20, 21, 22 100

Arithmetic Series

To find the sum of the first n terms:

$$a_1 + a_2 + a_3 + \ldots + a_n$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

*

IB Formula Packet:

$$S_n = \frac{n}{2}(u_1 + u_n) = \frac{n}{2}\left[u_1 + u_1 + d(n-1)\right]$$

$u_n = u_1 + d(n-1)$

$$S_n = \frac{n}{2}\left[2u_1 + d(n-1)\right]$$

The cool thing about his formula is that it works on an ODD number of terms

3 + 13 + 23 + 33 + 43

X

remember

Find the 85th term of the sequence
3, 8, 13, 18, ...

Instead, find the sum of all ~~83~~ 85

$S_{85} = \frac{85}{2}\left[2(3)+5(85-1)\right]$

$= 18105$

$= \underline{\underline{17850}}$

in one step

in two steps

$U_{85} = 3+5(85-1)$

$= 423$

$S_{85} = \frac{85}{2}(3+423)$

$= \cancel{17850}\ 18105$

Y

a) Determine the number of terms in the sequence

24, $23\frac{1}{4}$, $22\frac{1}{2}$, ..., -36

$d = -.75$

b) Then find the sum.

c) Then find only the 35^{th} term.

a) Determine the number of terms in the sequence

$24, 23\frac{1}{4}, 22\frac{1}{2}, \ldots, \ldots -36$

Best friend

$U_n = U_1 + d(n-1)$

$-36 = 24 - 0.75(n-1)$
$-24 \quad -24$

$-60 = -.75(n-1)$

$\frac{-60}{-.75} = n-1$

$80 = n-1 \quad n = 81$

$S_{81} = \frac{81}{2}(24 + {}^{-}36)$
$= \boxed{-486}$

$U_{35} = 24 - .75(35-1)$
$= -1.5$

b) Then find the sum.

c) Then find only the 35^{th} term.

Assignment

Worksheet

Use good notation

The solutions for yesterday's HW have been posted for you to check tonight.