

• Let me know about HW problems.



• Warm Up

Solve $30(2.1)^n = 6000$

It turns out you don't need your book until tomorrow.



Hi Ian.

Ha Ha ... you're missing
all of brain breaks

This week

- Lily

$$30(2.1)^n = 6000$$

divide by 30

$$(2.1)^n = 200$$

↓

$$\log(2.1^n) = \log(200)$$

$$n \cdot \log(2.1) = \log(200)$$

$$n = \frac{\log(200)}{\log(2.1)}$$

$$30(2.1)^n = 6000$$

divide by 30

$$(2.1)^n = 200$$

↓

$$\log(2.1^n) = \log(200)$$

$$n \cdot \log(2.1) = \log(200)$$

Log Form

base sam

$$\log_{2.1}(200) = n$$

$$n = \log_{2.1}(200)$$

$$n = \frac{\log(200)}{\log(2.1)}$$

=

#6 Solution does not match

deposit \$50
on first BD.

50, 75, 100, ...

a) How much \$ on 16th birthday?

$$U_n = U_1 + d(n-1)$$

$$U_{16} = 50 + 25(16-1)$$

$$= \boxed{\$425}$$

↓

b) How much would he have deposited in total on 16th Birthday?

Translation

Find the Sum of the first 16 terms

$$S_n = \frac{n}{2} [2u_1 + d(n-1)] \quad S_{16} = \frac{16}{2} (50 + 40)$$

$$= \frac{16}{2} [2(50) + 25(16-1)]$$

$$= \boxed{\$3,800}$$

Today

Finding the Sum of Geometric Sequences
(and of course the n^{th} term form those
same sequences)

Example

2, 10, 50, 250,



A sequence is geometric if
each term can be obtained from
the previous term by **multiplying**
by the same number.

This number is the constant ratio, r

$2, 10, 50, 250, \dots$

u_1 u_2 u_3

common ratio = $r = \frac{u_2}{u_1}$

$= \frac{u_3}{u_2}$

$= \frac{u_4}{u_3}$

$r = \frac{u_{n+1}}{u_n}$

$r = \frac{u_n}{u_{n-1}}$

u_{n-1}, u_n, u_{n+1}
 u_{100}, u_{101}

What is the common ratio of

$135, 90, 60, 40, \dots$

If the terms are getting smaller, then the common ratio must be less than 1

$$r = \frac{40}{60}$$

$$\frac{60}{90}$$

$$\frac{90}{135} = \frac{2}{3}$$

W

Note

**You are expected to show work,
using good notation in this unit.**

**Just fiddling around with
a calculator won't work out well
for you.**

Is the following sequence geometric?



0.5 , 1 , 2 , 4 , 8 , 16 ,
 (1) (2) (3)

If so, what is common ratio? $r = 2$

How many applications of 2 do you need to get
from the first term **to the 3rd term?**

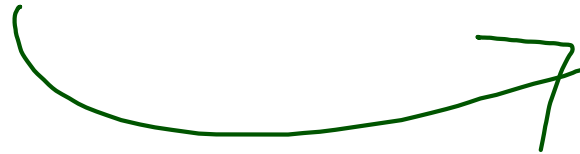
$$U_n = 0.5(2)^{n-1}$$

to the 4th term?

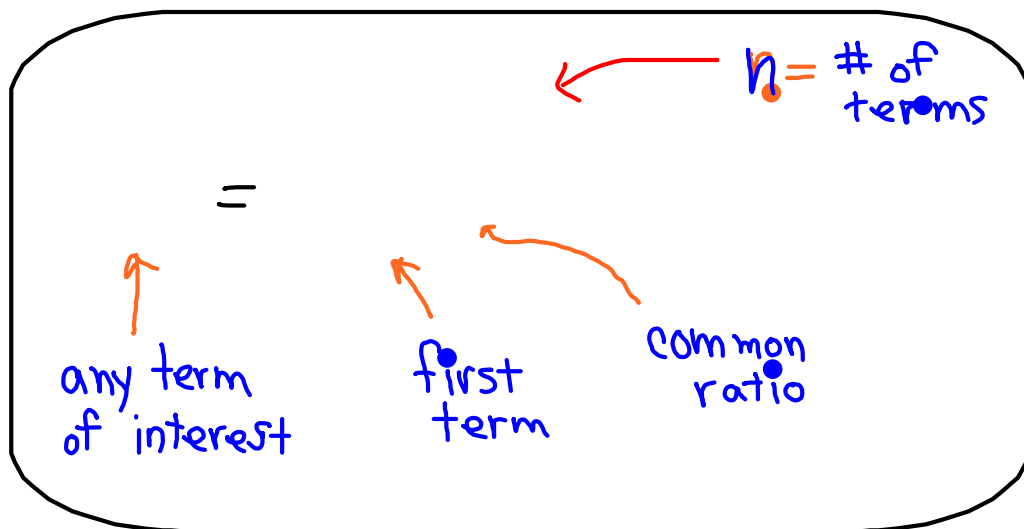
to the 85th term?

to the nth term?

if Geometric
Your best friend
will be



Explicit formula for Geometric Sequences



Explicit formula for Geometric Sequences



$$t_n = t_1(b)^{n-1}$$

$$u_n = u_1 r^{n-1}$$

$n = \#$ of terms
 $n-1$
 any term of interest
 first term
 common ratio

Let's try it!

What's the **23rd** term of this sequence?



$$\frac{1}{9}, -\frac{1}{3}, 1, -3, 9, \dots$$

Find the ratio:

$$r = \frac{-\frac{1}{3}}{\frac{1}{9}} \quad \text{or} \quad \frac{1}{-\frac{1}{3}} = -3 \quad \text{or} \quad \frac{-3}{1} = -3$$

$$u_n = \frac{1}{9}(-3)^{23-1} = 3,486,784,401$$

Z

The first term of a geometric sequence is 4 and the last term is 26,244. If there are 9 terms in the sequence, what is the common ratio?

Best Friend

$$u_n = u_1 \cdot r^{n-1}$$

\uparrow \uparrow \swarrow
 26244 4 9

$$4, -12, 36, 108$$

$$4, 12, 36, 108$$

$$4(r)^{9-1} = 26244$$

$$4r^8 = 26244$$

$$r^8 = \frac{26244}{4}$$

$$\sqrt[8]{\quad} \quad \sqrt[8]{\quad}$$

$$r = \sqrt[8]{\frac{26244}{4}} = \pm 3$$

The first term of a geometric sequence is 4 and the last term is 26,244. If there are 9 terms in the sequence, what is the common ratio?

Alternative wording for the same

↷

find r if $u_1 = 4$ and $u_9 = 26,244$

Best friend

$$U_n = U_1 (r)^{n-1}$$

Note

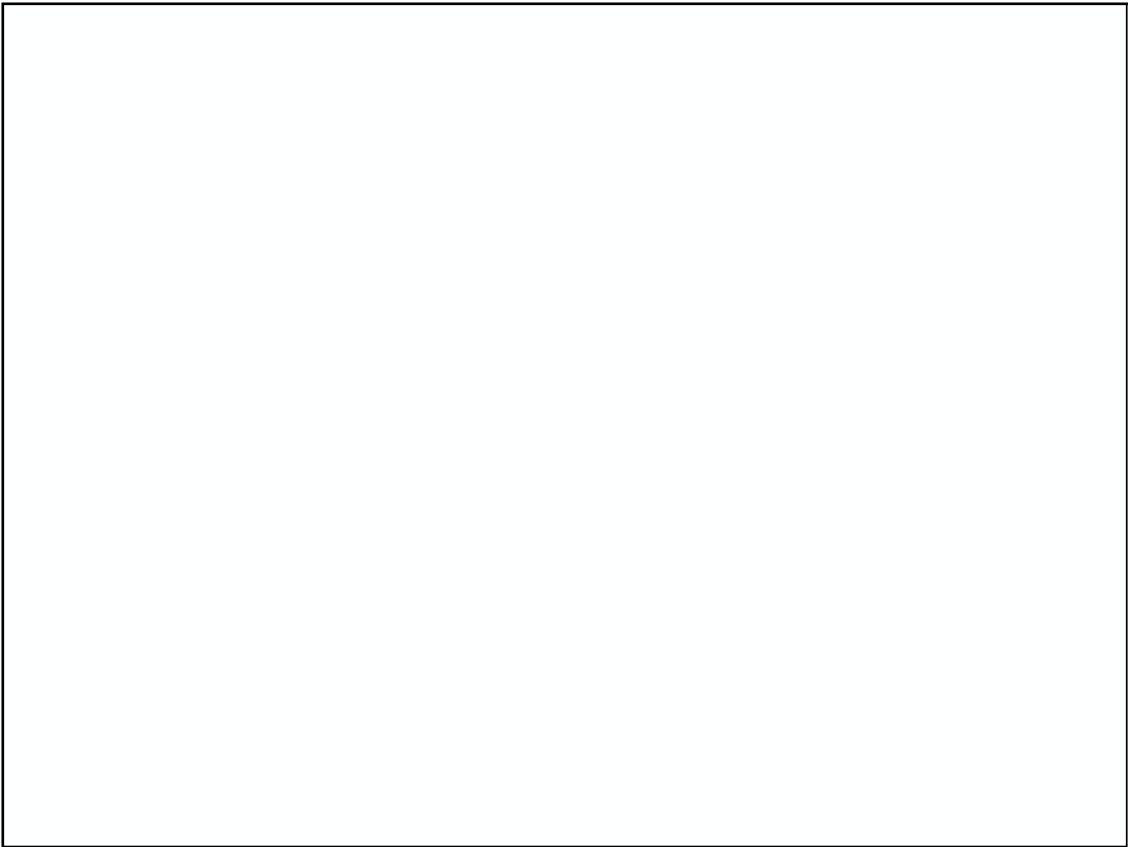
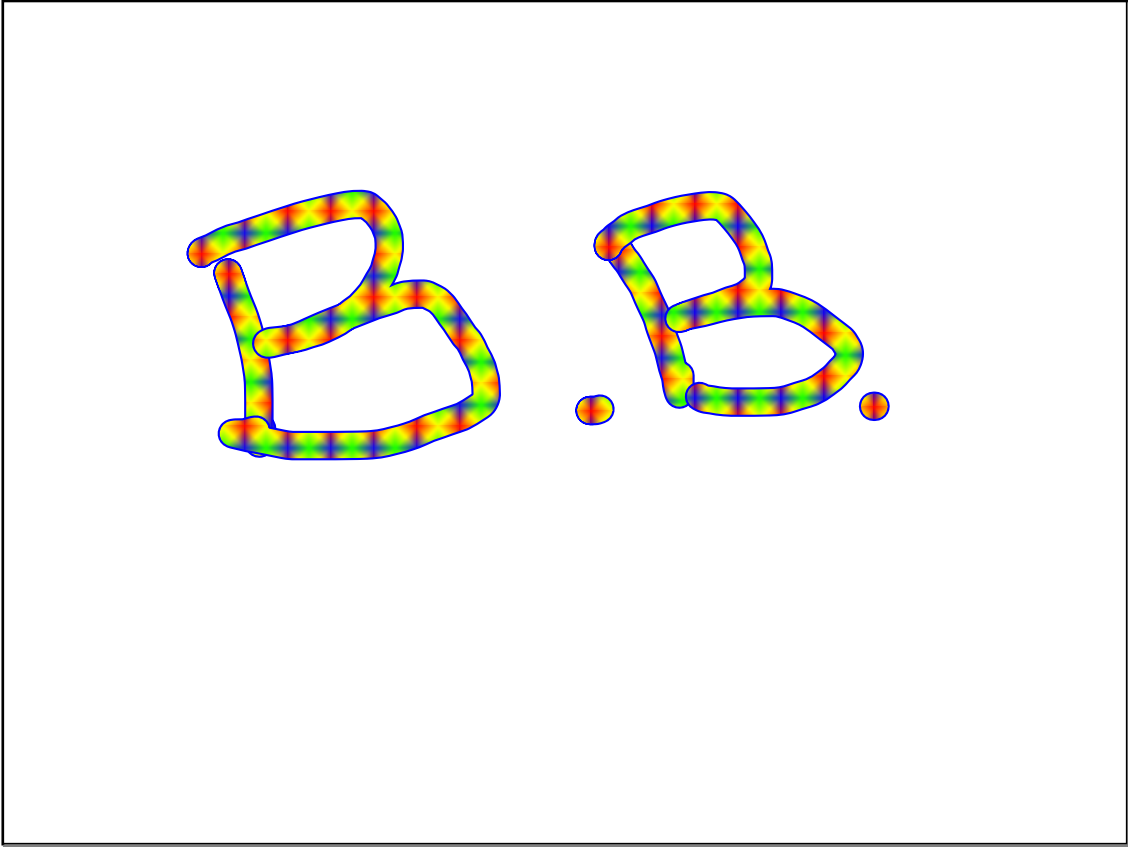
If something grows at 15%, then the common ratio would be $(1+.15)$ or 1.15

$$U_n = U_1 \cdot r^{n-1}$$

↖ ? ↗

$$y = ab^x$$

↑ 0 term



Geometric Series

the sum of a geometric sequence

Add the first 10 terms of

$$4, 2, 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8} \dots$$

$$S_{10} = 4 + 2 + 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} \dots$$

$$S_{10} = \frac{4(1 - \frac{1}{2}^{10})}{1 - \frac{1}{2}} =$$

Luckily, there's a formula!

To find the sum of the first n terms of a geometric sequence:

See your friendly
Formula
Packet

Luckily, there's a formula!

To find the sum of the first n terms of a geometric sequence:

$$S_n = \frac{u_1(1-r^n)}{1-r}$$

for finite sequences

24, 12, 6, 3, 1.5, ...

$$S_n = \frac{u_1(1-r^n)}{1-r} \quad \leftarrow \text{reverse}$$

$$r = \frac{-3}{7}$$

$$S_{10} = \frac{4 \left(1 - \left(\frac{-3}{7} \right)^{10} \right)}{1 - \left(\frac{-3}{7} \right)}$$

$$S_n = \frac{u_1(r^n - 1)}{r - 1} = \frac{4 \left[\left(\frac{-3}{7} \right)^{10} - 1 \right]}{\frac{-3}{7} - 1}$$

Formula Packet

The n^{th} term of an arithmetic sequence	$u_n = u_1 + (n-1)d$
The sum of n terms of an arithmetic sequence	$S_n = \frac{n}{2}(2u_1 + (n-1)d) = \frac{n}{2}(u_1 + u_n)$
The n^{th} term of a geometric sequence	$u_n = u_1 r^{n-1}$ ✓
The sum of n terms of a geometric sequence	$S_n = \frac{u_1(r^n - 1)}{r - 1} = \frac{u_1(1 - r^n)}{1 - r}, r \neq 1$

$$S_n = \frac{u_1(r^n - 1)}{r - 1}$$

$$S_n = \frac{u_1(1 - r^n)}{1 - r}$$

$$S_n = \frac{u_1(1 - r^n)}{1 - r}$$

$$S_n = \frac{u_1(r^n - 1)}{r - 1}$$

$$4 \left(\frac{1 - \left(\frac{-3}{4}\right)^{20}}{1 - \left(\frac{-3}{4}\right)} \right)$$

$$4 \left(\frac{\left(\frac{-3}{4}\right)^{20} - 1}{\left(\frac{-3}{4}\right) - 1} \right)$$

$$S_{10} = 4 + 2 + 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} \dots$$

$$S_{10} =$$

$$= \frac{4(1 - (\frac{1}{2})^{10})}{1 - \frac{1}{2}}$$

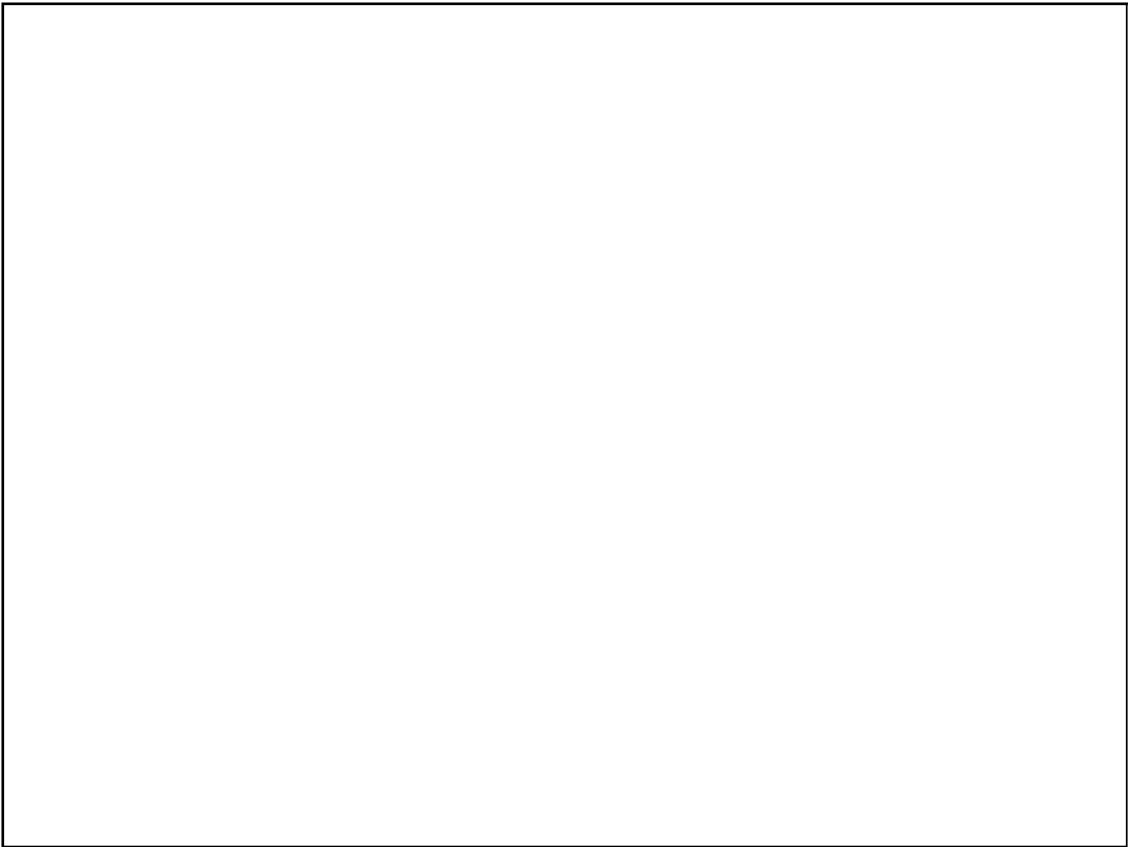
$$= 8(1 - (\frac{1}{2})^{10})$$

$$= \frac{1023}{128}$$

See your
test

Assignment #2

Worksheet



Why does $S_n = \frac{g_1(1-r^n)}{1-r}$ work?

$$S_n = g_1 + g_1 r + g_1 r^2 + \dots + g_1 r^{n-1}$$

Typical Geometric Series

↑
last (or Nth) term

multiply by r

$$S_n = g_1 + g_1 r + g_1 r^2 + \dots + g_1 r^{n-1}$$

Typical Geometric Series

↑
last (or Nth) term

multiply by r

$$rS_n = g_1 r + g_1 r^2 + g_1 r^3 + \dots + g_1 r^{n-1} + g_1 r^n$$

subtract preceding equation

$$S_n - rS_n = g_1 - g_1 r^n$$

$$S_n - rS_n = g_1 - g_1 r^n$$

$$(1-r)S_n = g_1$$

$$S_n = \frac{g_1(1-r^n)}{1-r} \quad \text{as long as } r \neq 1$$

Assignment 2 (Sequences/Series)

a worksheet