- Let me know about HW problems.

- Warm Up

Solve $30(2.1)^n = 6000$

It turns out you <u>don't</u> need your book until tomorrow.

Hi Ian.

Ha Ha .. you're missing all of brain breaks this week

~ Lily

$$30(2.1)^n = 6000$$

divide by 30

$$(2.1)^n = 200$$

$$\log(2.1^n) = \log(200)$$

$$n \cdot \log(2.1) = \log(200)$$

$$n = \frac{\log(200)}{\log(2.1)}$$

$$30(2.1)^n = 6000$$

divide by 30

$$\boxed{(2.1)^n = 200}$$

base sam

Log Form

$$\log_{2.1}(200) = n$$

$$\log(2.1^n) = \log(200)$$

$$n \cdot \log(2.1) = \log(200)$$

$$n = \log_{2.1}(200)$$

$$\boxed{n = \frac{\log(200)}{\log(2.1)}}$$

$$=$$

#6 Solution does not match

deposit \$50 on first BD.

50, 75, 100, ...
(1) (2) (3)

a) How much \$ on 16^{th} birthday?

$$U_n = U_1 + d(n-1)$$

$$U_{16} = 50 + 25(16-1)$$

$$= \boxed{\$425}$$

b) How much would he have deposited in total on 16^{th} Birthday?

Translation: Find the Sum of the first 16 terms

$$S_n = \frac{n}{2}\left[2u_1 + d(n-1)\right]$$

$S_{16} = \frac{16}{2}(50 + 425)$

$$= \frac{16}{2}\left[2(50) + 25(16-1)\right]$$

$$\boxed{= \$3,800}$$

Today

Finding the Sum of Geometric Sequences
(and of course the n^{th} term form those same sequences)

example

2, 10, 50, 250, ...

U

A sequence is geometric if each term can be obtained from the previous term by multiplying by the same number.

This number is the constant ratio, r

2, 10, 50, 250, ... u_{n-1}, u_n, u_{n+1}

u_{100} u_{101}

u_1 u_2 u_3

$$\text{common ratio} = r = \frac{u_2}{u_1} = \frac{u_3}{u_2} = \frac{u_4}{u_3}$$

$$r = \frac{u_{n+1}}{u_n}$$

$$r = \frac{u_n}{u_{n-1}}$$

W

What is the common ratio of

135, 90, 60, 40, ...

If the terms are getting smaller, then the common ratio must be less than 1

$$r = \frac{40}{60} \qquad \frac{60}{90} \qquad \frac{90}{135} = \frac{2}{3}$$

Note

You are expected to show work, using good notation in this unit.

Just fiddling around with a calculator won't work out well for you.

Is the following sequence geometric?

0.5 , 1 , 2 , 4 , 8 , 16 , ...

① ② ③

If so, what is common ratio? $r = 2$

How many applications of 2 do you need to get from the first term to the 3rd term?

to the 4th term?

to the 85th term?

to the nth term?

$U_n = 0.5(2)^{n-1}$

if Geometric ....

Your best friend

will be

Explicit formula for Geometric Sequences

Q

n = # of terms

=

any term of interest

first term

common ratio

Explicit formula for Geometric Sequences

Q

$t_n = t_1(b)^{n-1}$

n = # of terms

$$U_n = U_1 r^{n-1}$$

any term of interest

first term

common ratio

Y

Let's try it!

What's the 23rd term of this sequence?

$\frac{1}{9}, -\frac{1}{3}, 1, -3, 9, \ldots$

Find the ratio:

$r = \frac{-\frac{1}{3}}{\frac{1}{9}}$ or $\frac{1}{-\frac{1}{3}} =$ or $\frac{-3}{1} = -3$

$U_n = \frac{1}{9}(-3)^{23-1} = 3,486,784,401$

Z

The first term of a geometric sequence is 4 and the last term is 26,244. If there are 9 terms in the sequence, what is the common ratio?

Best Friend

$U_n = U_1 \cdot r^{n-1}$

26244 4 9

$4(r)^{9-1} = 26244$

$4r^8 = 26244$

$r^8 = \frac{26244}{4}$

$\sqrt[8]{}$ $\sqrt[8]{}$

$r = \sqrt[8]{\frac{26244}{4}} = \pm 3$

4, -12, 36, -108

4, 12, 36, 108

The first term of a geometric sequence is 4 and the last term is 26,244. If there are 9 terms in the sequence, what is the common ratio?

Alternative wording for the same

find r if $u_1 = 4$ and $u_9 = 26{,}244$

Best friend

$$U_n = u_1(r)^{n-1}$$

Note

If something grows at 15%, then the common ratio would be (1+.15) or 1.15

$$U_n = U_1 \cdot r^{n-1}$$

?

$$y = ab^x$$

0 term

B.B.

Geometric Series

the sum of a geometric sequence

Add the first 10 terms of

$4, 2, 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8} \ldots$

$S_{10} = 4 + 2 + 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} \ldots$

$S_{10} = \frac{4\left(1 - \frac{1}{2}^{10}\right)}{1 - \frac{1}{2}} =$

Luckily, there's a formula!

To find the sum of the first n terms of a geometric sequence:

See your friendly Formula Packet

Luckily, there's a formula!

To find the sum of the first n terms of a geometric sequence:

$$S_n = \frac{u_1(1-r^n)}{1-r}$$

for finite sequences

24, 12, 6, 3, 1.5 . .075 .

$$S_n = \frac{u_1(1-r^n)}{1-r}$$ ← reverse ← reverse

$r = \frac{-3}{7}$

$$S_{10} = \frac{4\left(1-\left(\frac{-3}{7}\right)^{10}\right)}{1-\left(\frac{-3}{7}\right)}$$

$$S_n = \frac{u_1(r^n - 1)}{r-1} = \frac{4\left[\left(\frac{-3}{7}\right)^{10} - 1\right]}{\frac{-3}{7} - 1}$$

Formula Packet

The n^{th} term of an arithmetic sequence	$u_n = u_1 + (n-1)d$
The sum of n terms of an arithmetic sequence	$S_n = \frac{n}{2}(2u_1 + (n-1)d) = \frac{n}{2}(u_1 + u_n)$
The n^{th} term of a geometric sequence	$u_n = u_1 r^{n-1}$ ✓
The sum of n terms of a geometric sequence	$S_n = \frac{u_1(r^n - 1)}{r-1} = \frac{u_1(1-r^n)}{1-r}$, $r \neq 1$

$$S_n = \frac{u_1(r^n - 1)}{r - 1}$$

$$S_n = \frac{u_1(1 - r^n)}{1 - r}$$

$$S_n \frac{u_1(1 - r^n)}{1 - r}$$

$$S_n = \frac{u_1(r^n - 1)}{r - 1}$$

$$4\left(\frac{1 - \left(-\frac{3}{4}\right)^{20}}{1 - \left(-\frac{3}{4}\right)}\right)$$

$$4\left(\frac{\left(\frac{-3}{4}\right)^{20} - 1}{\left(\frac{-3}{4}\right) - 1}\right)$$

$S_{10} = 4 + 2 + 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} \cdots$

$S_{10} =$

$$= \frac{4\left(1-\left(\frac{1}{2}\right)^{10}\right)}{1-\frac{1}{2}}$$

$$= 8\left(1-\left(\frac{1}{2}\right)^{10}\right)$$

$$= \frac{1023}{128}$$

See your test

Assignment #2

Worksheet

Why does $S_n = \frac{g_1(1-r^n)}{1-r}$ work?

$$S_n = g_1 + g_1 r + g_1 r^2 + \ldots\ldots + g_1 r^{n-1}$$

Typical Geometric Series

last (or N^{th}) term

multiply by r

$$S_n = g_1 + g_1 r + g_1 r^2 + \ldots\ldots + g_1 r^{n-1}$$

Typical Geometric Series

last (or N^{th}) term

multiply by r

$$rS_n = g_1 r + g_1 r^2 + g_1 r^3 + \ldots\ldots g_1 r^{n-1} + g_1 r^n$$

subtract preceeding equation

$$S_n - rS_n = g_1 - g_1 r^n$$

$$S_n - rS_n = g_1 - g_1 r^n$$

$$(1-r)S_n = g_1$$

$$S_n = \frac{g_1(1-r^n)}{1-r}$$ as long as $r \neq 1$

Assignment 2 (Sequences/Series)

a worksheet